Modular Construction of Models

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Outline

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2. The Common Algebraic Specification Language (CASL)
3. DOLCE
4. Model Finders
5. Models along architectural specifications

Motivation

- Foundational ontologies provide the language and semantics for domain ontologies
- they are specified in many cases in **FOL**: like DOLCE and SUMO
- important question: Do they have a model? Are they consistent?
- Model-finders often fail to find a model for them directly
- several inconsistencies have already been found in SUMO [Ian Horrocks, Andrei Voronkov (2006)]
- SUMO-challenge on `http://www.tptp.org` has a first winner of $100
- we can construct a global model from smaller ones using CASL architectural specifications

The Common Algebraic Specification Language (CASL)

CASL

- CASL is a first order language designed by CoFI and approved by IFIP WG 1.3

Example (Basic spec)

spec TEMPORARY_STRICT_PARTIAL_ORDER =
  esorts $s < EDorPDorQ; T$
  pred $Rel : s \times s \times T$
  $\forall\, x, y, z : s; t : T$
  $\bullet\ Rel(x, y, t) \Rightarrow \neg\, Rel(y, x, t)$
  $\bullet\ Rel(x, y, t) \wedge Rel(y, z, t) \Rightarrow Rel(x, z, t)$
end

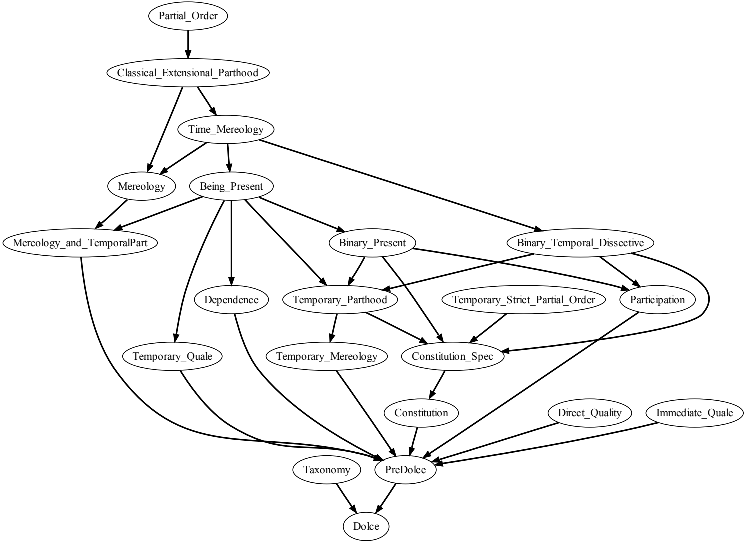
- SP ::= BasicSP | SP then SP | SP and SP | SP with σ | SP hide σ
- tool support is available via HETS (the Heterogeneous Tool Set)

Descriptive Ontology for Linguistic and Cognitive Engineering (DOLCE)

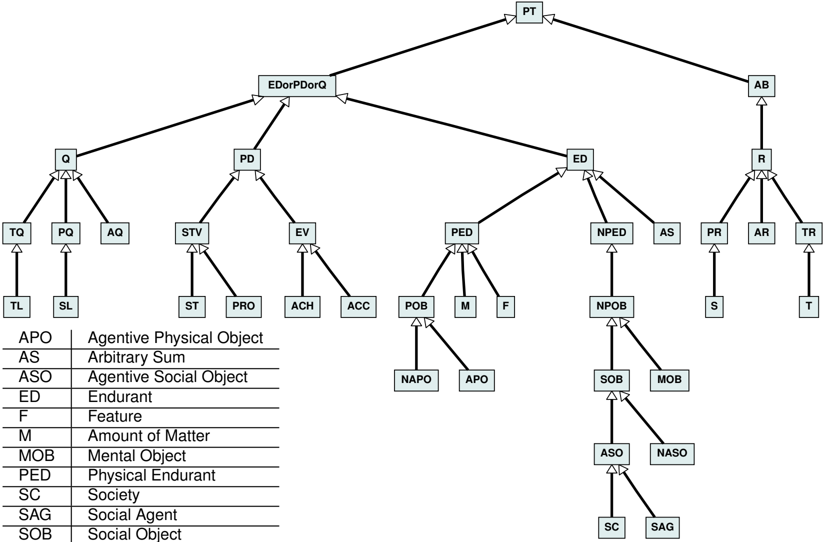
Dolci?

- DOLCE: Descriptive Ontology for Linguistic and Cognitive Engineering
- developed at the LOA in Trento
- contains several hundreds of axioms
- initially formalized in KIF (some variant of first-order logic)
- modularized formalization in CASL is also available and the starting point of our work
- complexity of DOLCE stems from the fact that it combines several (non-trivial) formalised ontological theories into one theory
  - theories of essence and identity
  - parts and wholes (mereology)
  - dependence
  - composition and constitution
  - properties and qualities

DOLCE's Modules



DOLCE's Taxonomy



APO	Agentive Physical Object
AS	Arbitrary Sum
ASO	Agentive Social Object
ED	Endurant
F	Feature
M	Amount of Matter
MOB	Mental Object
PED	Physical Endurant
SC	Society
SAG	Social Agent
SOB	Social Object
POB	Physical Object

Model Finders

Model finders

- We have made experiments with several Model finders on DOLCE
  - Darwin
  - SPASS
  - Isabelle-refute

CEP, a part of DOLCE

Example (Classical extensional parthood ⟨*CEP*⟩)

sort s; **pred** $At : s$; **pred** $AtP : s \times s$; **pred** $Ov : s \times s$
pred $P : s \times s$; **pred** $PP : s \times s$; **pred** $Sum : s \times s \times s$

$\forall\, x : s \bullet P(x, x)$ %(reflexivity)%

$\forall\, x, y : s \bullet P(x, y) \wedge P(y, x) \Rightarrow x = y$ %(antisymmetry)%

$\forall\, x, y, z : s \bullet P(x, y) \wedge P(y, z) \Rightarrow P(x, z)$ %(transitivity)%

$\forall\, x : s; y : s \bullet PP(x, y) \Leftrightarrow P(x, y) \wedge \neg\, P(y, x)$ %(Dd14)%

$\forall\, x : s; y : s$
$\bullet\; Ov(x, y) \Leftrightarrow \exists\, z : s \bullet P(z, x) \wedge P(z, y)$ %(Dd15)%

$\forall\, x : s \bullet At(x) \Leftrightarrow \neg\, \exists\, y : s \bullet PP(y, x)$ %(Dd16)%

$\forall\, x : s; y : s \bullet AtP(x, y) \Leftrightarrow P(x, y) \wedge At(x)$ %(Dd17)%

$\forall\, z : s; x : s; y : s$
$\bullet\; Sum(z, x, y) \Leftrightarrow \forall\, w : s \bullet Ov(w, z) \Leftrightarrow Ov(w, x) \vee Ov(w, y)$

$\forall\, x, y : s \bullet \exists\, z : s \bullet Sum(z, x, y)$ %(Existence of the sum)%

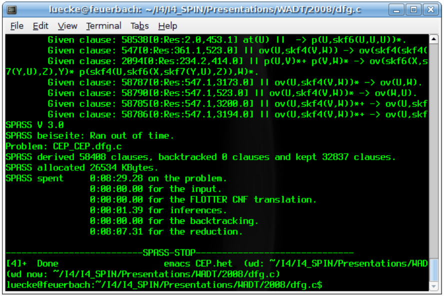
CEP, a part of DOLCE

With a bit of meta-reasoning, we can see that

finite CEP-models = finite powersets without $\emptyset$

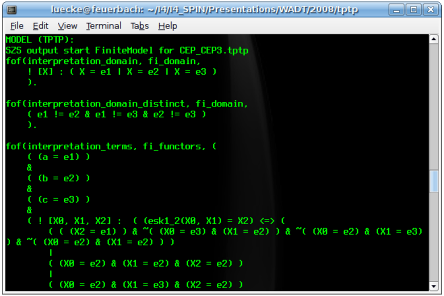
SPASS

- SPASS is a first order theorem prover based on resolution
- can check consistency if for a theory the *Th* the problem is given as $Th \vdash False$
- *Th* is consistent if SPASS reaches saturated set of clauses in such a problem
- could not verify consistency of CEP



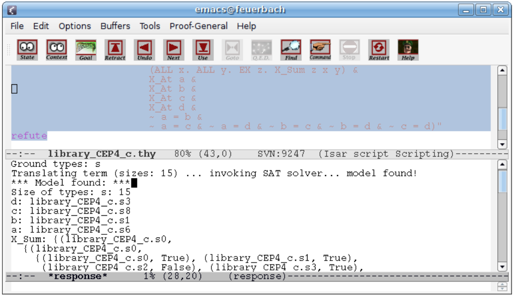
Darwin

- Darwin is a first order theorem prover/model finder based on the model evolution calculus
- it can find a counter-model for $Th \vdash False$ as well as a model for Th in a constructive way
- output in: TPTP, DIG
- scored quite well at the CASC in the last years
- could find a model with 3 atoms for CEP



Isabelle-refute

- part of the Isabelle interactive theorem prover
- uses SAT solver to find finite counter-models for first order specifications, so negation of the actual theory is used
- could find a model with 4 atoms for CEP; with some help: expected size of the model had to be supplied
- drawback: CASL sub-sorting is not supported directly



Comparison

	CEP_1	CEP_2	CEP_3	CEP_4
SPASS	×	×	×	×
Darwin	✓	✓	✓	×
Isabelle-refute	✓	✓	✓	✓

- Isabelle needed help in form of the specification of the actual size of the model for CEP_4
- none of the model-finders was able to find a model for DOLCE within several days/weeks

Models along architectural specifications

Unit-Specifications

Units are:

- named models $U : USP$

Unit specifications *USP* are:

- structured specifications *SP* of single units, for which a model has to be found directly (after flattening)
- specifications $SP_1 \times \cdots \times SP_n \xrightarrow{\tau} SP_{n+1}$ of parameterized units (roughly theory-extensions)

Syntax of architectural specifications in a nutshell

Definition (arch spec)

arch spec ASP $=$

units $U_1 : USP_1$;

...

$U_n : USP_n$

result UT

end

with U_i being the names of unit-models or parameterized unit-functions that map models to models, USP_i being their specifications

Definition (Syntax.)

- Unit Declarations: $U : SP \mid U_F : SP_1 \times \cdots \times SP_n \xrightarrow{\tau} SP_{n+1}$
- Unit Terms: $U \mid T_1 \, \textbf{and} \, T_2 \mid U_F[T_1] \ldots [T_n]$

Semantics of architectural specifications in a nutshell

arch spec ASP $=$
    units $U_1 : USP_1$;
        ...
        $U_n : USP_n$
    result UT
end

$[\![\mathrm{ASP}]\!] =$
$E \mapsto [\![UT]\!]_E$

Unit environments:
$E = (F_1, \ldots, F_n) \in \mathbf{Mod}(USP_1) \in \times \cdots \times \mathbf{Mod}(USP_n)$

Semantics of unit terms:

- $[\![U_i]\!]_E = F_i$
- $[\![T_1 \text{ **and** } T_2]\!]_E = [\![T_1]\!]_E \oplus [\![T_2]\!]_E$ (amalgamation)
- extended static semantics of arch specs guarantees that amalgamability is always ensured (using sharing analysis)

The consistency proof

- write an arch spec for the decomposition of DOLCE
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-
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        -

The consistency proof

- write an arch spec for the decomposition of DOLCE
- check its well-formedness using HETS
-
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- write an arch spec for the decomposition of DOLCE
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- check its well-formedness using HETS
- prove consistency of the architectural specification:
  - prove consistency of non-parameterised unit specs
    - all of them are small ⇒ find models using e.g. Darwin
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  - prove consistency of parameterised unit specs:
    - show that result spec is conservative over parameter spec:
    - construct a free extension of parameter spec, with recursive definitions (this is known to be conservative)
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    - use proof calculus presented by Mihai at WADT 2010
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  - construct a unit spec for the architectural spec
    - use proof calculus presented by Mihai at WADT 2010
  - prove that DOLCE refines to this unit spec
    - can be proved using structural development graph rules alone

Some data and lessons learned

- arch spec has 38 units
- well-formedness check using HETS not feasible
- after split into four arch specs, well-formedness check using HETS took 35h on i7
  - for chosing the split, unit dependency diagrams needed
  - often, only parts linked by several arrows can be found
    $\Rightarrow$ appropriate restriction of units needed
- unit dependency diagrams also needed in order to understand amalgamability problems
  $\Rightarrow$ display of diagrams of extended static semantics implemented

Lessons learned (cont'd)

- first attempt: arch spec structure follows that of structured spec ⇒ failed (due to DEPENDENCE)
- second attempt followed structured of taxonomy ⇒ successful
- by using a strengthening of DEPENDENCE, we could rely on stronger assumptions for the interpretation of DEPENDENCE for various subconcepts when extending it to a superconcept.
- only subsorted logic allows for the architectural decomposition, single-sorted logic does not (universe has to be fixed at once)

Conclusion

- Standard model finders cannot cope with DOLCE
- Developed a CASL architectural specification for DOLCE, hence we have split the task of constructing a DOLCE model into several independent subtasks
- Use of subsorting has been crucial for obtaining the decomposition

Future Work

- Checking if all extensions in the arch spec are conservative
- Using our approach for other large theories like the SUMO ontology
- Deriving a toolkit for model-finding for large theories
- Adding support for semi-automatic derivation of arch specs from structured specifications to HETS

The End

Thank You