Preliminaries

On-the-fly Strategy Synthesis for Event-Clock Linear Temporal Logic on Timed Games

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Realizability problem in a real-time setting

Realizability problem in a real-time setting

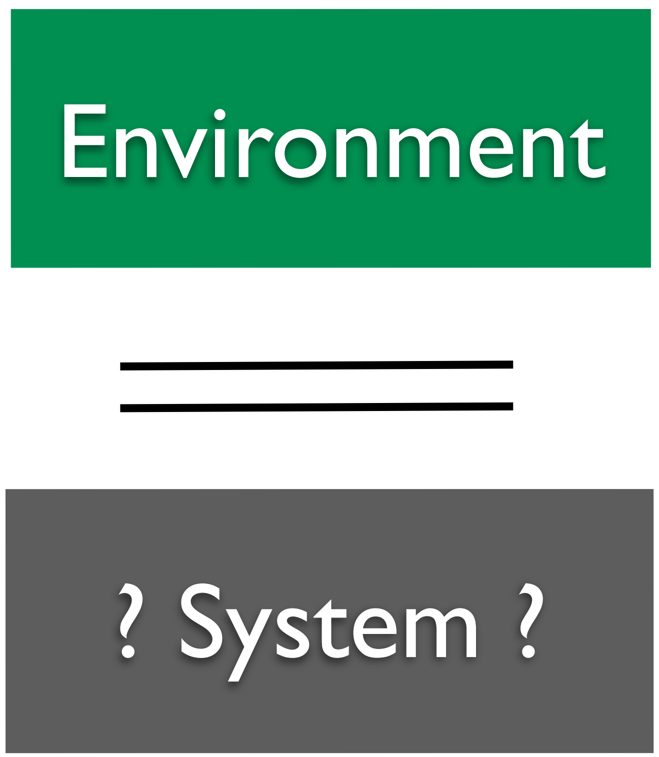
Environment

Realizability problem in a real-time setting

Environment

? System ?

Realizability problem in a real-time setting



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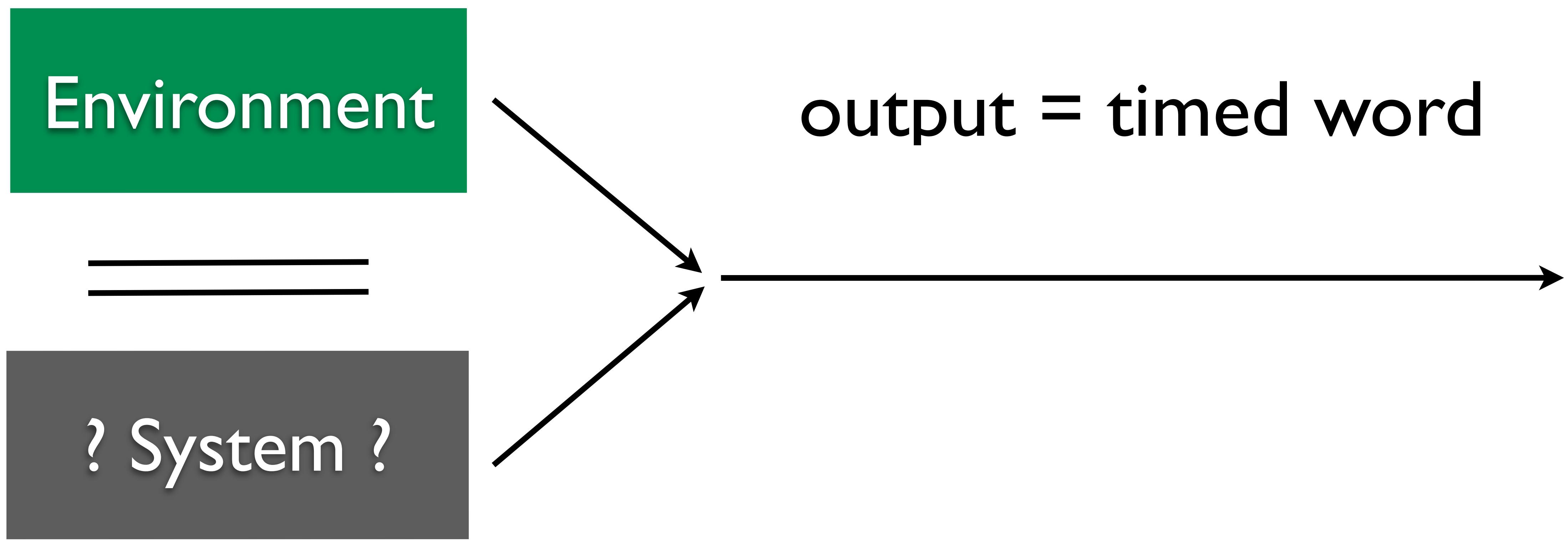

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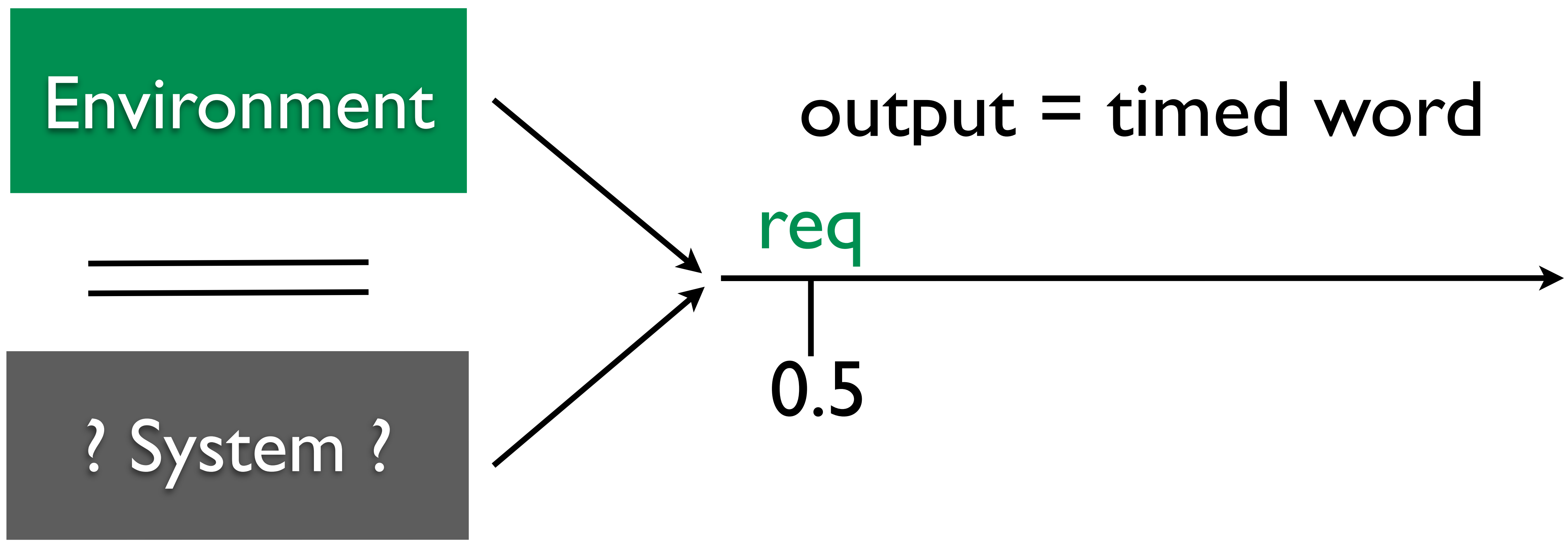

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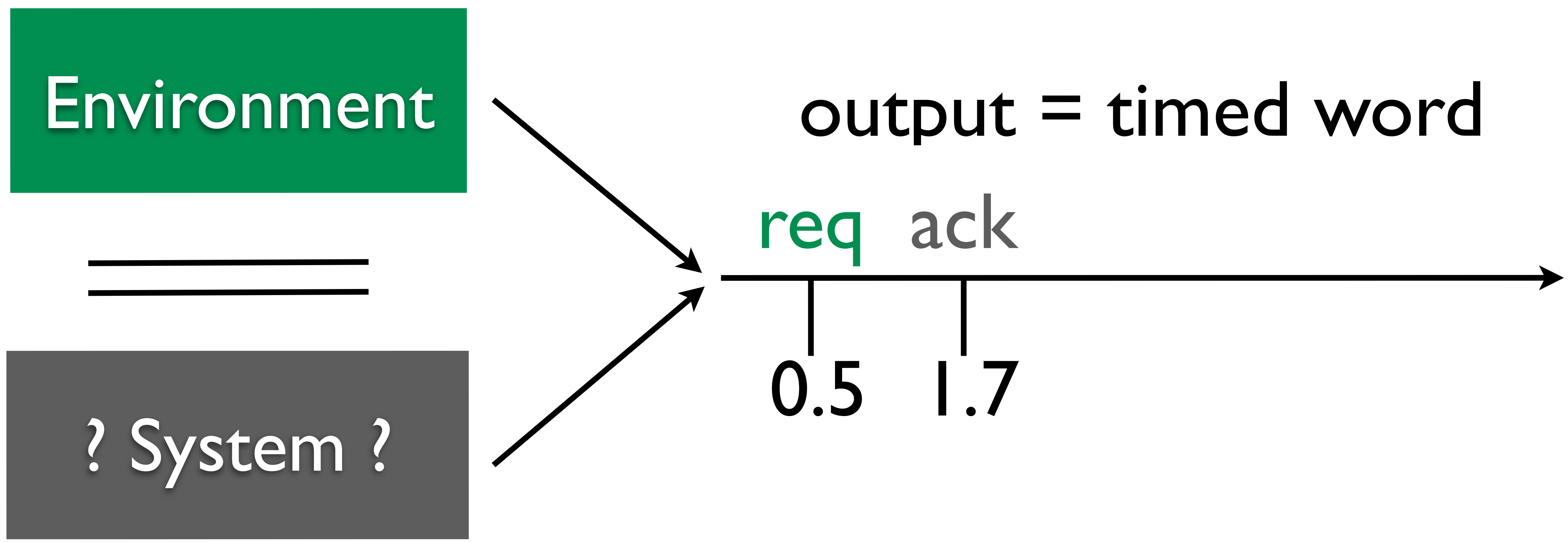

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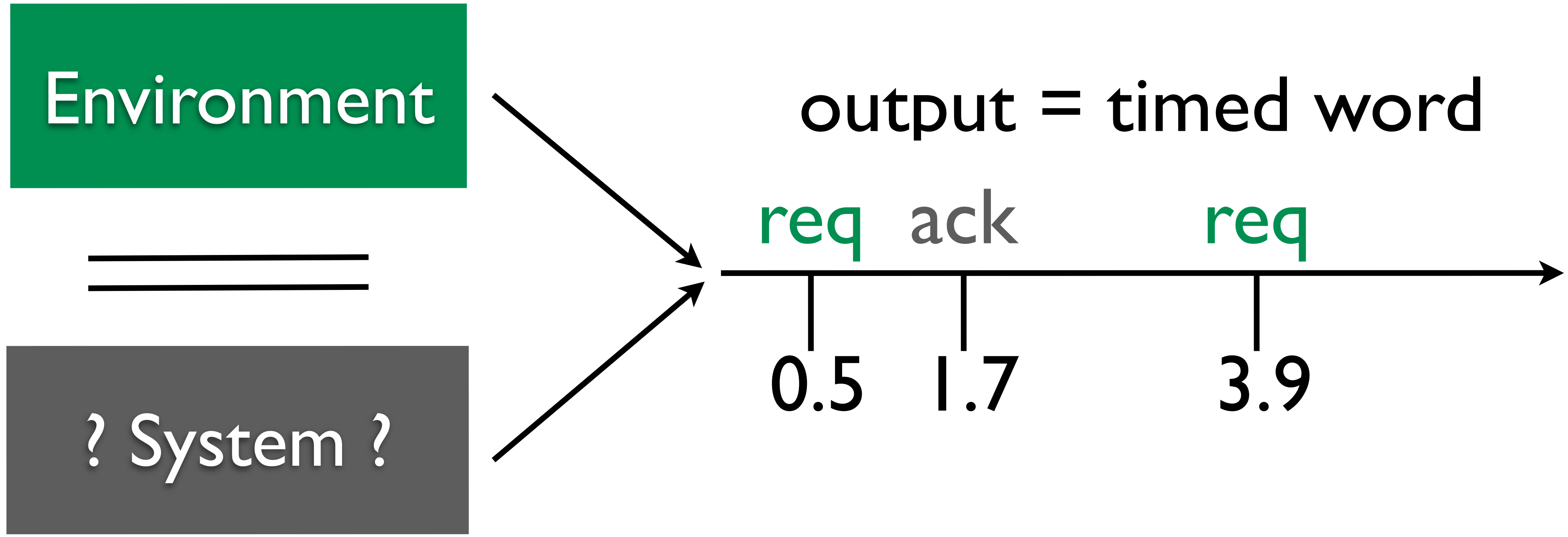

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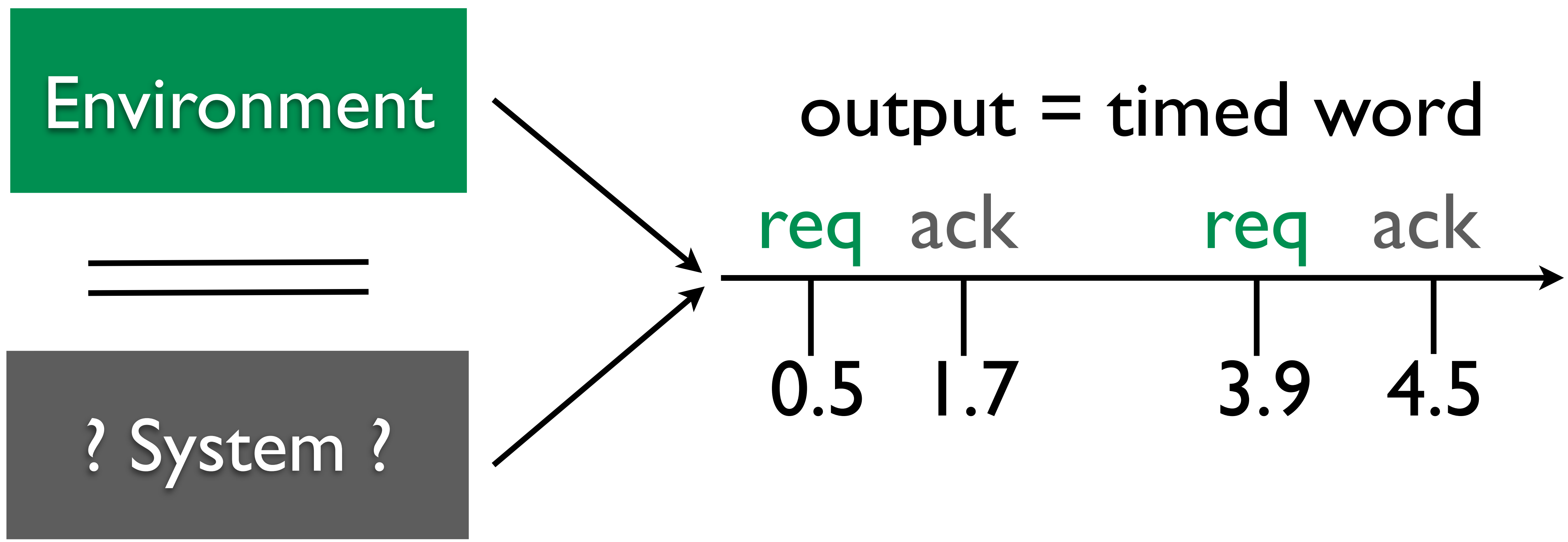

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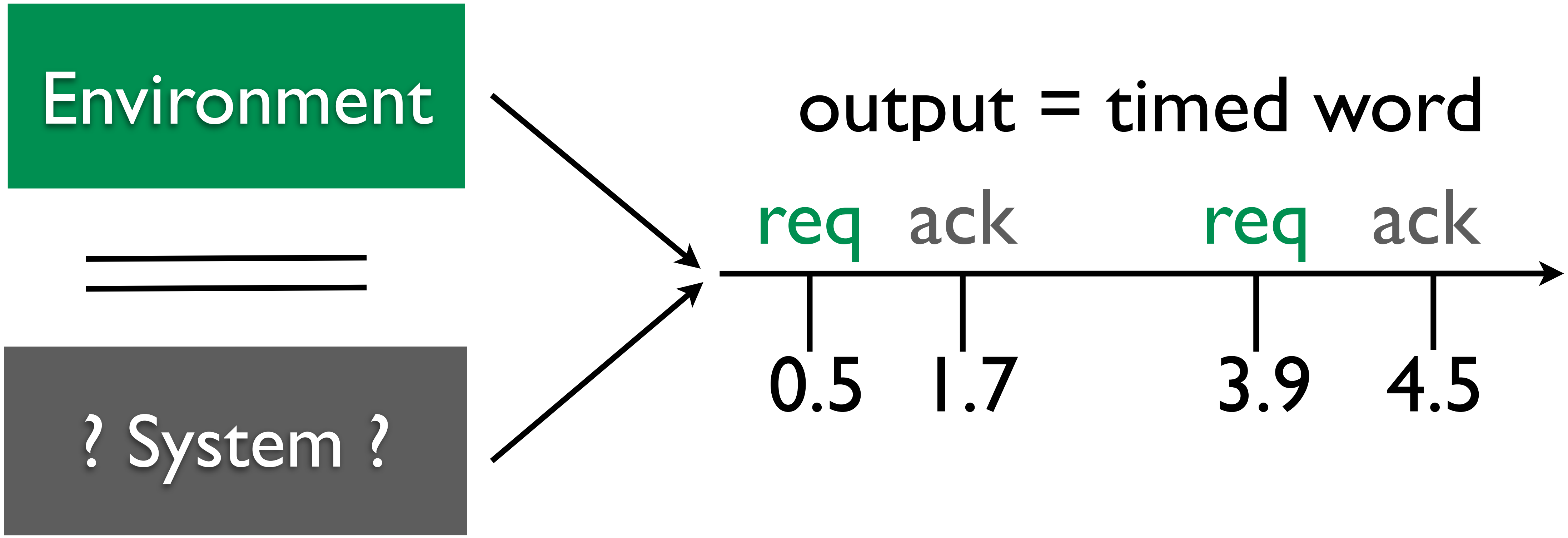
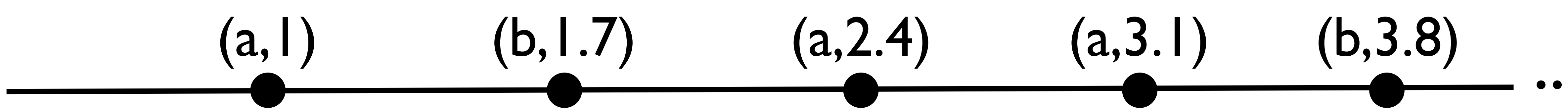


Problem

Given a spec Φ, does there exist a way for the System to choose its signals along time so that, **no matter how** the environment chooses its signals, the resulting execution satisfies the formula Φ ?

Timed words

Timed word on $\Sigma=\{a,b\}$:



= infinite sequence of elements in $\Sigma\times\mathbb{R}^{\geq 0}$

$(\sigma_0,t_0)\ (\sigma_1,t_1)\ (\sigma_2,t_2)\ \ldots\ (\sigma_n,t_n)\ \ldots$

such that $\sigma_i\in\Sigma$ and $t_i \leq t_{i+1}$, for all $i\in\mathbb{N}$.

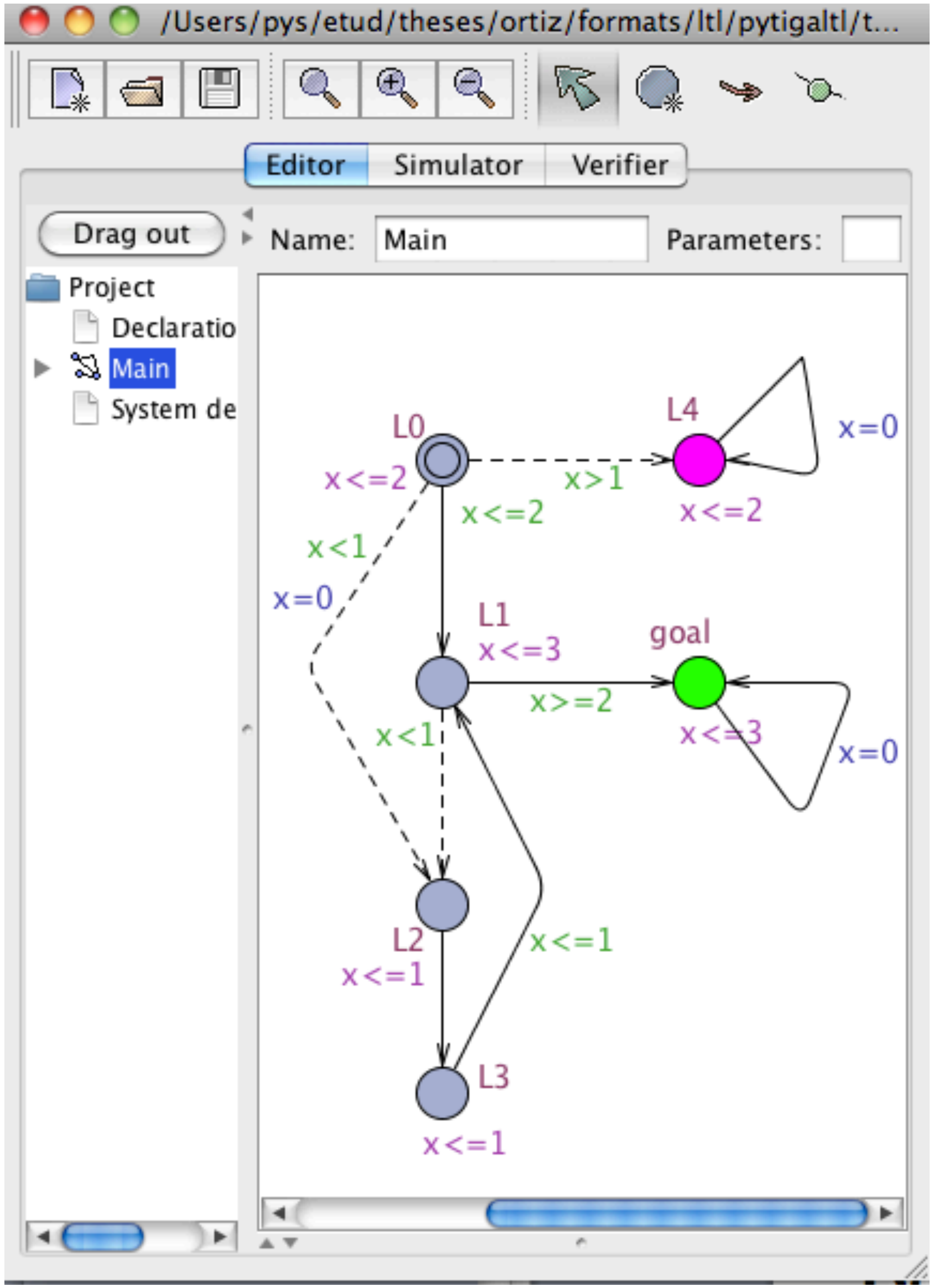
Timed Games

± Timed Automaton

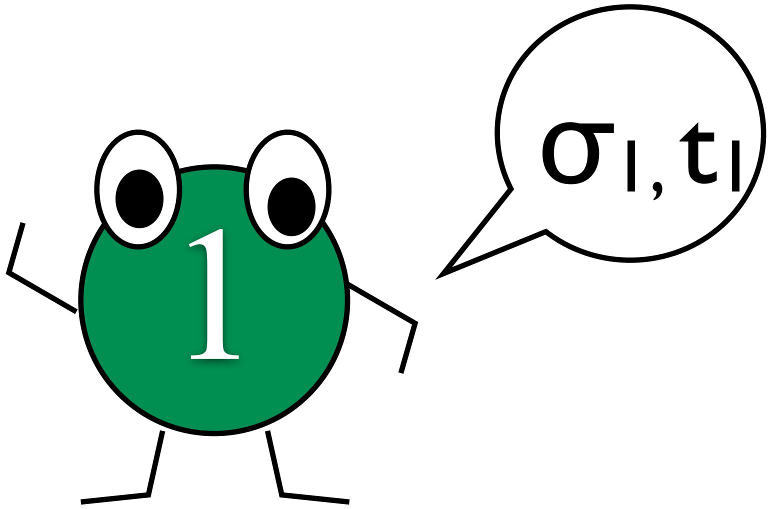
2 players: Sys and Env

Own transitions

Both players can agree to wait (as long as the location invariant stays true)



One round of the game



Player 1 chooses an action and a delay t_1

$(\sigma^1,\tau^1),\ldots,(\sigma^n,\tau^n),$

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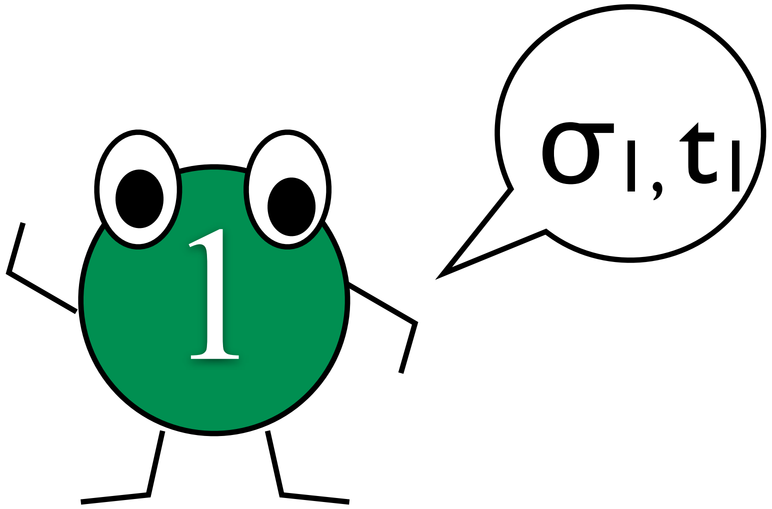

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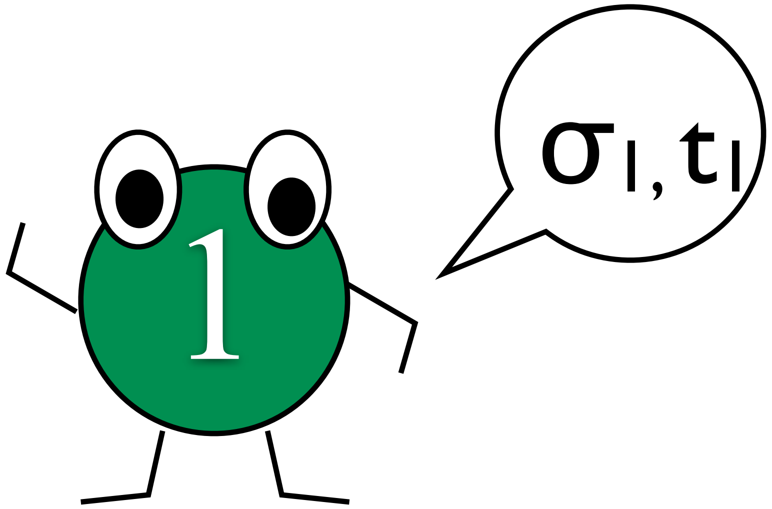
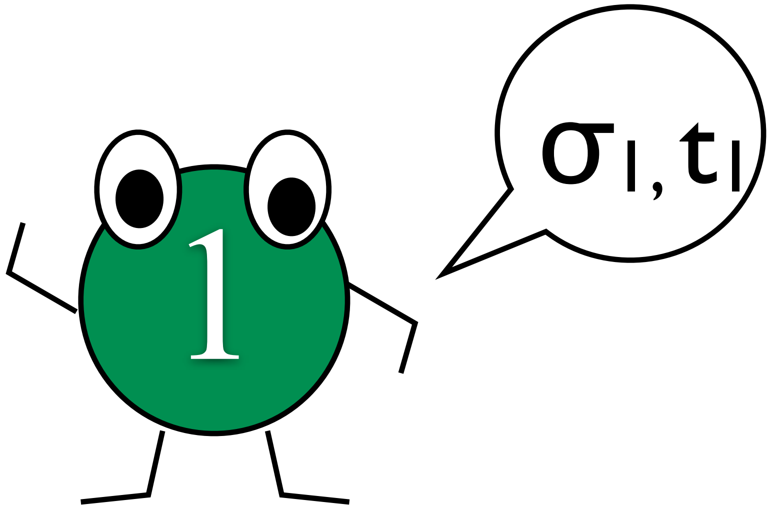


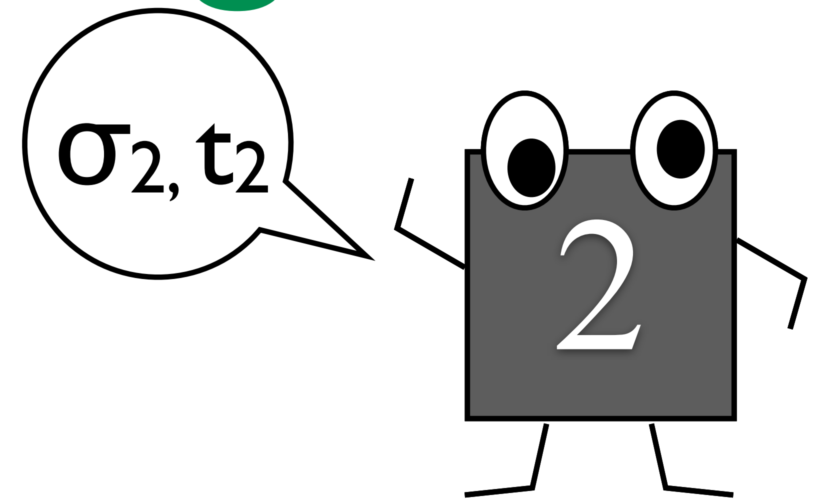
Player 1 chooses an action and a delay t_1

Player 2 may let Player 1 play

$(\sigma^1,\tau^1),\ldots,(\sigma^n,\tau^n),(\sigma_1, \tau^n+t_1)$

One round of the game





Player 1 chooses an action and a delay t_1

or chooses an action and a delay t_2, $t_2 \leq t_1$

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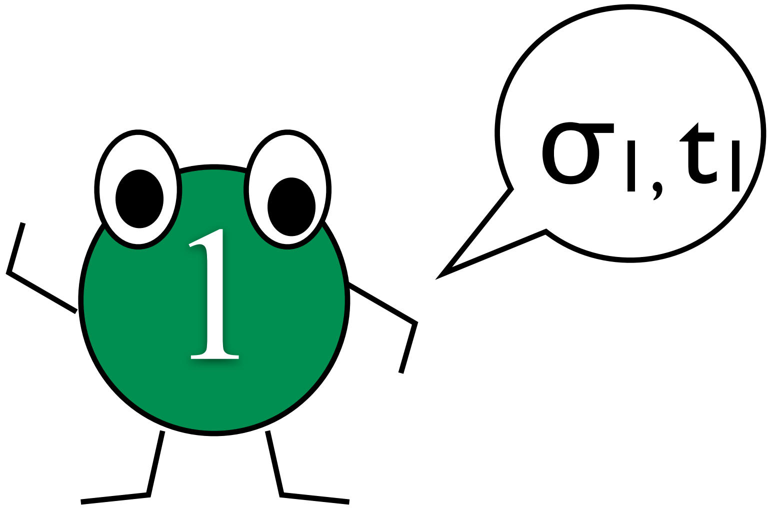

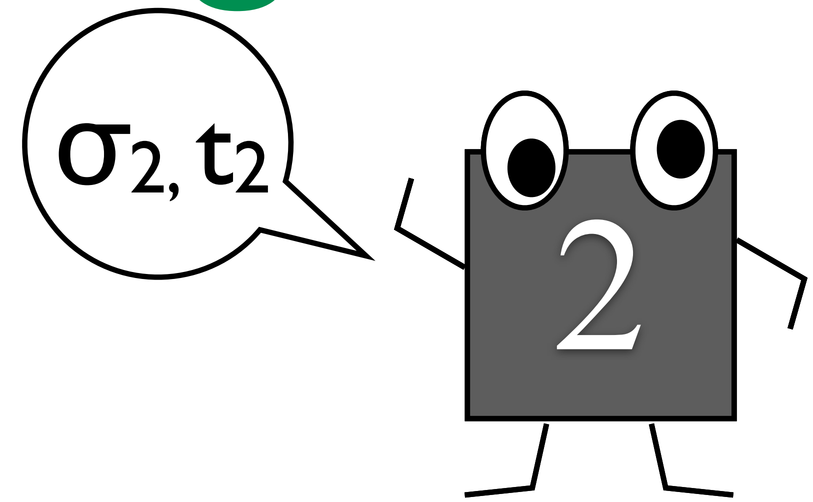

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$(\sigma^1,\tau^1),\ldots,(\sigma^n,\tau^n), (\sigma_2, \tau^n+t_2)$

Timed strategies

- Player 1's strategies: $\lambda_1: (\Sigma\times\mathbb{R}^{\geq 0})^* \rightarrow (\Sigma_1\times\mathbb{R}^{\geq 0})$

ex: $\lambda_1((a,0.6),(b,0.9))=(a,0.5)$

then **either** Player 2 let Player 1 play, and we obtain:

(a,0.6),(b,0.9)(a,1.4)

or it overtakes Player 1, for example by playing (b,0.3), and we get

(a,0.6),(b,0.9)(b,1.2)

➢➢ λ_1 is winning in $\langle\Sigma_1,\Sigma_2,$**Win**$\rangle$ if Outcome(λ_1)$\subseteq$**Win**

Event-Clock Logic (ECL)

$$\varphi \in \text{ECL} ::= a \mid \neg\varphi \mid \varphi \vee \varphi \mid \varphi\,\mathcal{S}\,\varphi \mid \varphi\,\mathcal{U}\,\varphi \mid \triangleleft_{\mathrm{I}}\,\varphi \mid \triangleright_{\mathrm{I}}\,\varphi$$

with I an interval of $\mathbb{R}^{\geq 0}$ with integer bounds

$\triangleright_{[a,b]}\ \varphi$

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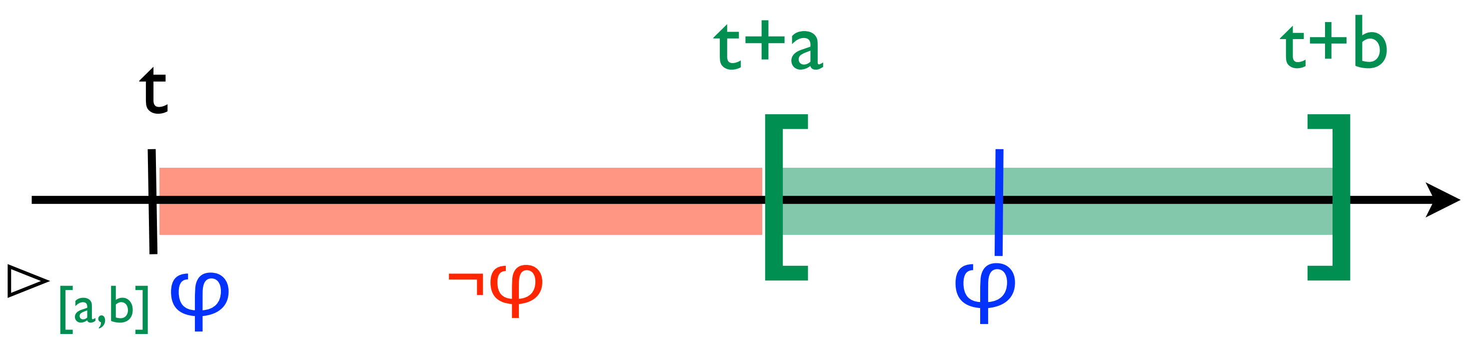

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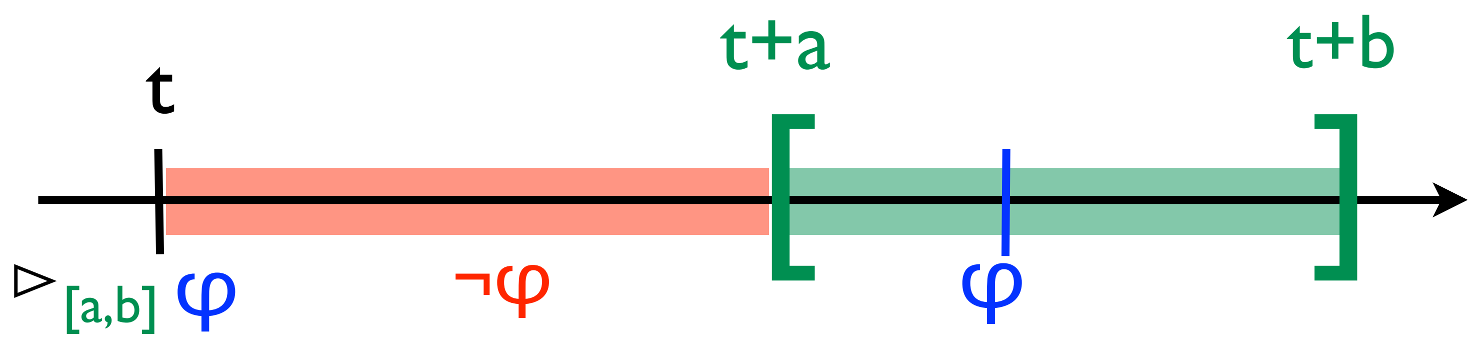

Remark: it is different from:

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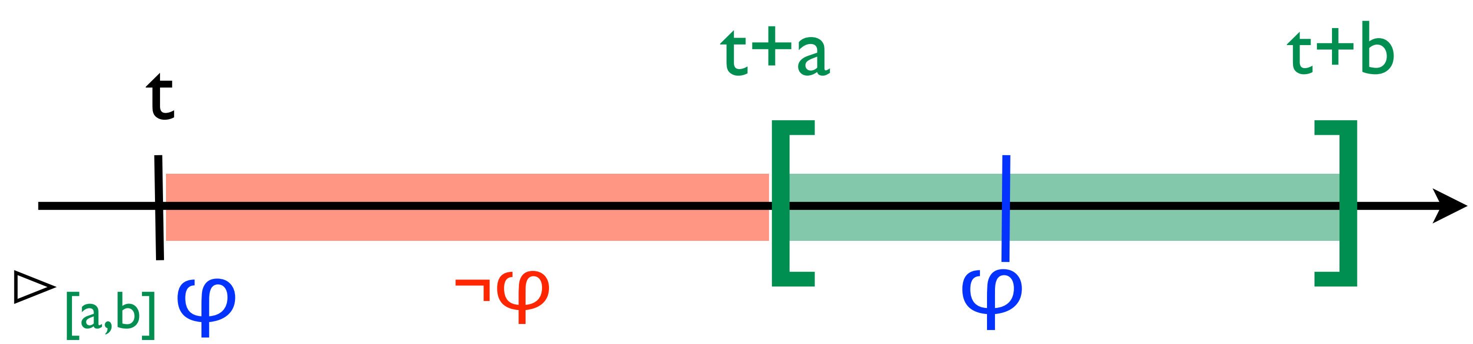

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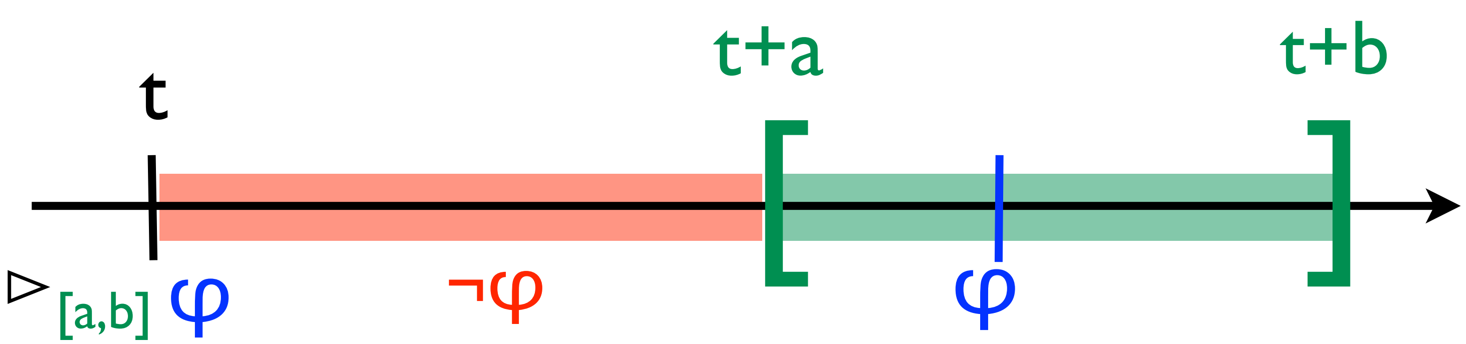

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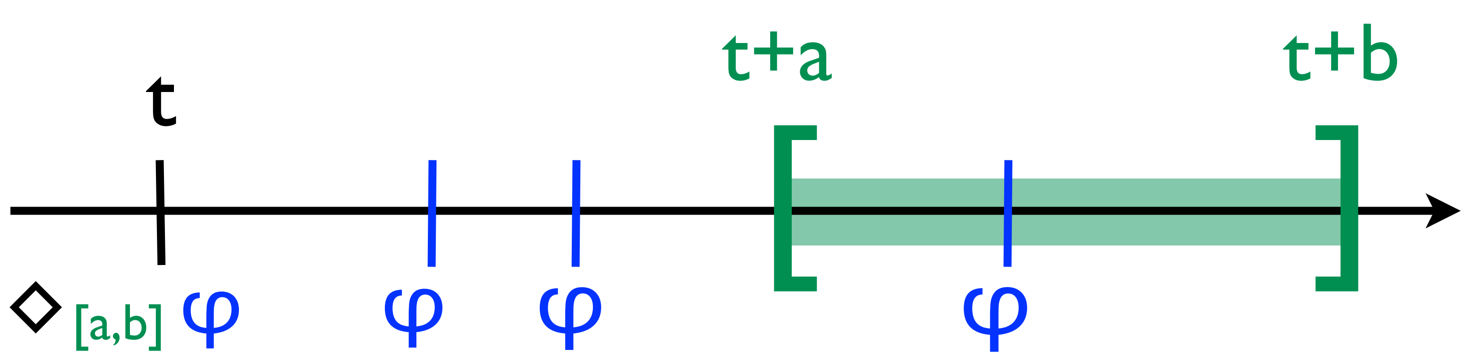

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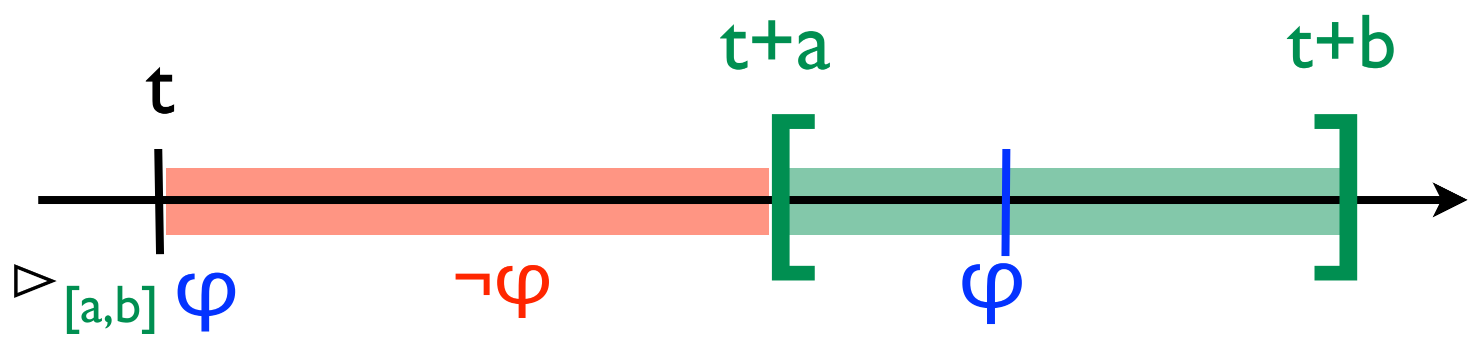

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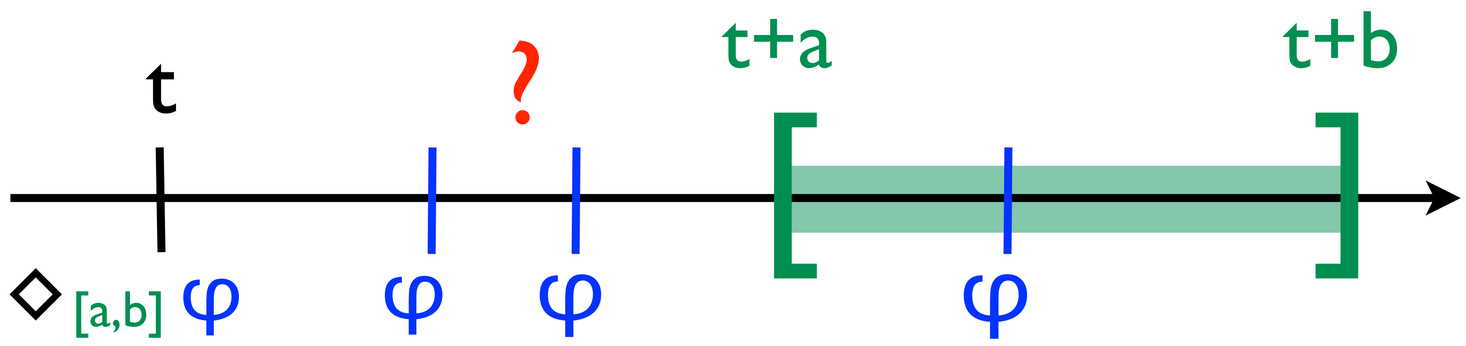

Event-Clock Logic (ECL)

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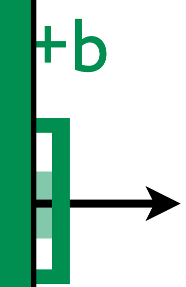
We consider timed games of the form

$\langle \Sigma_1, \Sigma_2, [\![\varphi]\!] \rangle$

where

φ is an ECL formula



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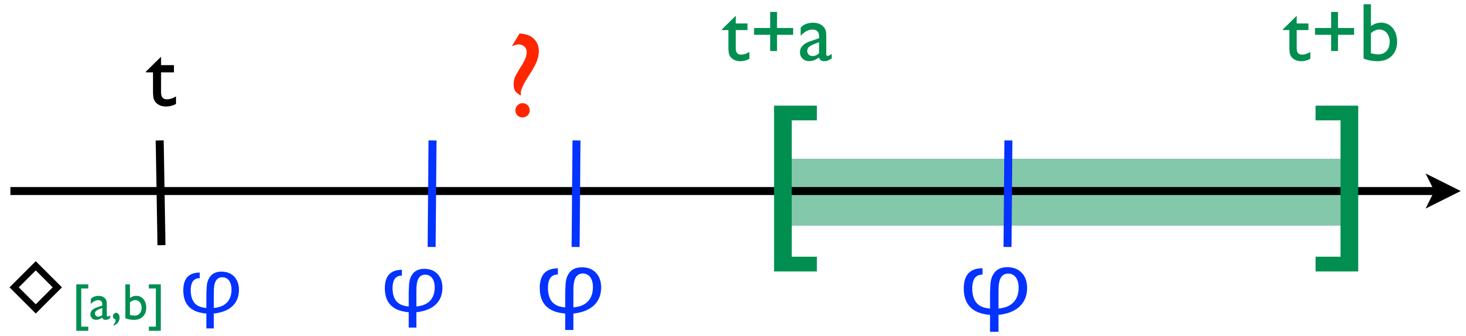

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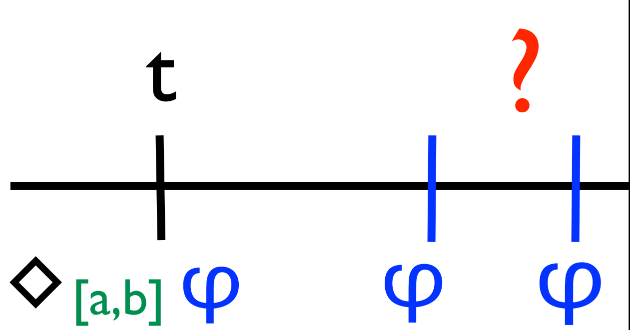
This problem is called

ECL «realizability»

$\Diamond_{[a,b]}\,\varphi$



Why ECL?

- Satisfiability of MTL **undecidable** on infinite words.
  $\Longrightarrow$ Realizability is thus **undecidable** too !
- ECL is an interesting subcase of MITL (equivalent to $MITL_{0,\infty}$).

Results

Undecidability of ECL realizability

Theorem: ECL realizability is undecidable

- Idea of the proof: encode computations of lossy three counters machines into timed words
- Build a game s.t. Player 1 has a winning strategy iff the machine admits an infinite bounded run
  - One has to use the interaction of the Players to check that the encoding is correct.

LTL$_{\triangleleft}$ realizability is decidable

$$\varphi \in \text{ECL} ::= a \mid \neg\varphi \mid \varphi \vee \varphi \mid \varphi\, \mathcal{S}\, \varphi \mid \varphi\, \mathcal{U}\, \varphi \mid \triangleleft_{\mathrm{I}}\, \varphi \mid \triangleright_{\mathrm{I}}\, \varphi$$

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$$\psi \in \mathsf{LTL}_\lhd ::= a \mid \neg\psi \mid \psi \vee \psi \mid \psi\,\mathcal{S}\,\psi \mid \psi\,\mathcal{U}\,\psi \mid \lhd_{\mathrm{I}}\,a$$

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The real-time modality can «speak» about past events only

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- **Theorem**: The realizability problem for LTL$_\lhd$ is 2EXPTIME-complete

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The real-time modality can «speak» about past events only

- **Theorem**: The realizability problem for LTL$_\triangleleft$ is 2EXPTIME-complete

- **Idea**: from ψ, build a deterministic timed automaton with parity condition

LTL$_{\lhd}$ realizability is decidable

Determinization of Büchi automata is already hard in practice !

$\ldots \varphi \mid \lhd_{\mathrm{I}}\, \varphi \mid \rhd_{\mathrm{I}}\, \varphi$

$\ldots \psi \mid \lhd_{\mathrm{I}}\, a$

…peak»

- **Theorem**: The realizability problem for LTL$_{\lhd}$ is 2EXPTIME-complete

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$\varphi \mid \triangleleft_I \varphi \mid \triangleright_I \varphi$

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Determinization of Büchi automata is already hard in practice !

Can we find «Safraless» procedures that avoid Safra's determinization ?

- **Theorem**
  LTL$_\triangleleft$ is 2EX

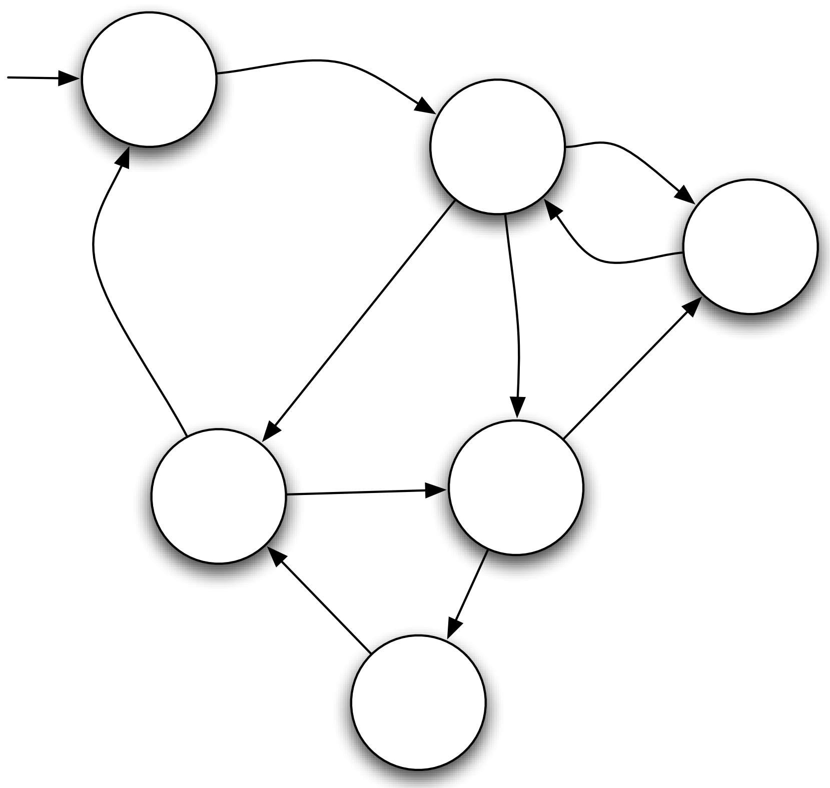
- **Idea**: from ψ, build a deterministic timed automaton with parity condition

Safraless procedures

- Safraless realizability/synthesis (untimed setting):
  - ★ Rank construction [KupfermanVardi05]:
    LTL → UcoBW → ABT → NBT → Büchi game
  - ★ K-co-Büchi condition:
    [ScheweFinkbeiner07] application to distributed synthesis,
    [FiliotJinRaskin09] application to LTL synthesis.
    LTL → UcoBW → UKcoBW → Safety game

Idea of procedure

- **Reduce** the realizability problem to a **safety (timed) game**
  - Game played on a graph
  - Goal: avoid bad states
- Not a Büchi condition: avoid Safra !
- Allows incremental procedure
- Tools and algorithms exist to solve safety (timed) games
  - e.g.: UppAal TiGa



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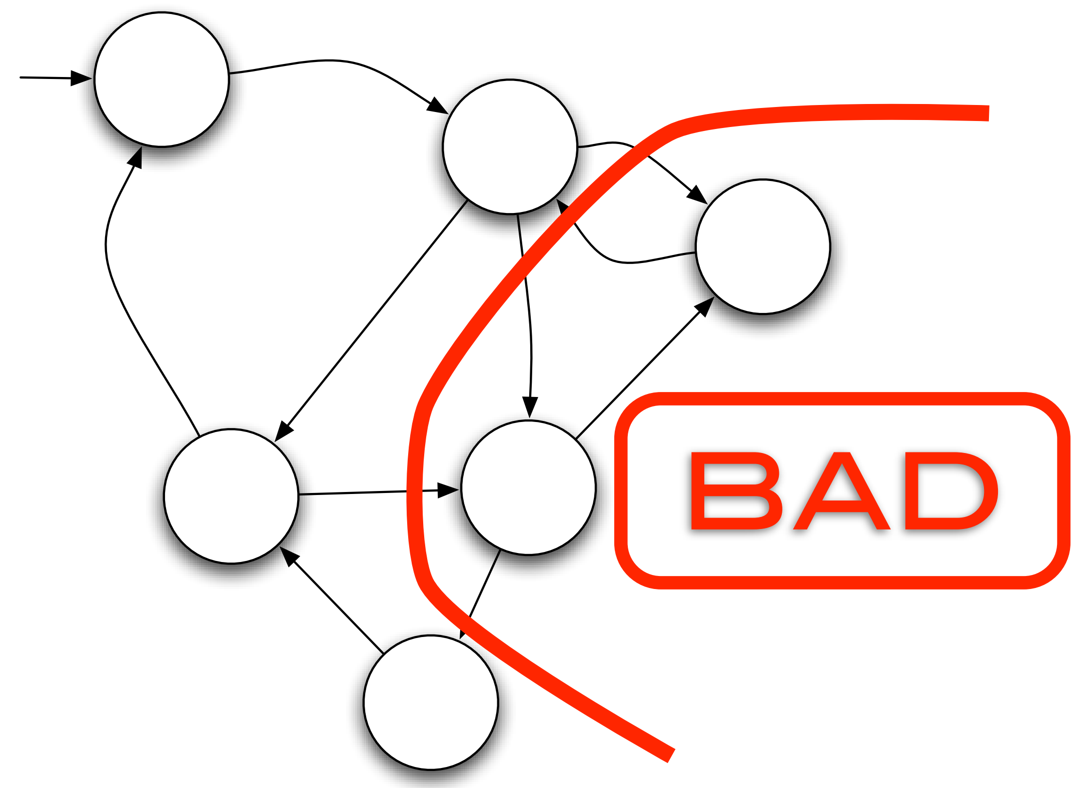

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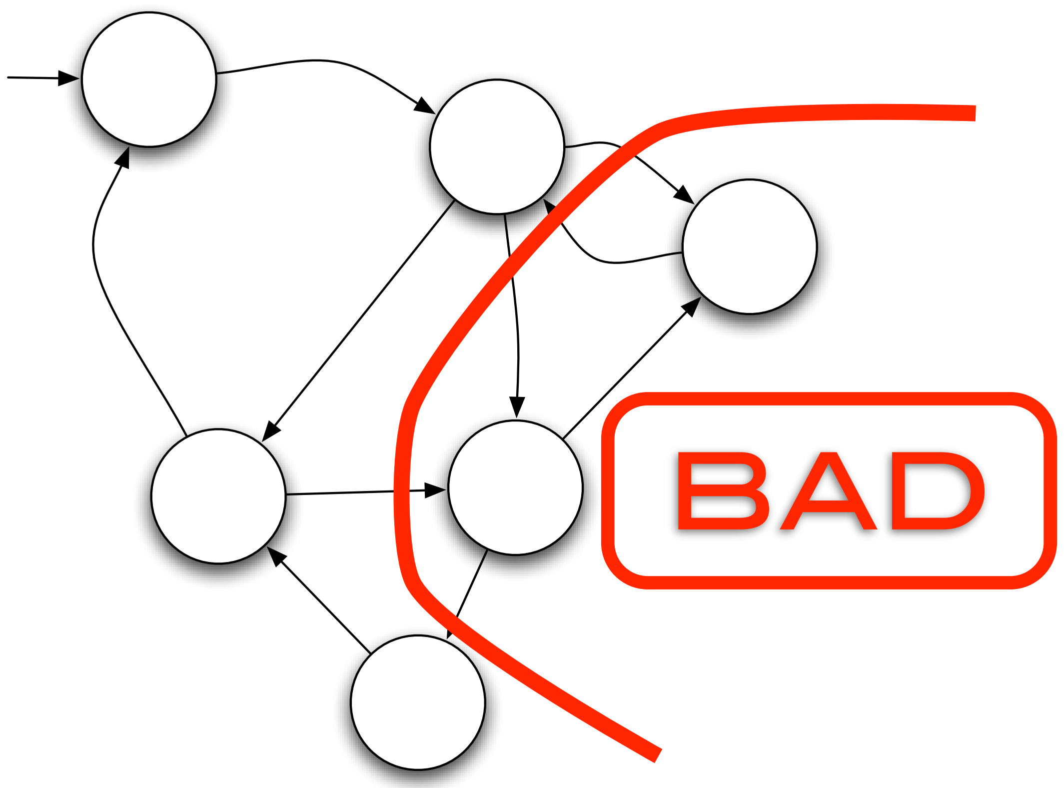
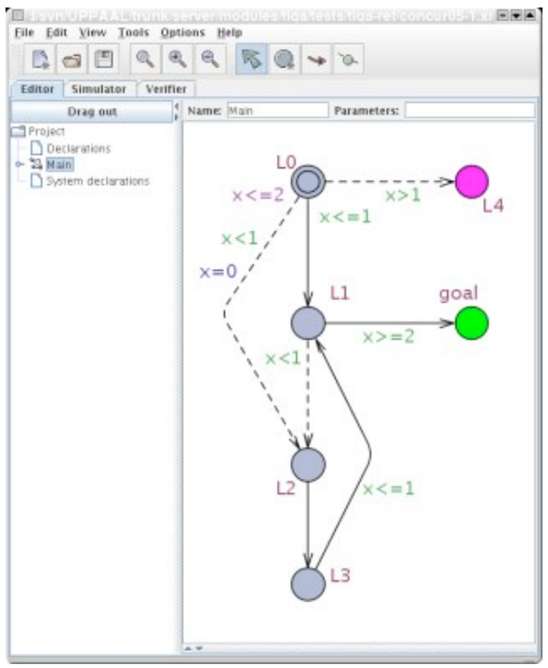
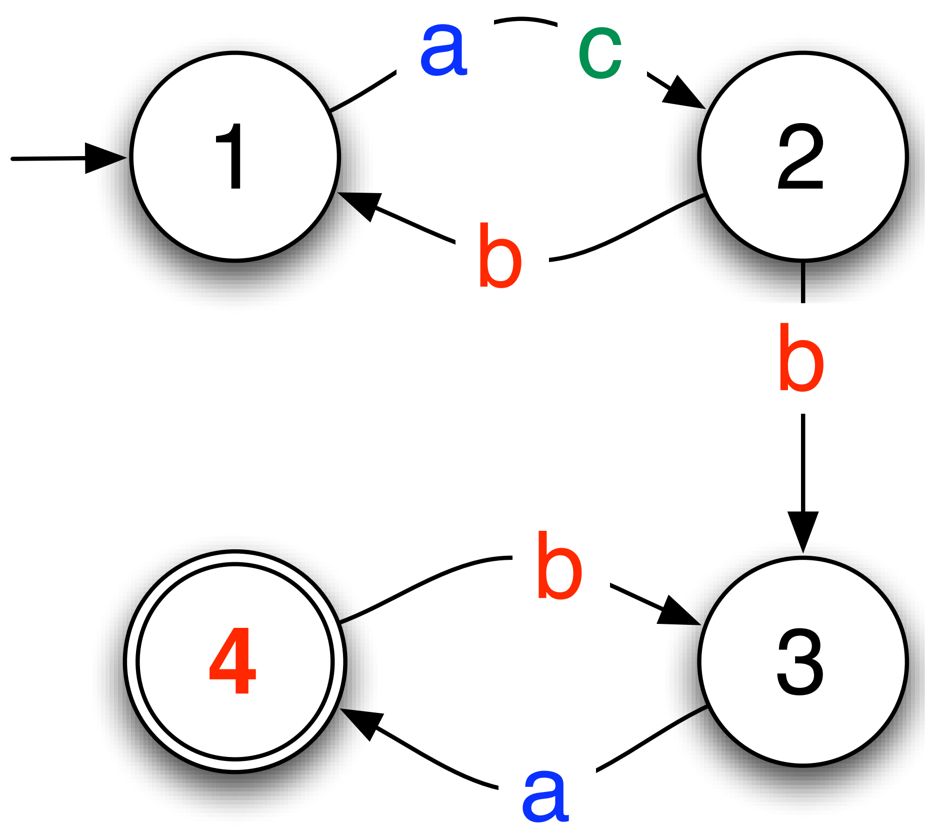


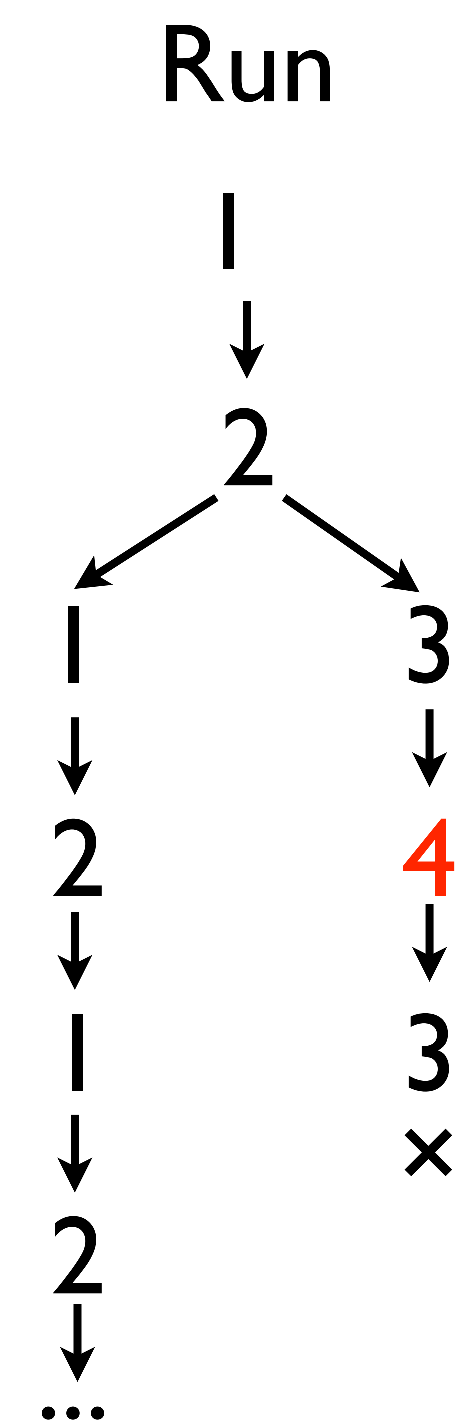


Universal coBüchi **Word** Automata



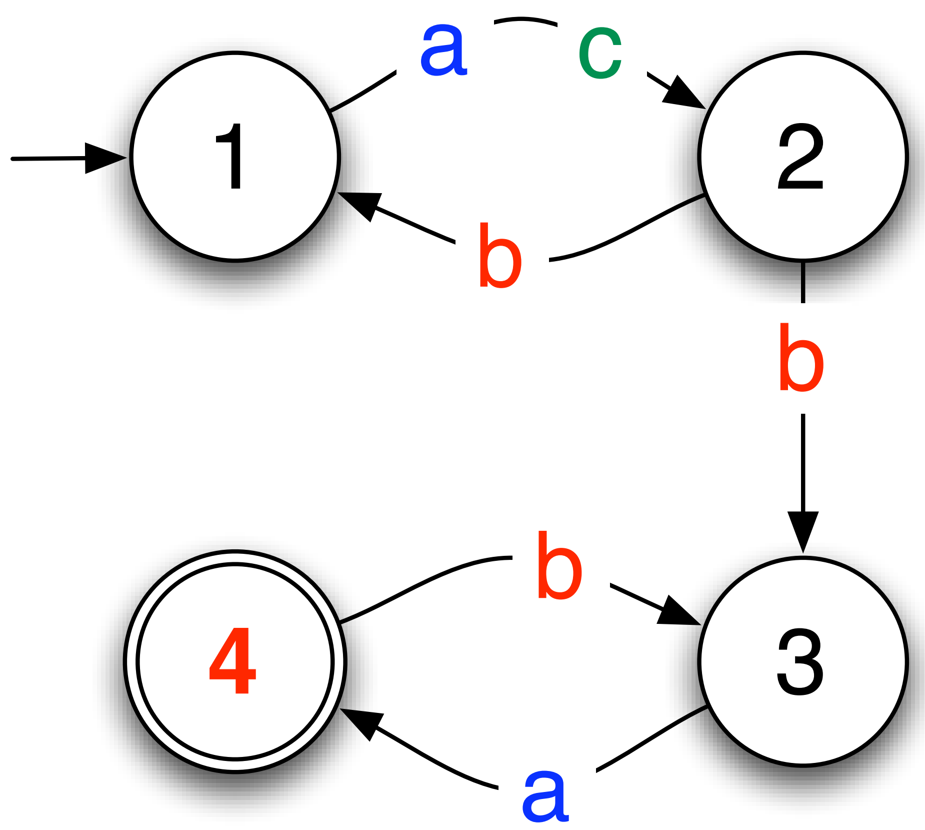
Σ^ω

a
b
a
b
c
...



w ∈ L_{UcoB}(A)
iff
all runs of A on w visit
finitely many times α.

Universal KcoBüchi **Word** Automata



Σ^ω

a

b

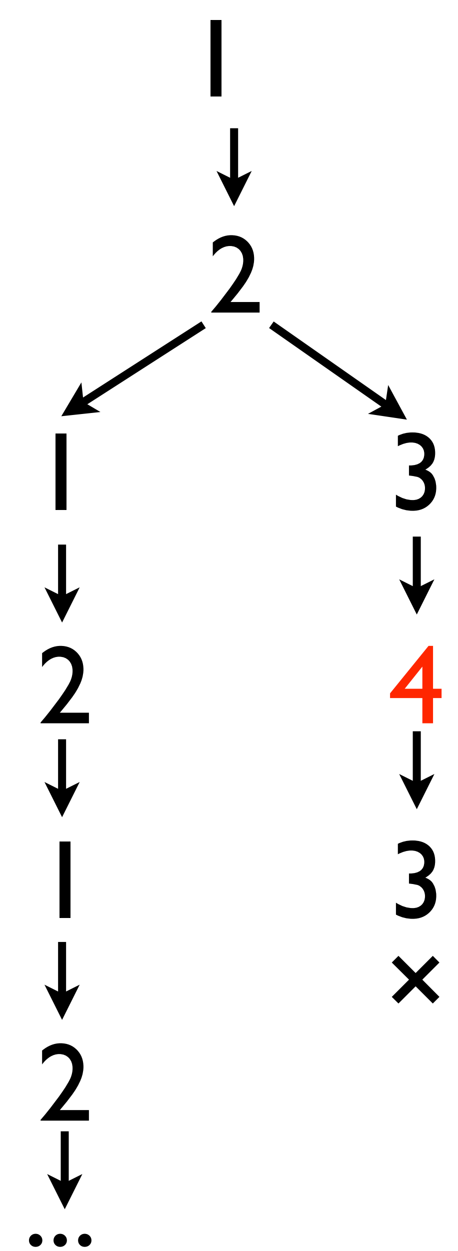
a

b

c

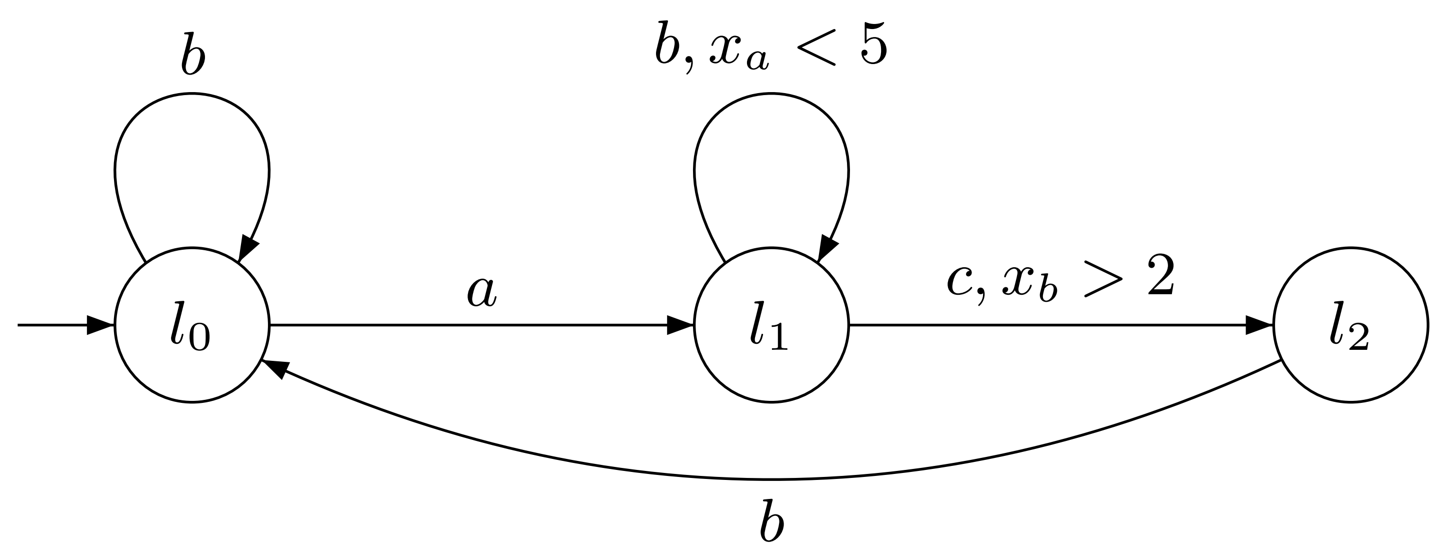
...

Run

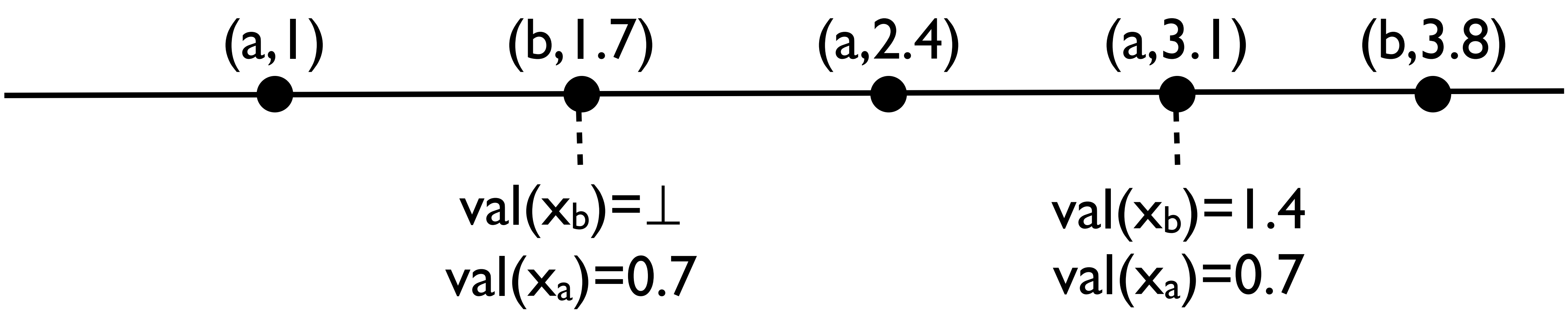


$w \in L_{UKcoB}(A)$
iff
all runs of A on w visit
at most K times α.

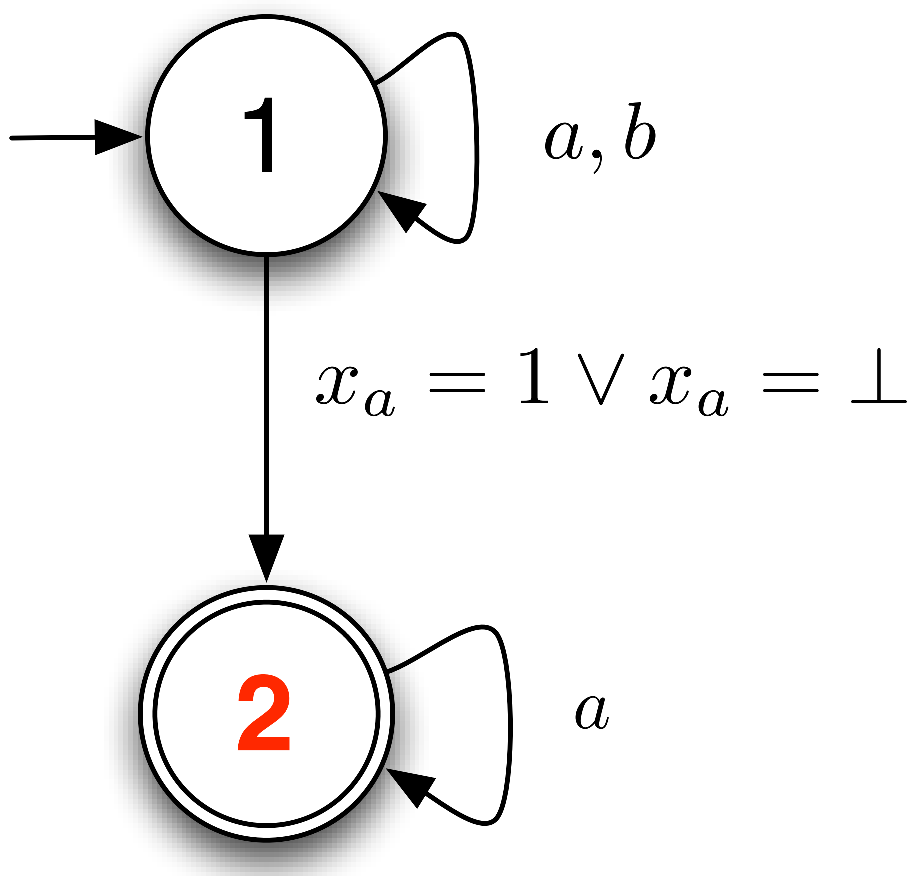
Event-recording automata

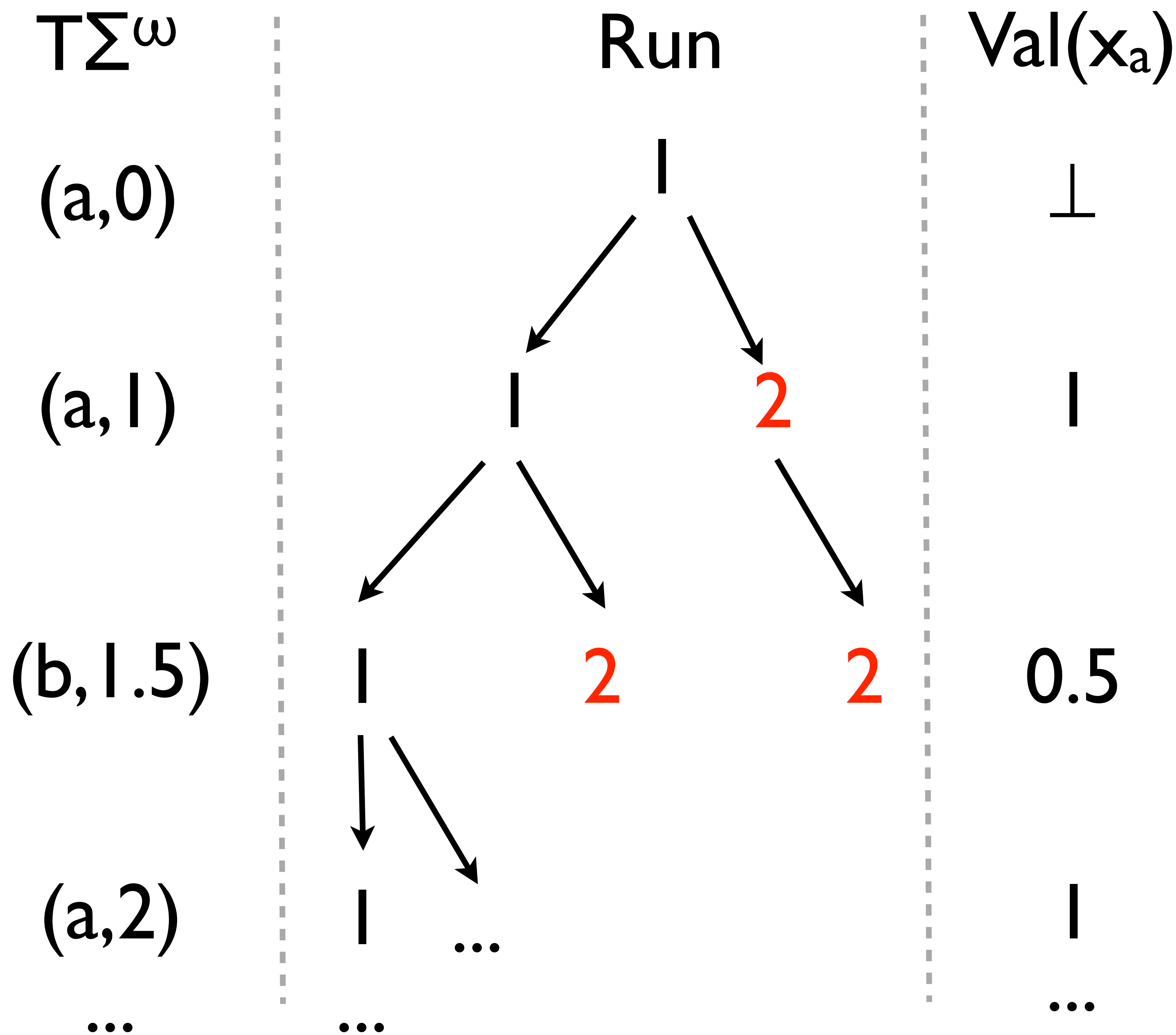


Clock are not reset and are associated to events: $\{ x_\sigma \mid \sigma \in \Sigma \}$
Each clock monitors the last occurence of the associated letter
Values of event-clocks are input determined:



Universal ERA with coBüchi a. c.





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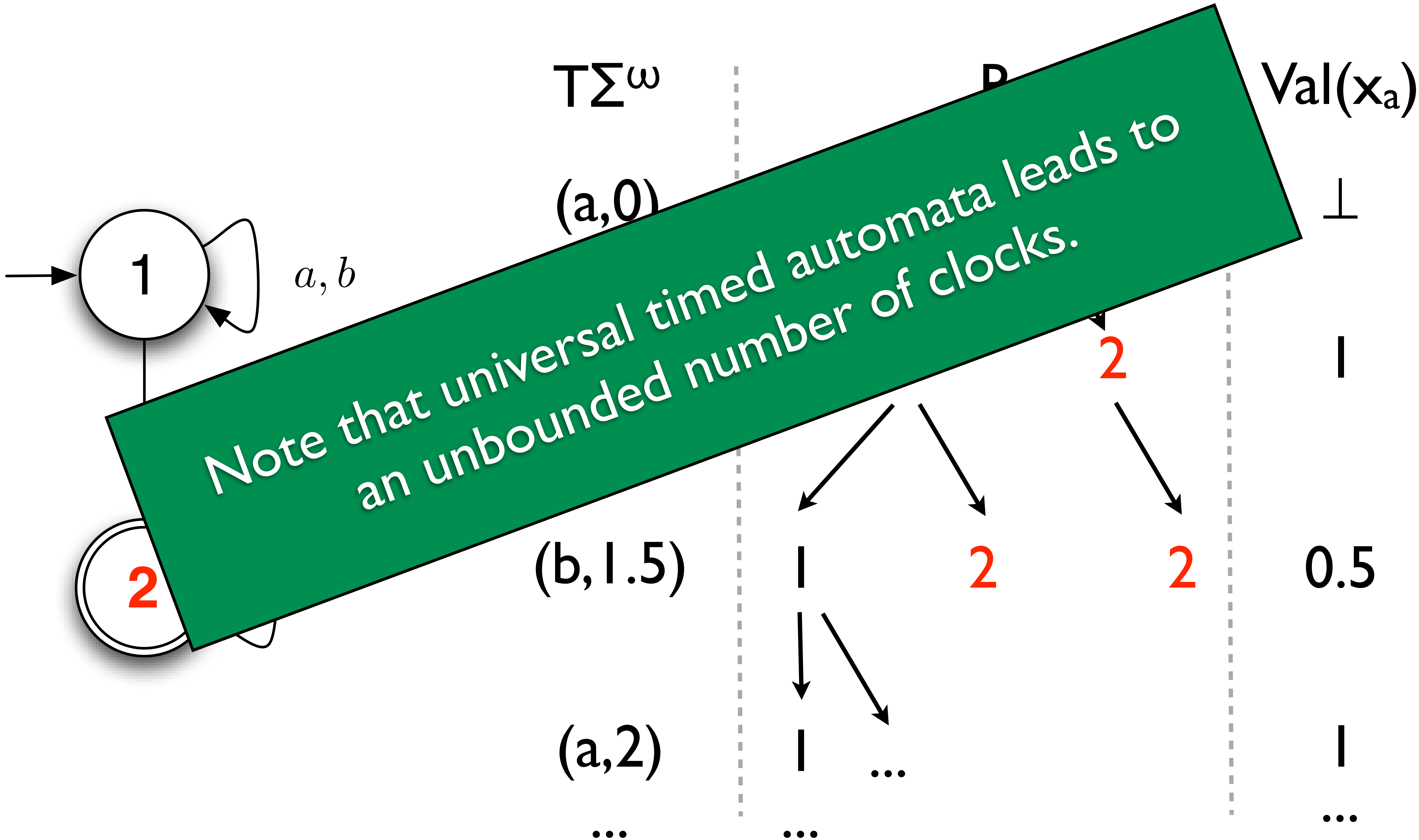

Back to LTL$_\triangleleft$ realizability

Back to $LTL_{\triangleleft}$ realizability

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We are now playing the game on A_φ

Goal of Player 1: ensure that every run on the outcome visits accepting states **finitely often**

From UCoB to UKCoB

Theorem: Winning strategies of Player 1 on the UCoB automaton can be represented by a finite machine (with m states)

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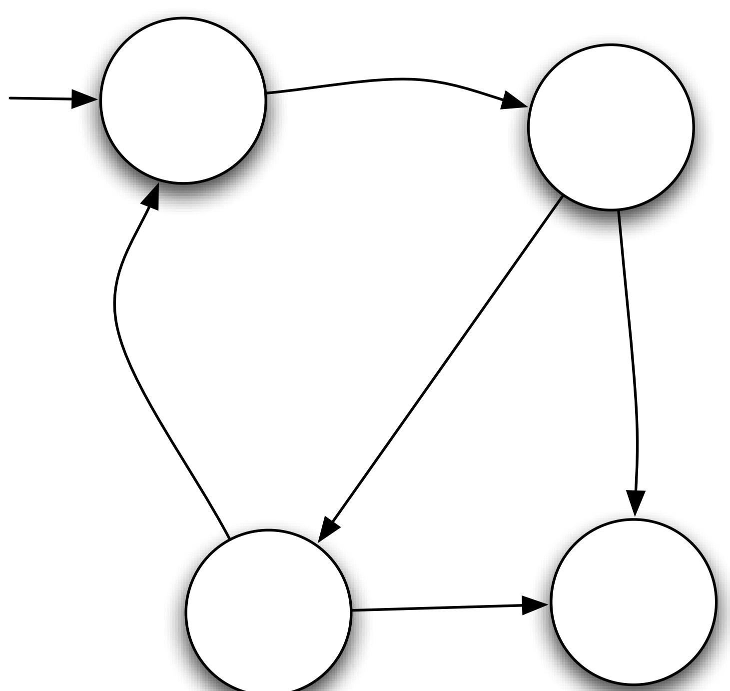

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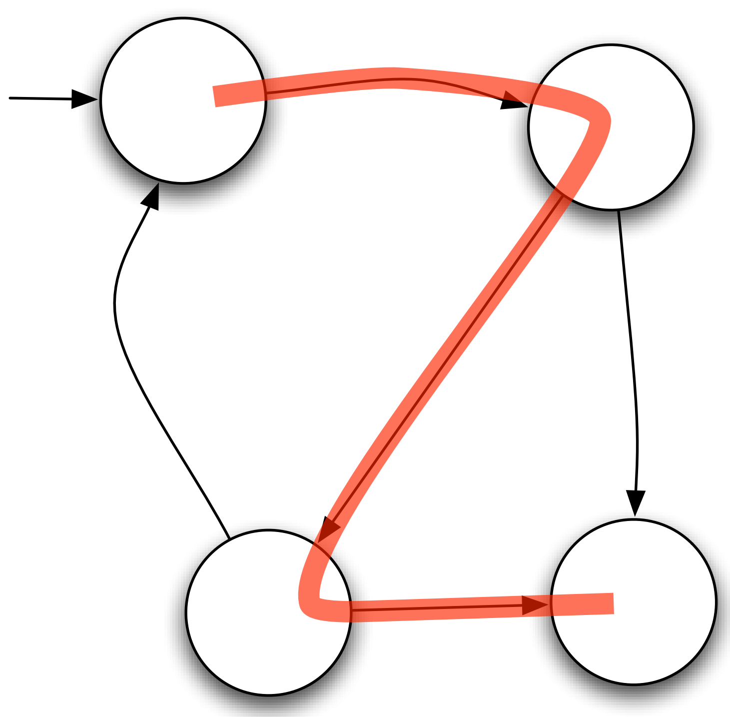

$(\sigma_1, t_1)\ (\sigma_2, t_2)\ (\sigma_3, t_3)\ldots$

From UCoB to UKCoB

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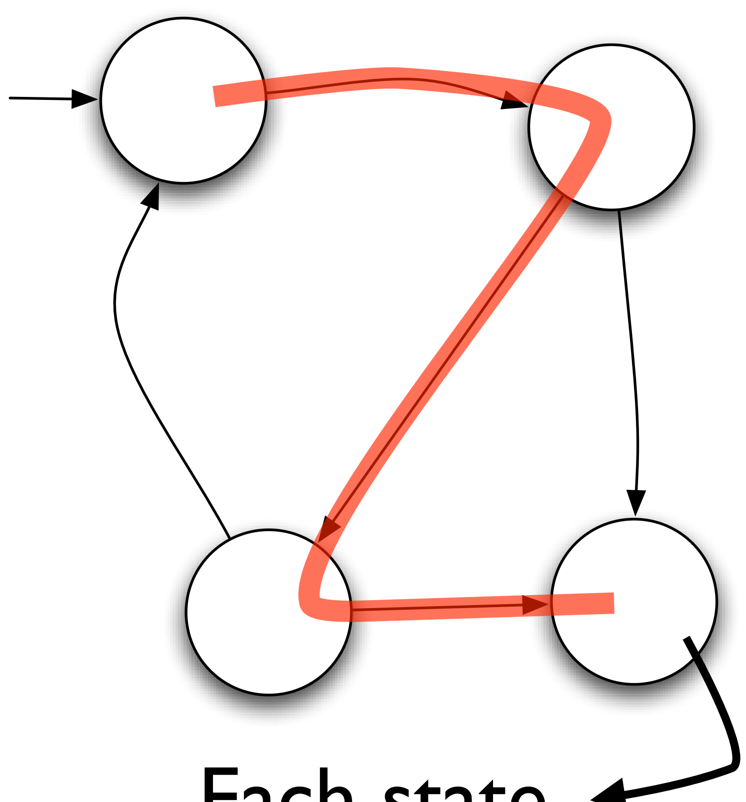


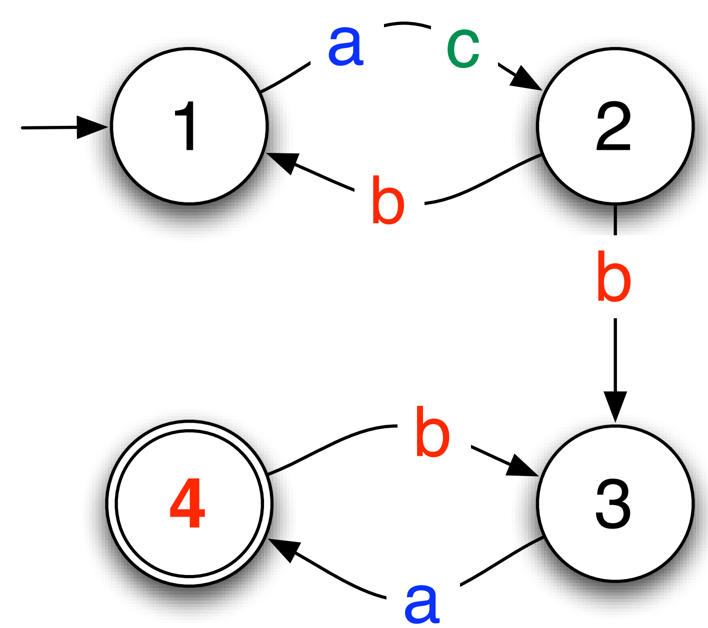
(σ_1, t_1) (σ_2, t_2) (σ_3, t_3)...

From UCoB to UKCoB

Strategy

m states

UCoB

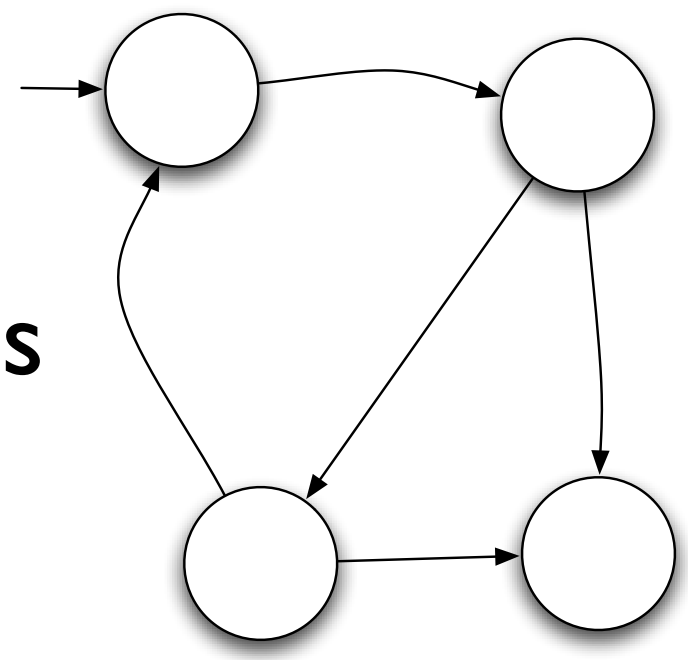


n states

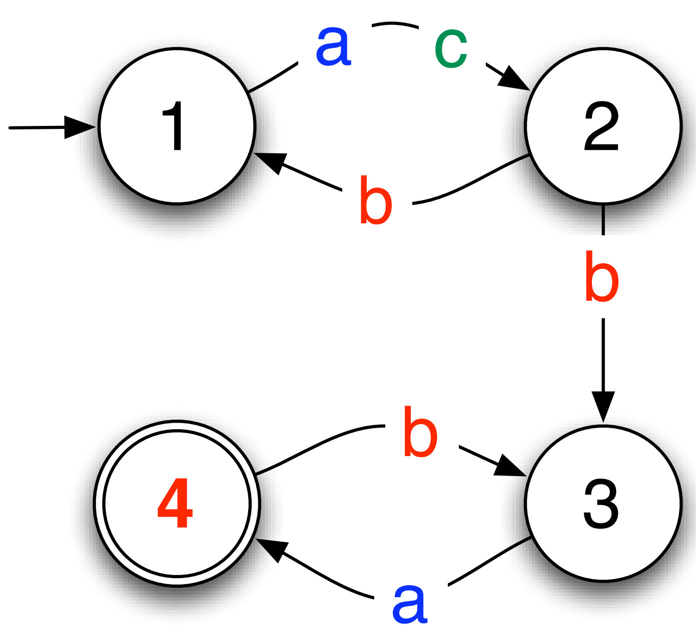
From UCoB to UKCoB

Strategy

m states



UCoB



n states

From UCoB to UKCoB

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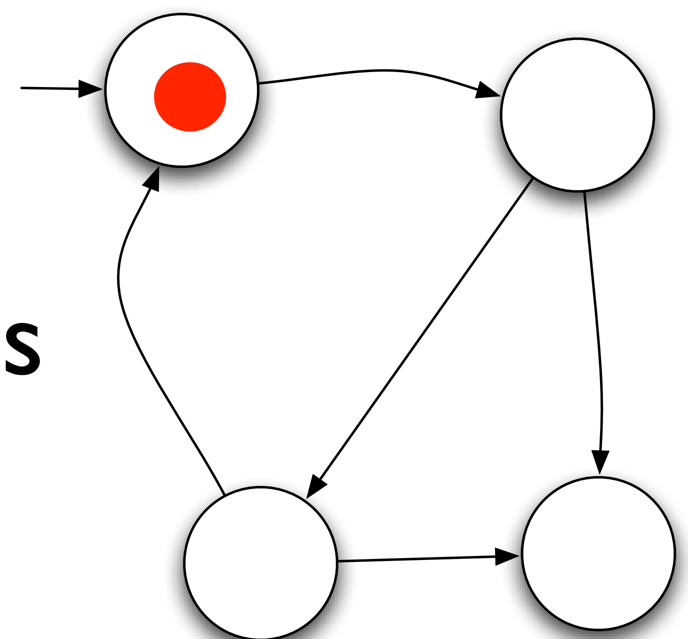
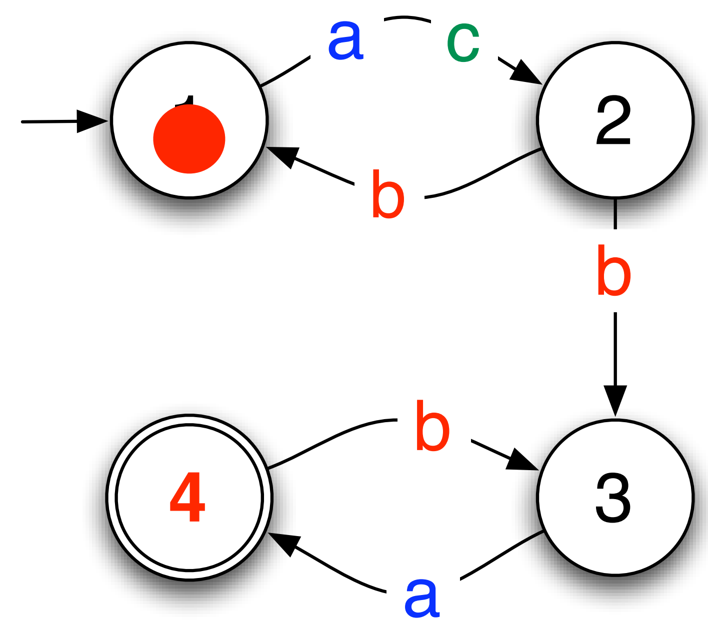


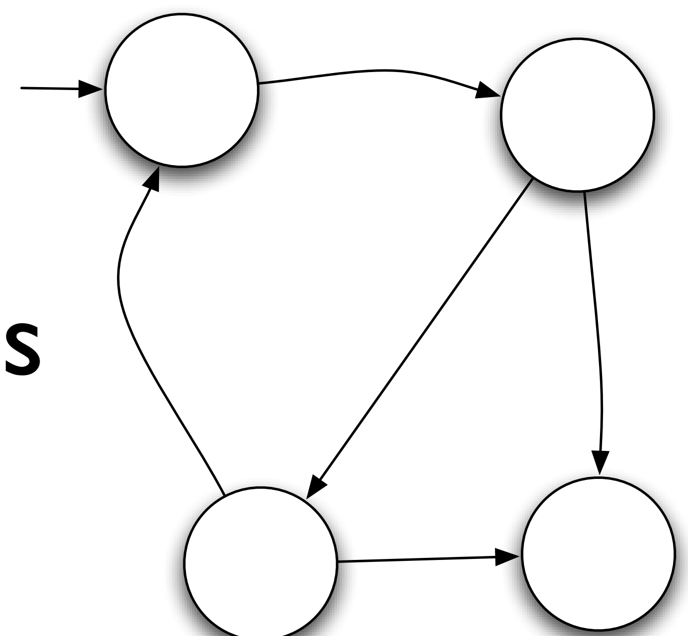
UCoB



n states

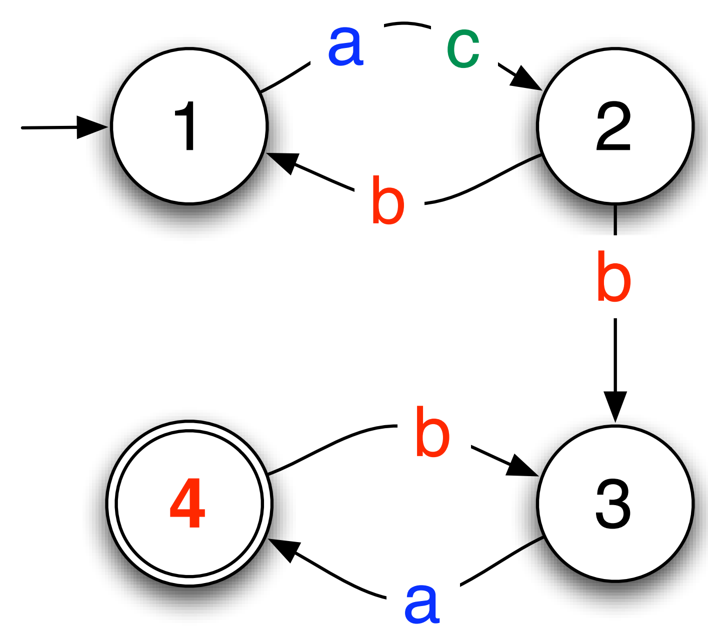
From UCoB to UKCoB

Strategy



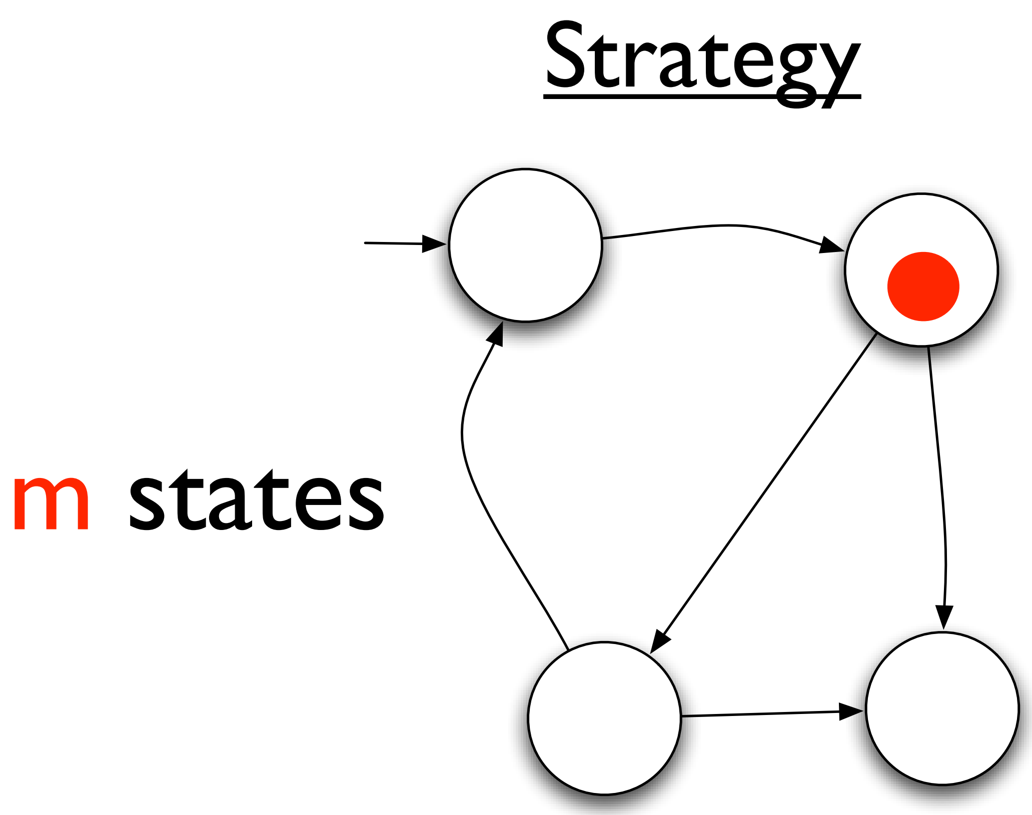
m states

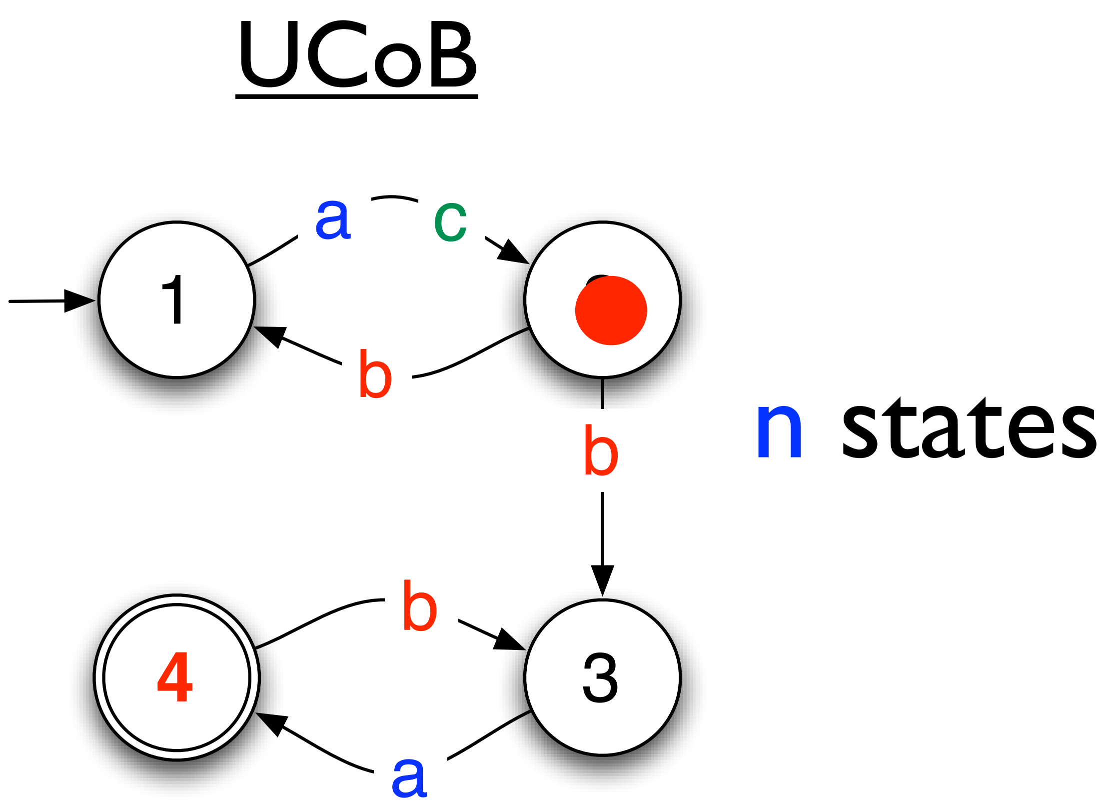
UCoB



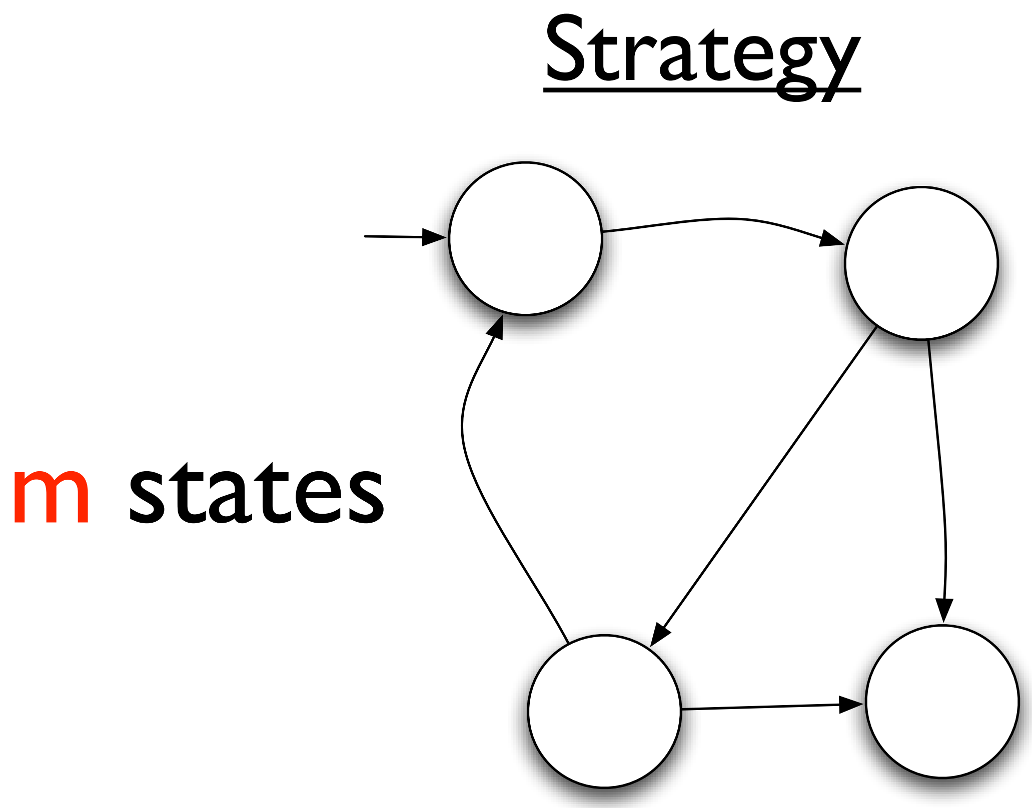
n states

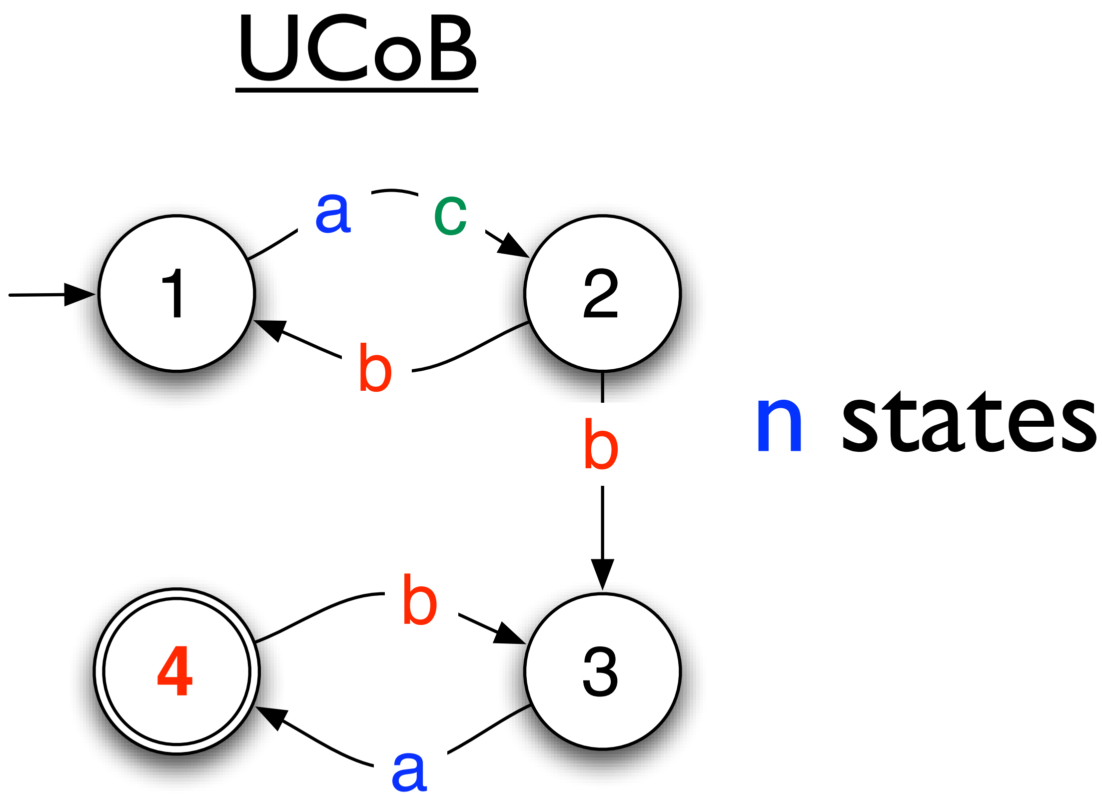
From UCoB to UKCoB





From UCoB to UKCoB

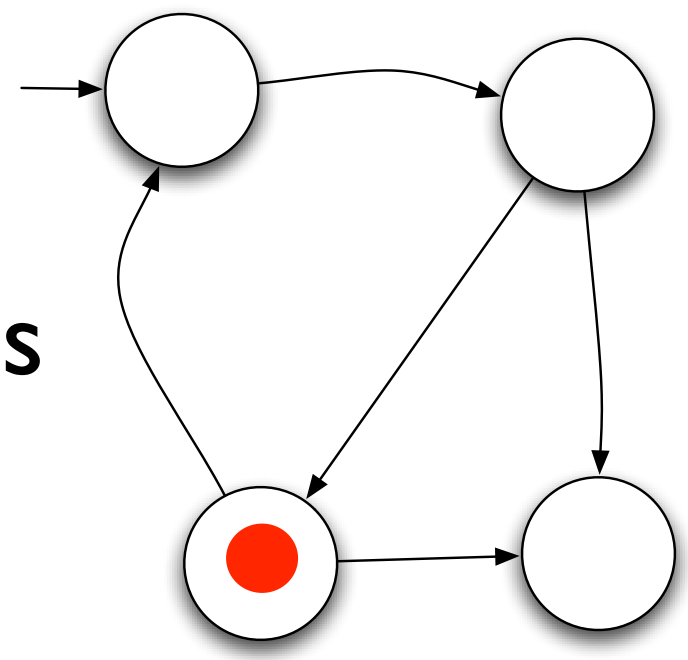




From UCoB to UKCoB

Strategy

m states



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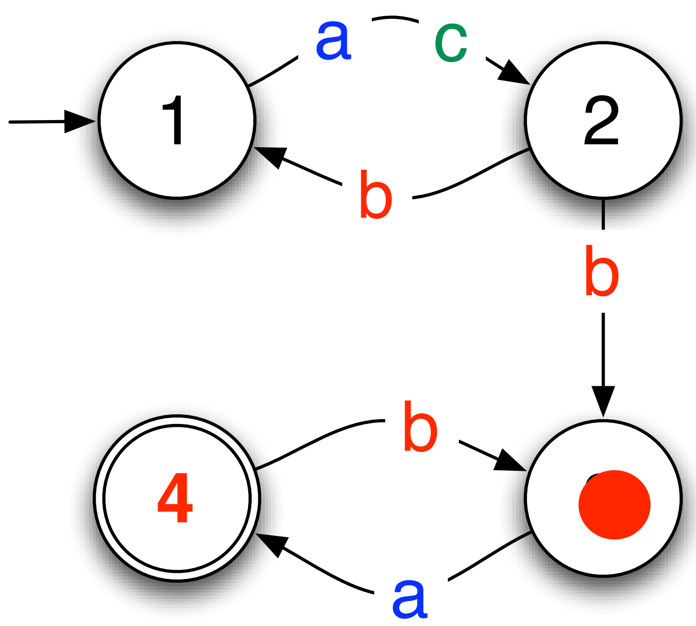
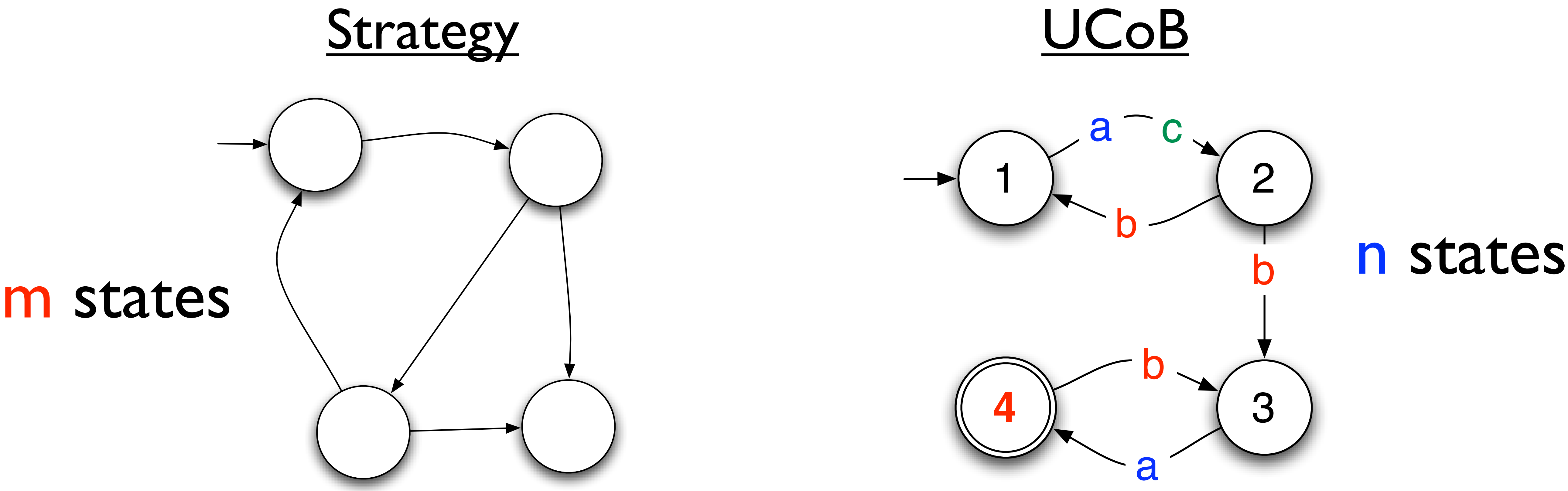


n states

From UCoB to UKCoB



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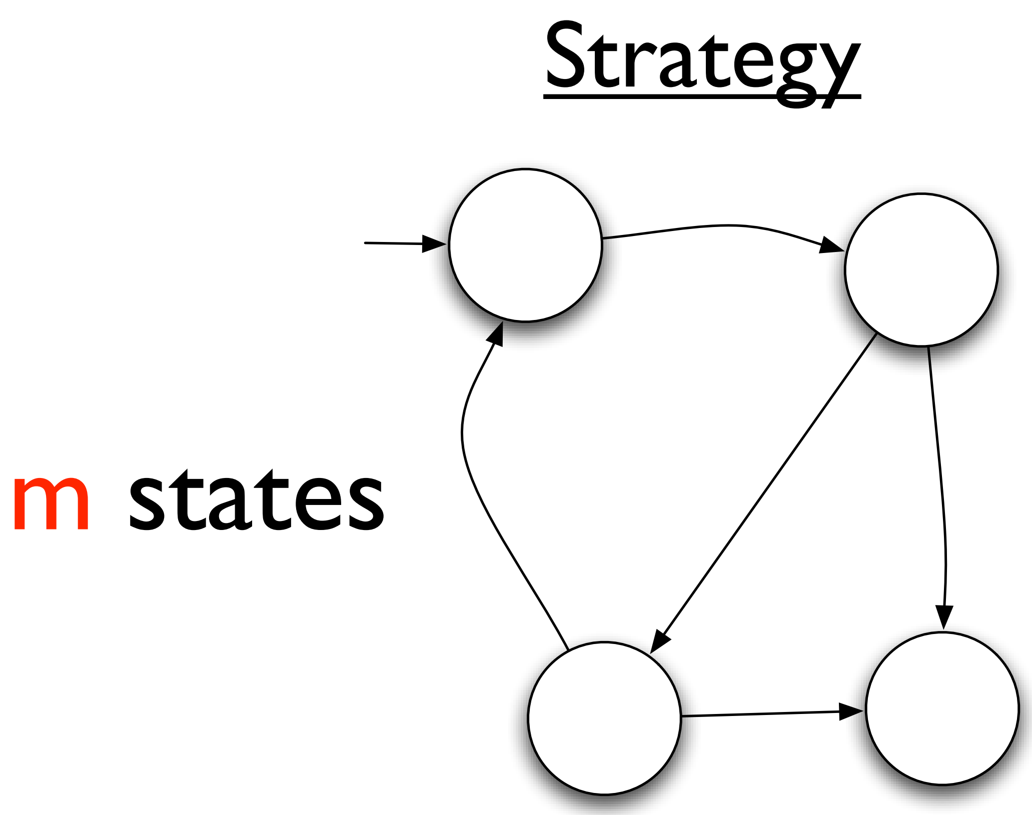
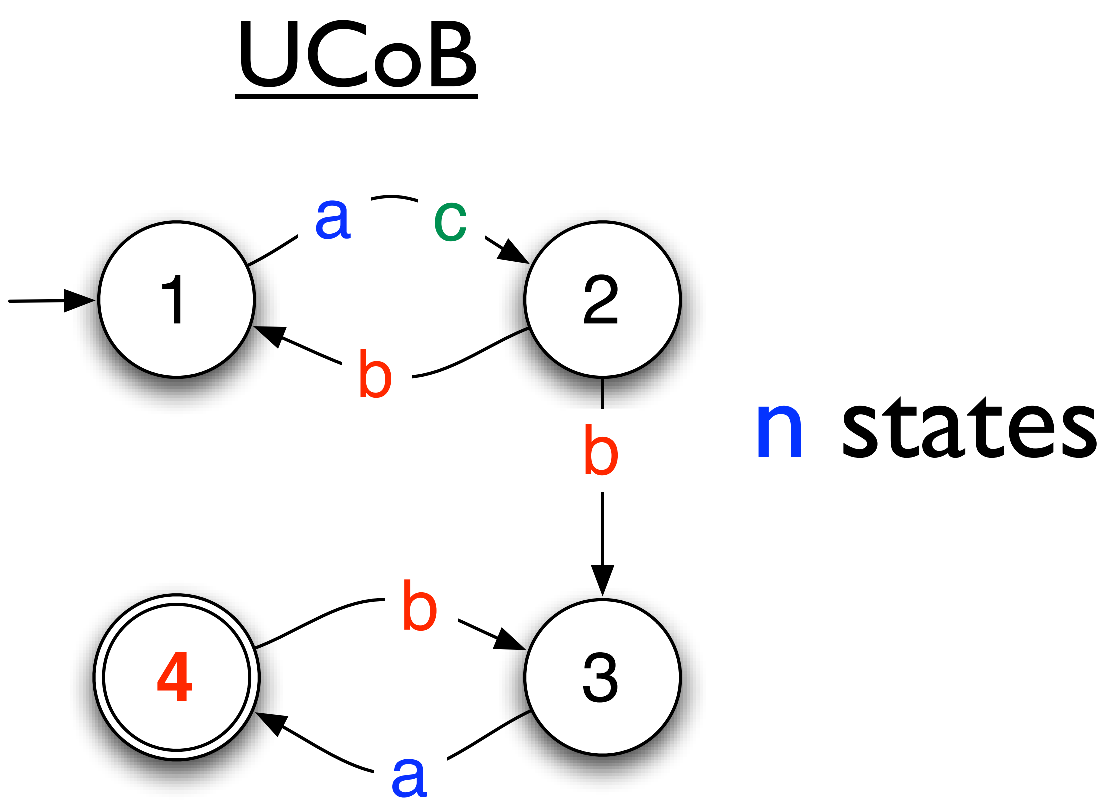


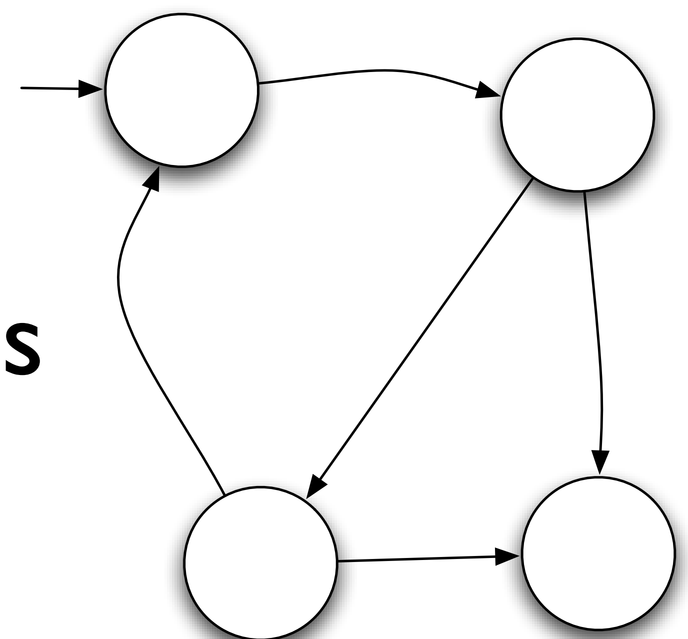


Assume the strategy lets us visit an accepting state more than n×m times

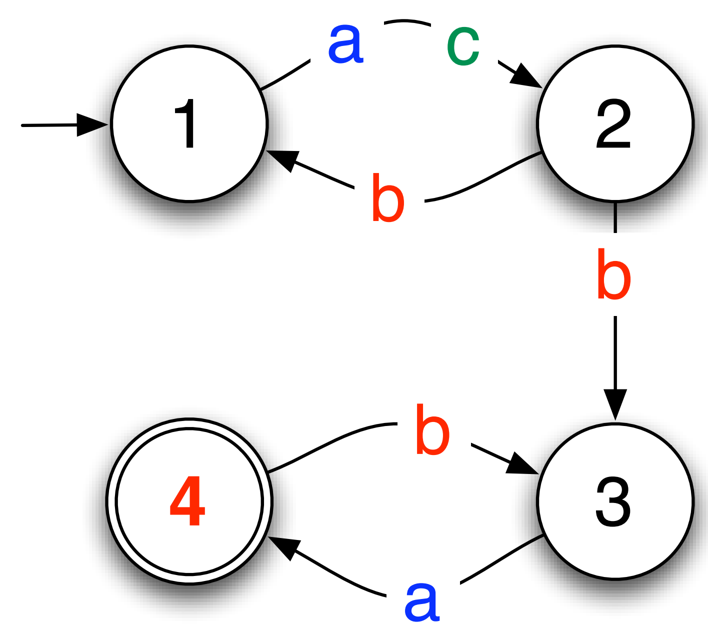
From UCoB to UKCoB

Strategy

m states



UCoB



n states

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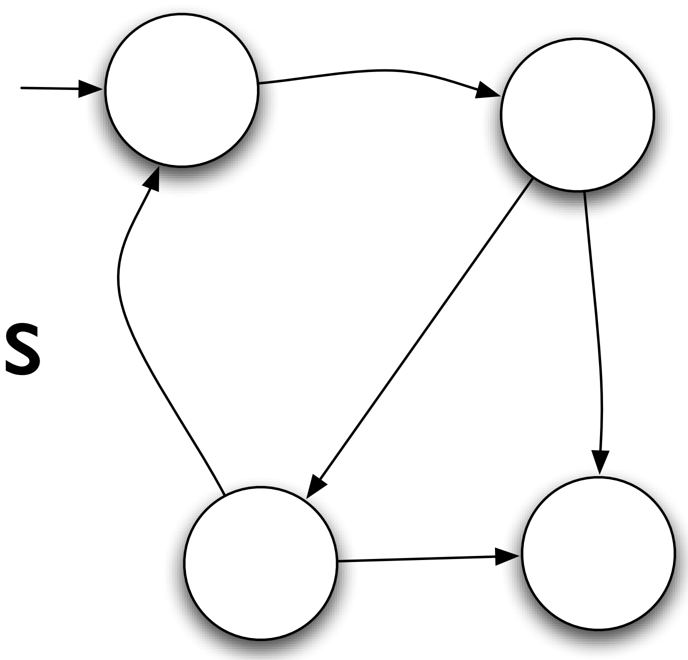
☞ cycle in the product of the strategy and the UCoB

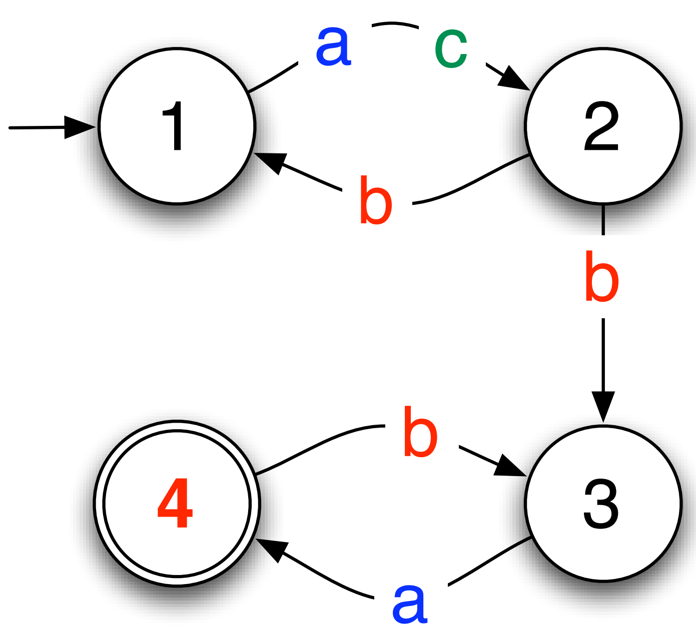
From UCoB to UKCoB

Strategy



m states

UCoB



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From UCoB to UKCoB

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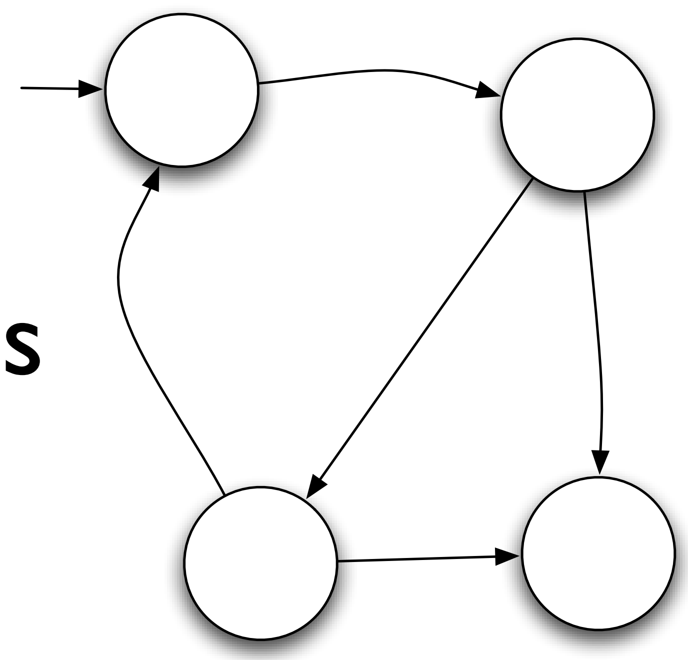
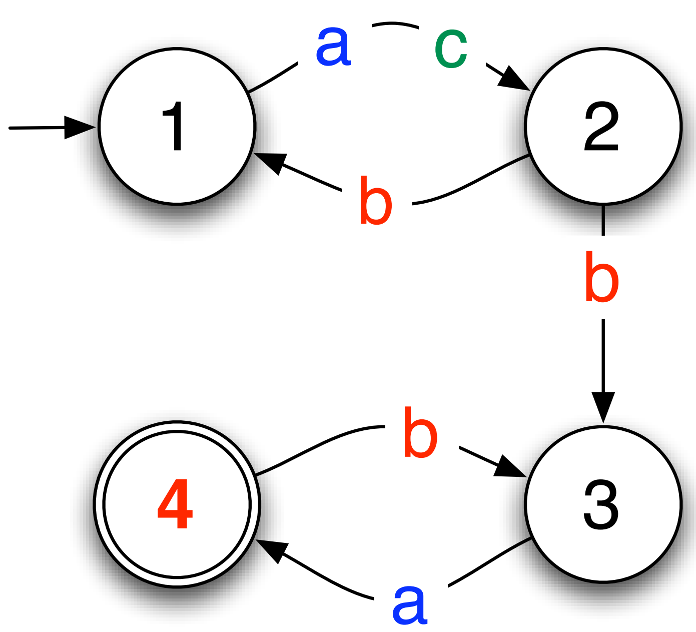


m states

UCoB



n states

Assume the strategy lets us visit an accepting state more than n×m times

☞ cycle in the product of the strategy and the UCoB

☞ accepting states are visited infinitely often

☞ the strategy is **not winning**

From UCoB to UKCoB

Theorem: Player 1 has a winning strategy in
$\langle \Sigma_1, \Sigma_2, L_{UcoB}(A_\varphi) \rangle$

iff

she has a winning strategy in
$\langle \Sigma_1, \Sigma_2, L_{UKcoB}(A_\varphi) \rangle$ for $K=n \times m$

We can thus solve the game by playing with the (weaker) **K-Co-Büchi** acceptance condition

K-Co-Büchi = avoid visiting accepting states too often
= **safety condition** !

Incremental procedure

Theorem: **If** Player 1 has a winning strategy in

$\langle \Sigma_1, \Sigma_2, L_{UKcoB}(A_\varphi) \rangle$

then

she has a winning strategy in

$\langle \Sigma_1, \Sigma_2, L_{UK'coB}(A_\varphi) \rangle$ for $K' \geq K$

Incremental procedure

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```
i := 0
While(true)
  If P1 wins on LUicoB(Aφ) return «win»
  Else if P2 wins on LUicoB(A¬φ) return «lose»
  Else i:=i+1
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Incremental procedure

Theorem: If Player I has a winning strategy in

Each step can be computed by **solving** a safety game

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Incremental procedure

Theorem: **If** Player I has a winning strategy in

she has a winning strategy in

$\langle \Sigma_I,$

Each step can be computed by **solving** a safety game

In practice this algorithm **might terminate** with **small** values of **i**

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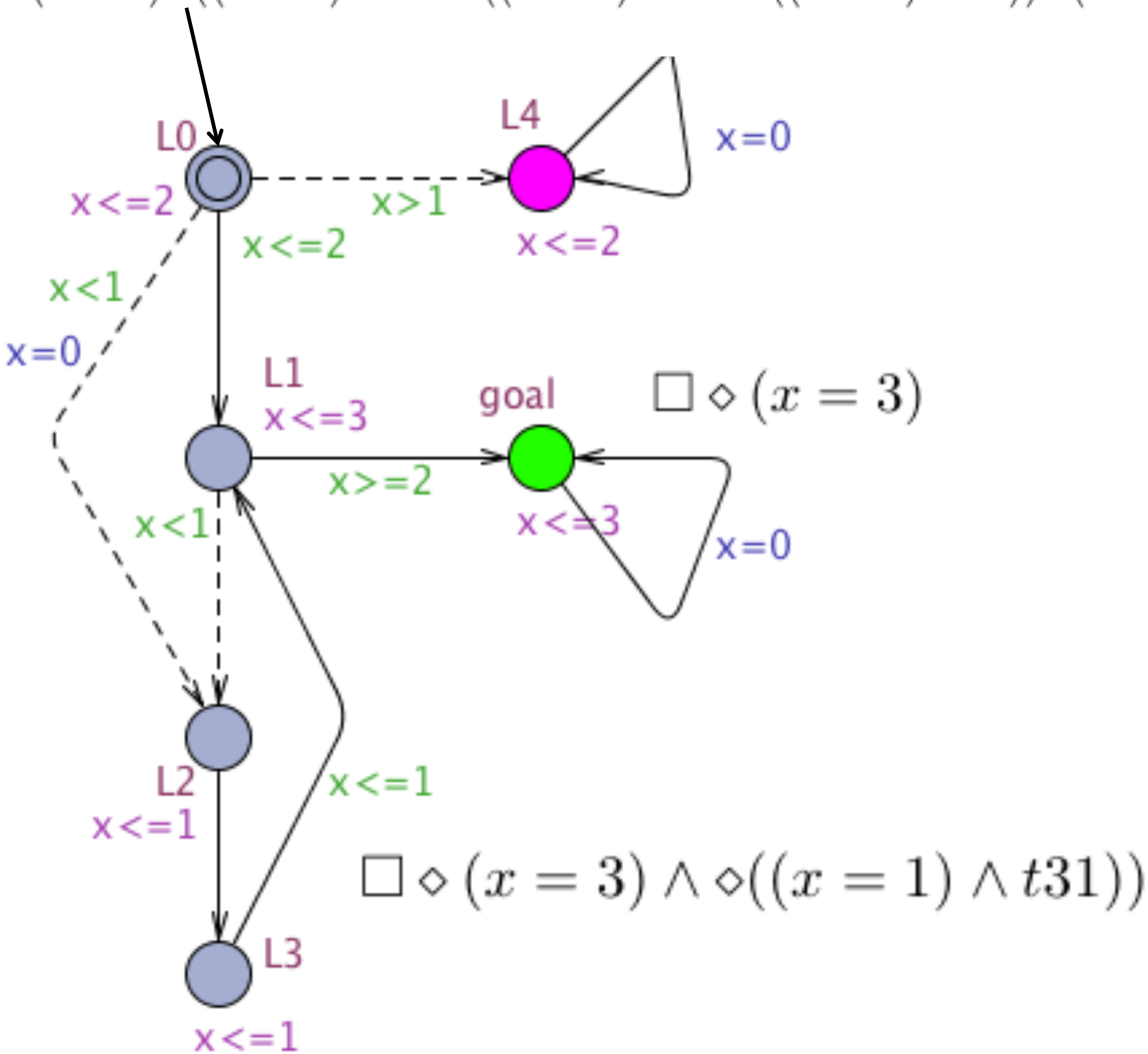
Initial example

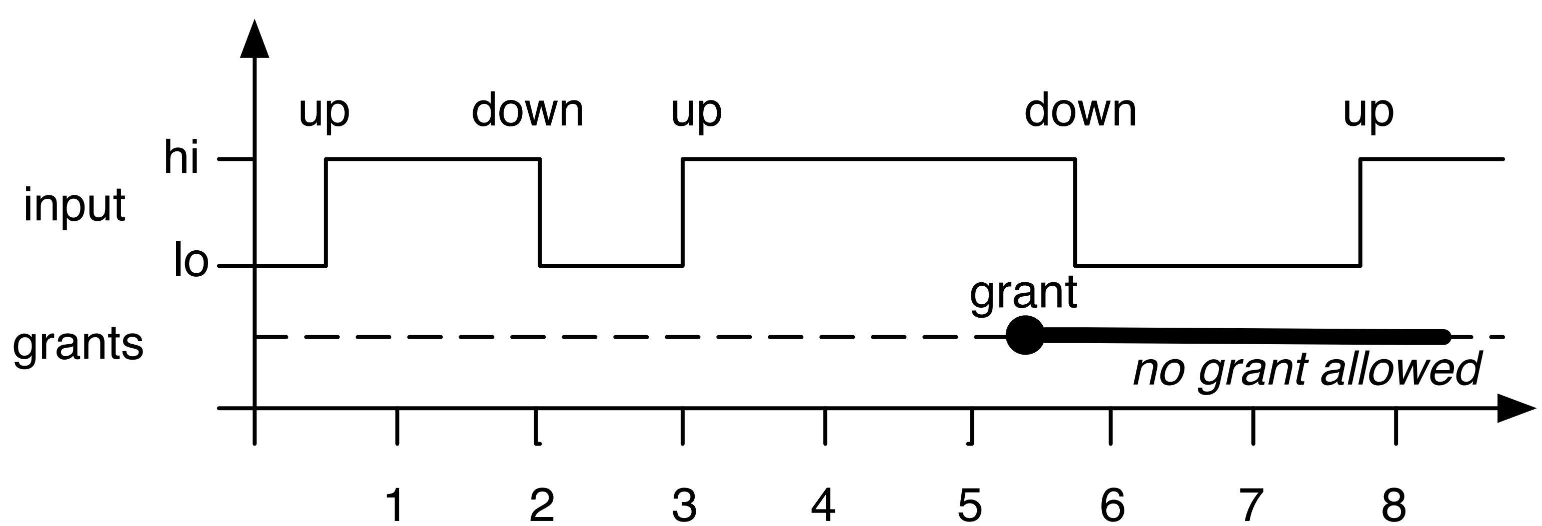
$\Box\Diamond(x=3)\wedge((x<1)\wedge t02\wedge\Diamond((x=1)\wedge t23\wedge\Diamond((x=1)\wedge t31))\vee(t01\wedge(x=1))\wedge\Diamond((x=1)\mathcal{U}(t1g\wedge(x=2))))$



Example

$\Sigma_1 = \{grant\}$, $\Sigma_2 = \{up, down\}$

$$\text{Hyp} \equiv \Box\Big(up \rightarrow \big(\neg down\,\mathcal{U}(down \wedge \lhd_{\geq 1}\, up)\big)\Big) \wedge$$
$$\Box\Big(down \rightarrow \big(\neg up\,\mathcal{U}(up \wedge \lhd_{\geq 1}\, down)\big)\Big)$$
$$\text{Req}_1 \equiv \Box\Big(\big(down \wedge \lhd_{>2}\, up\big) \rightarrow \big(\neg up\,\mathcal{U}\, grant\big)\Big)$$
$$\text{Req}_2 \equiv \Box\big(grant \rightarrow \neg \lhd_{<3}\, grant\big)$$



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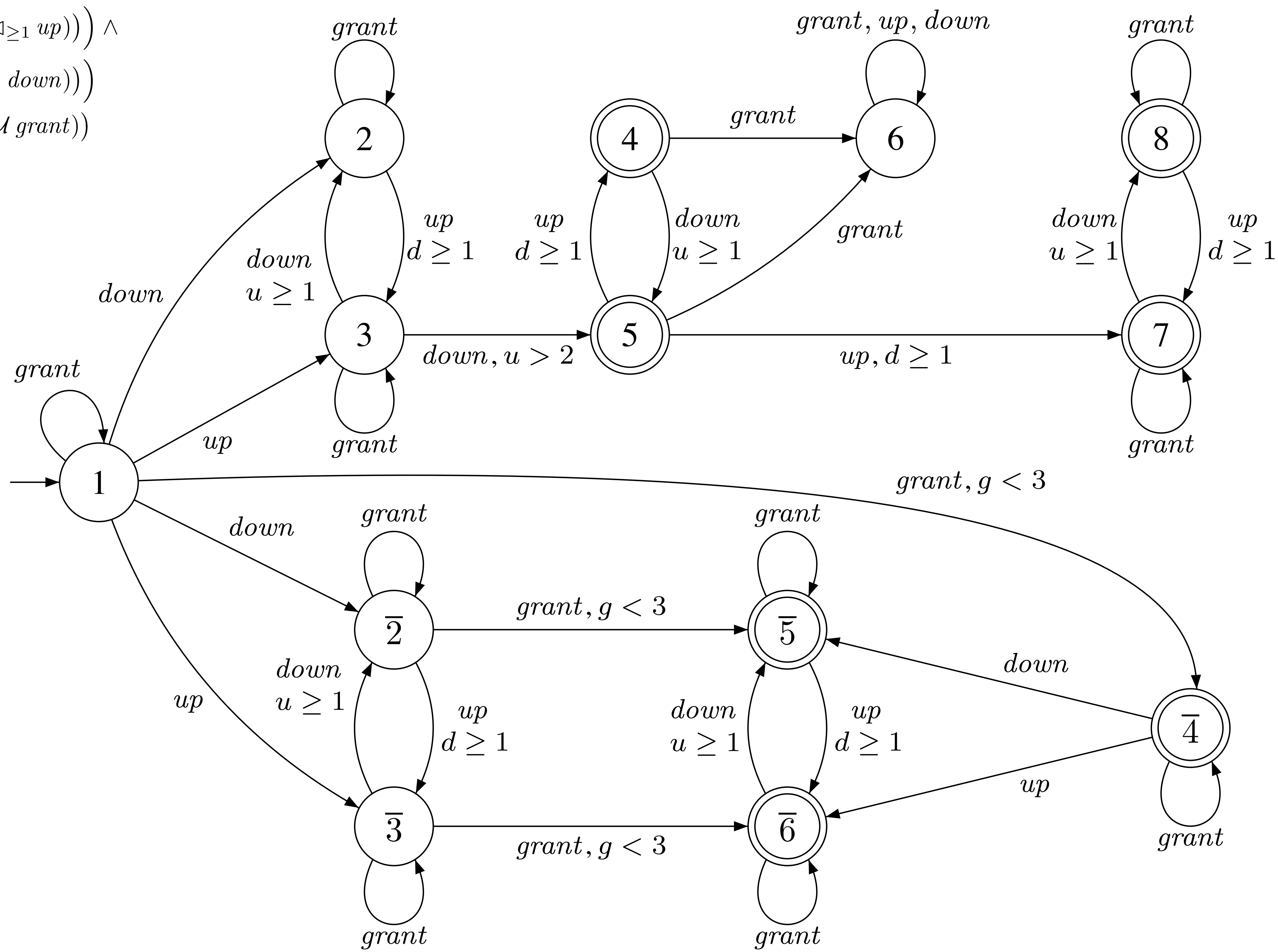
$\quad \Box\Big(down \rightarrow \big(\neg up\, \mathcal{U}(up \wedge \lhd_{\geq 1}\, down)\big)\Big)$

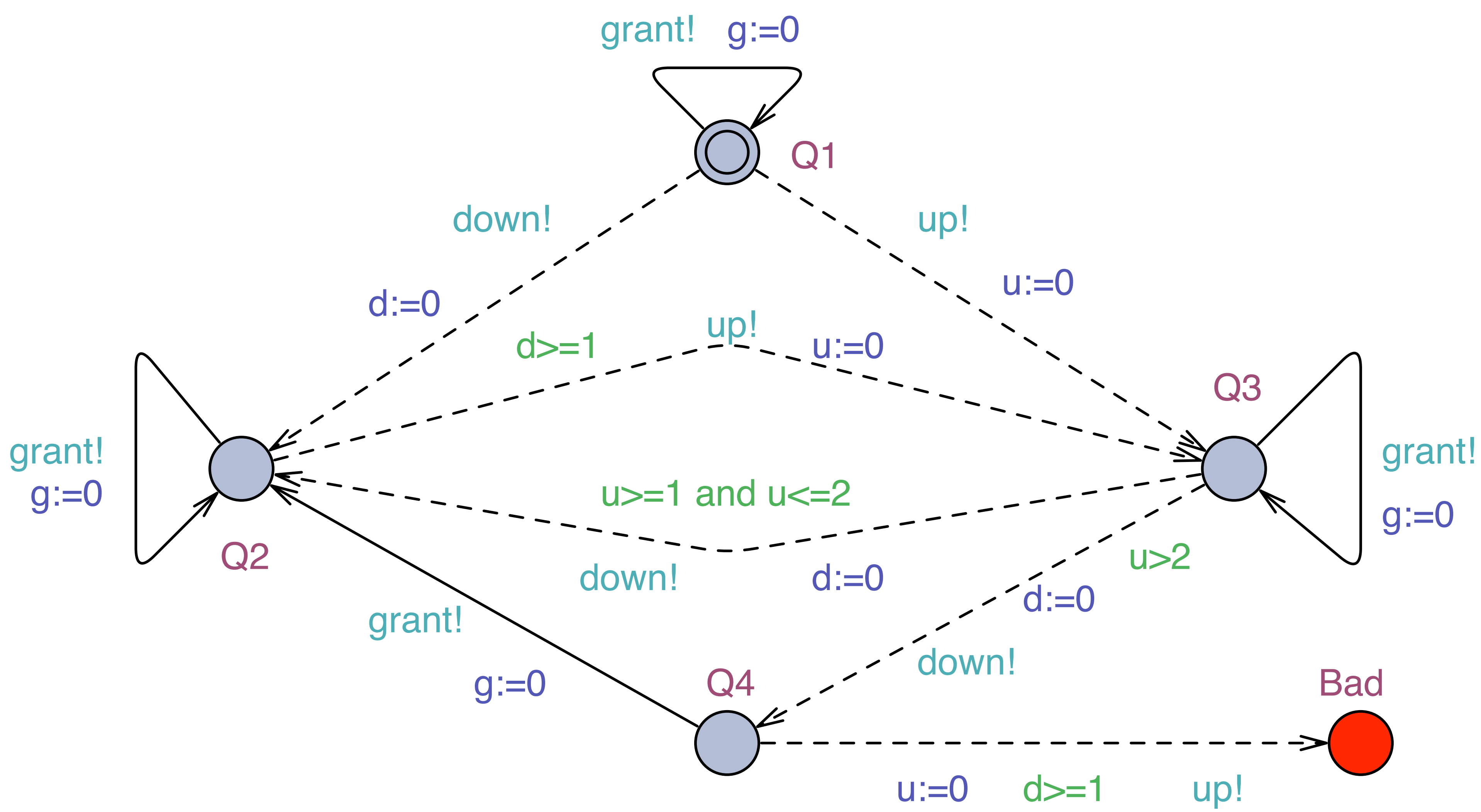
$\mathsf{Req}_1 \equiv \Box\big((down \wedge \lhd_{>2}\, up) \rightarrow (\neg up\, \mathcal{U}\, grant)\big)$

$\mathsf{Req}_2 \equiv \Box(grant \rightarrow \neg \lhd_{<3}\, grant)$

using extension of classical constructions

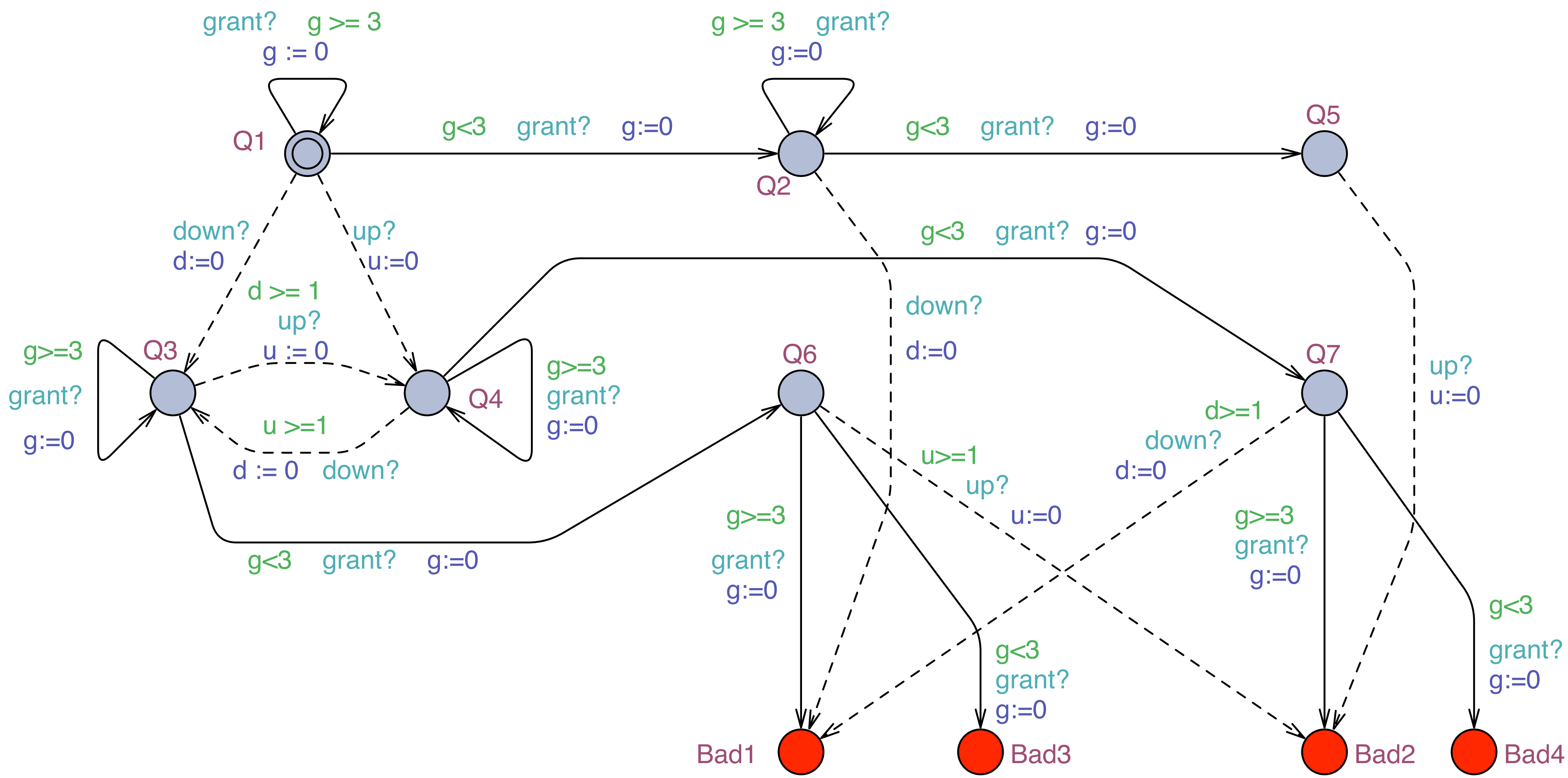


Illustration



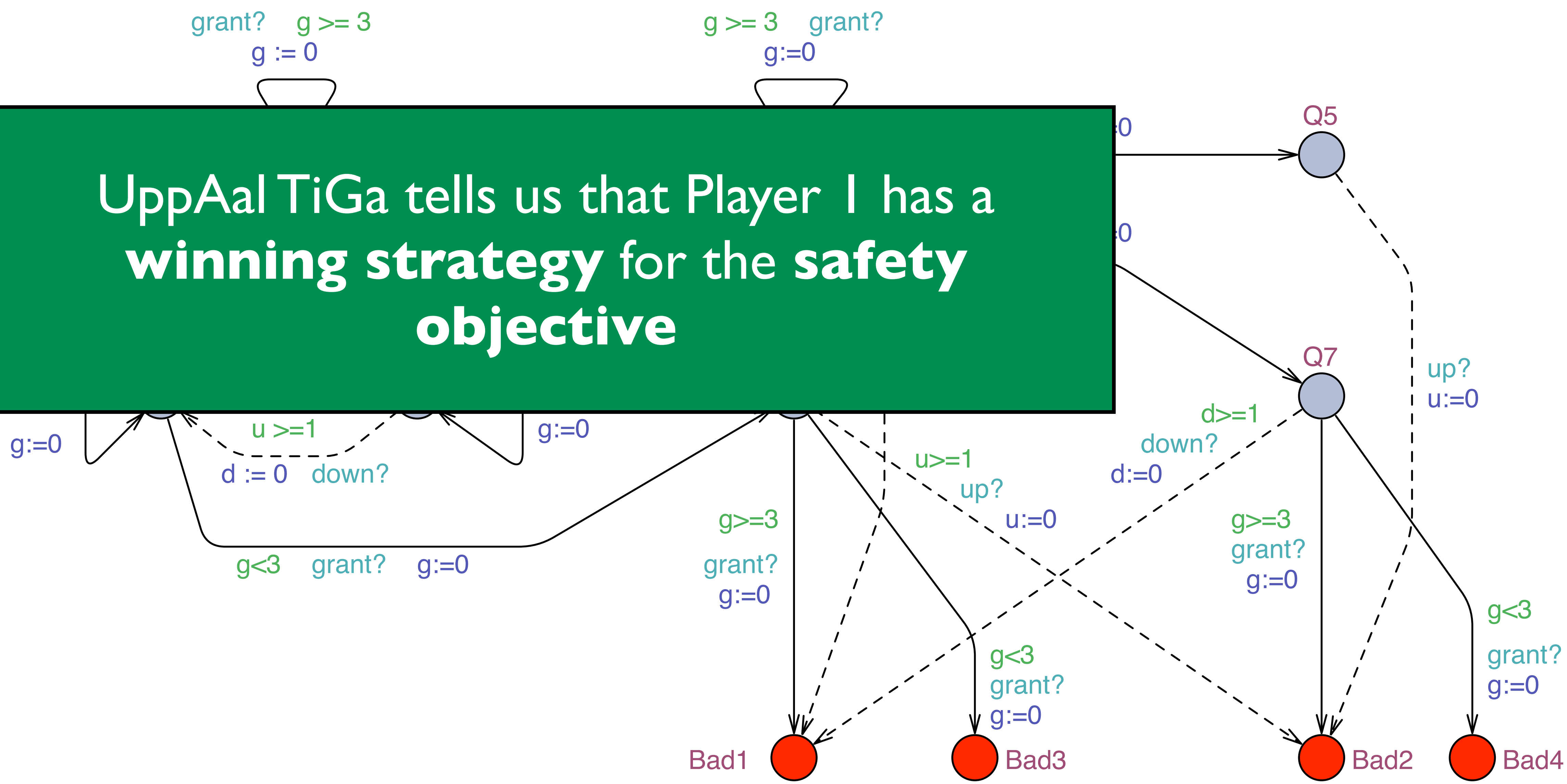
For K=1

Illustration



For K=1

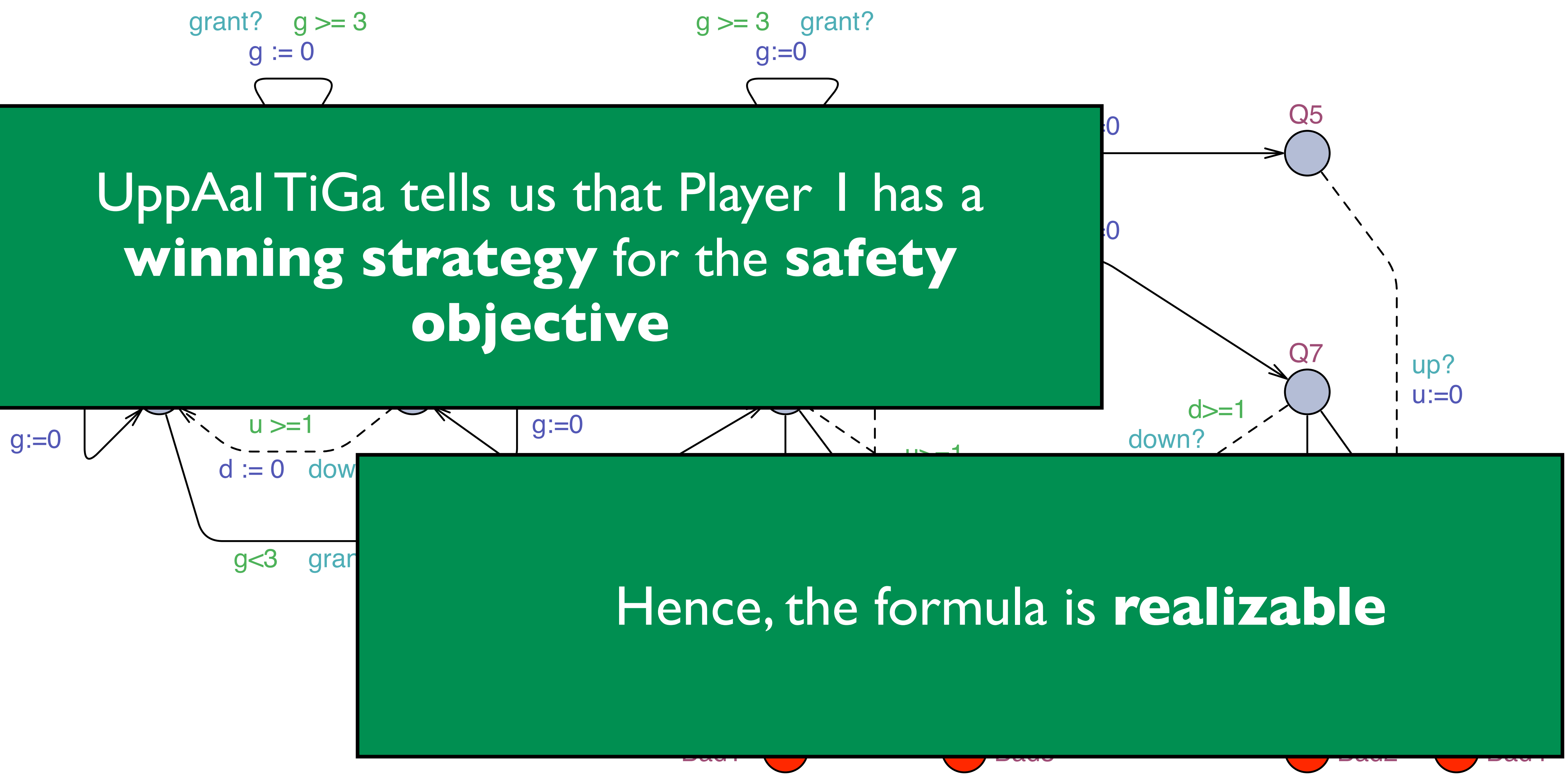
Illustration



UppAal TiGa tells us that Player 1 has a **winning strategy** for the **safety objective**

For K=1

Illustration



UppAal TiGa tells us that Player 1 has a **winning strategy** for the **safety objective**

Hence, the formula is **realizable**

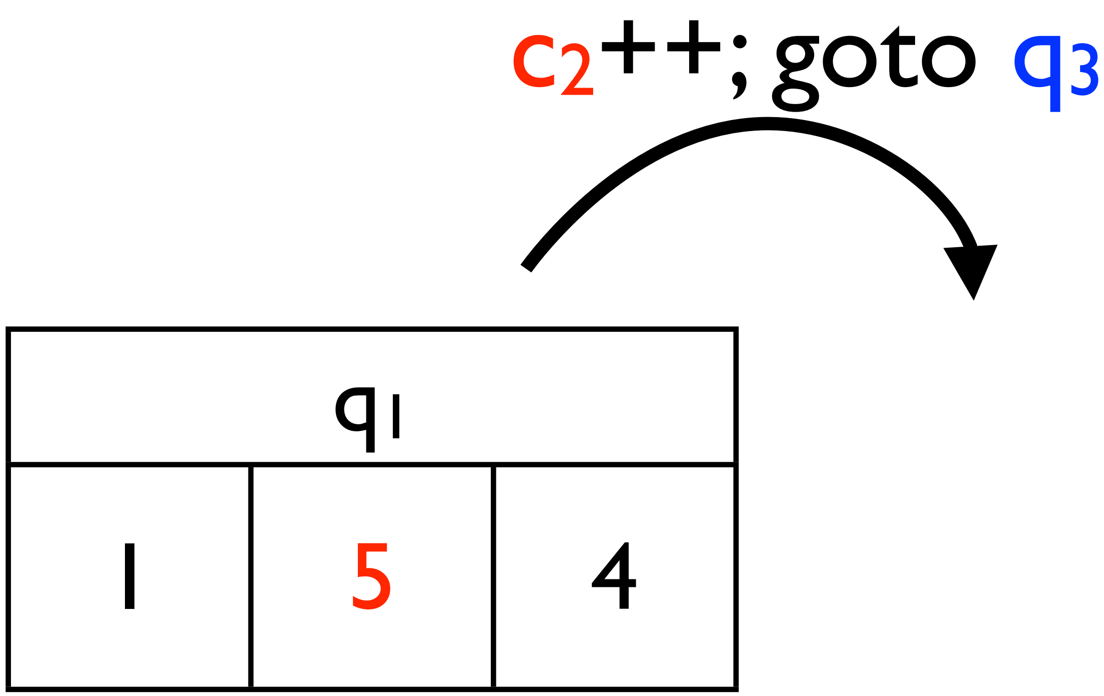
For K=1

Questions ?

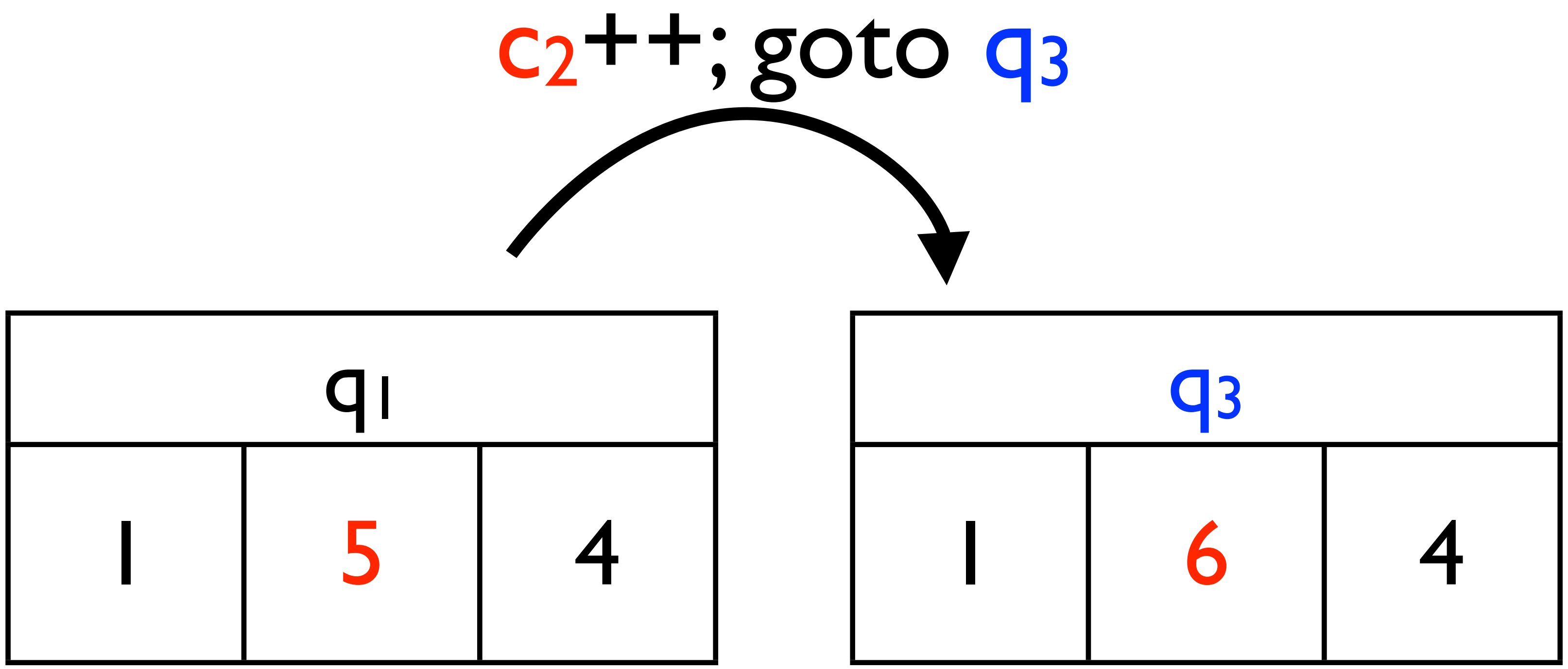
Counter machine - run

q_1		
1	5	4

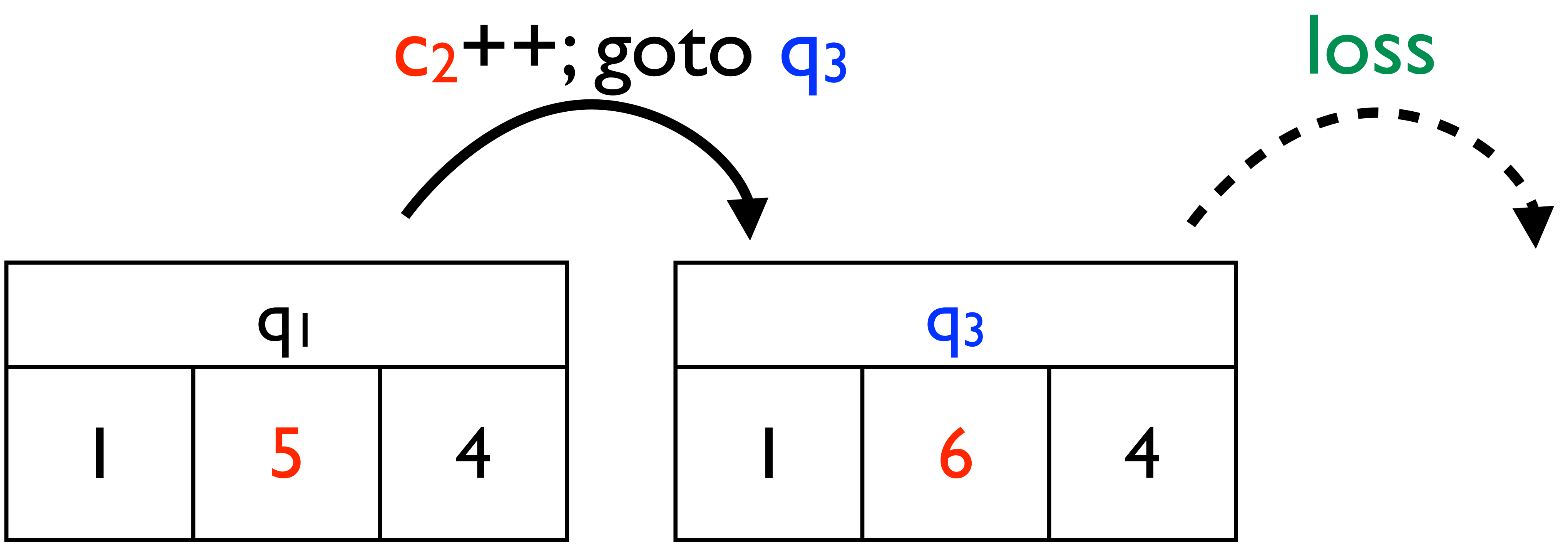
Counter machine - run



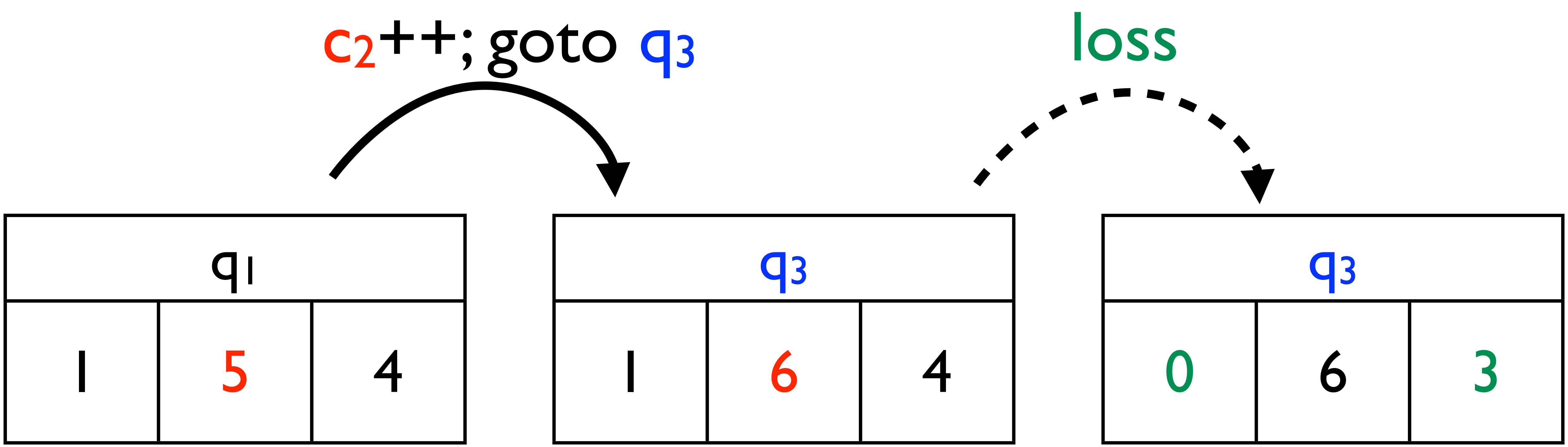
Counter machine - run



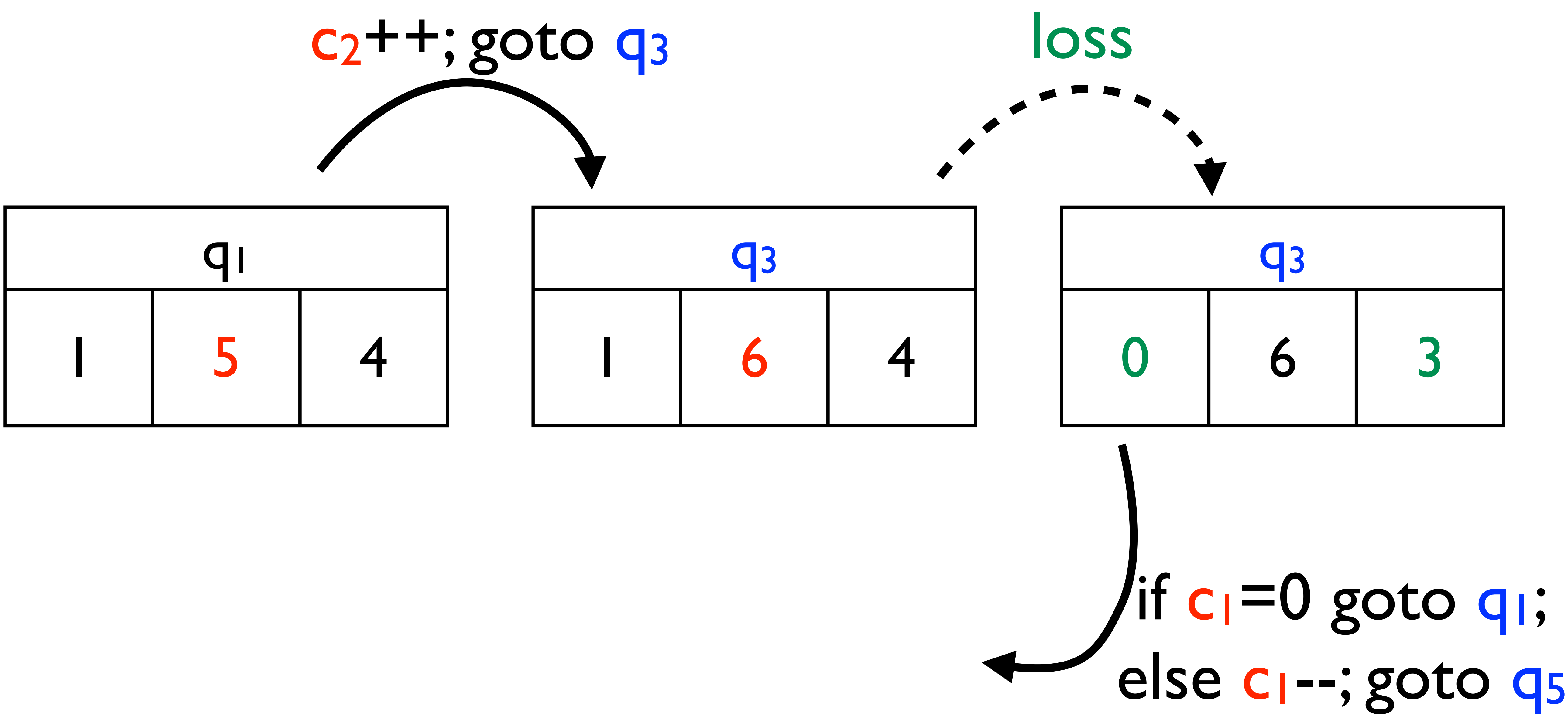
Counter machine - run



Counter machine - run



Counter machine - run



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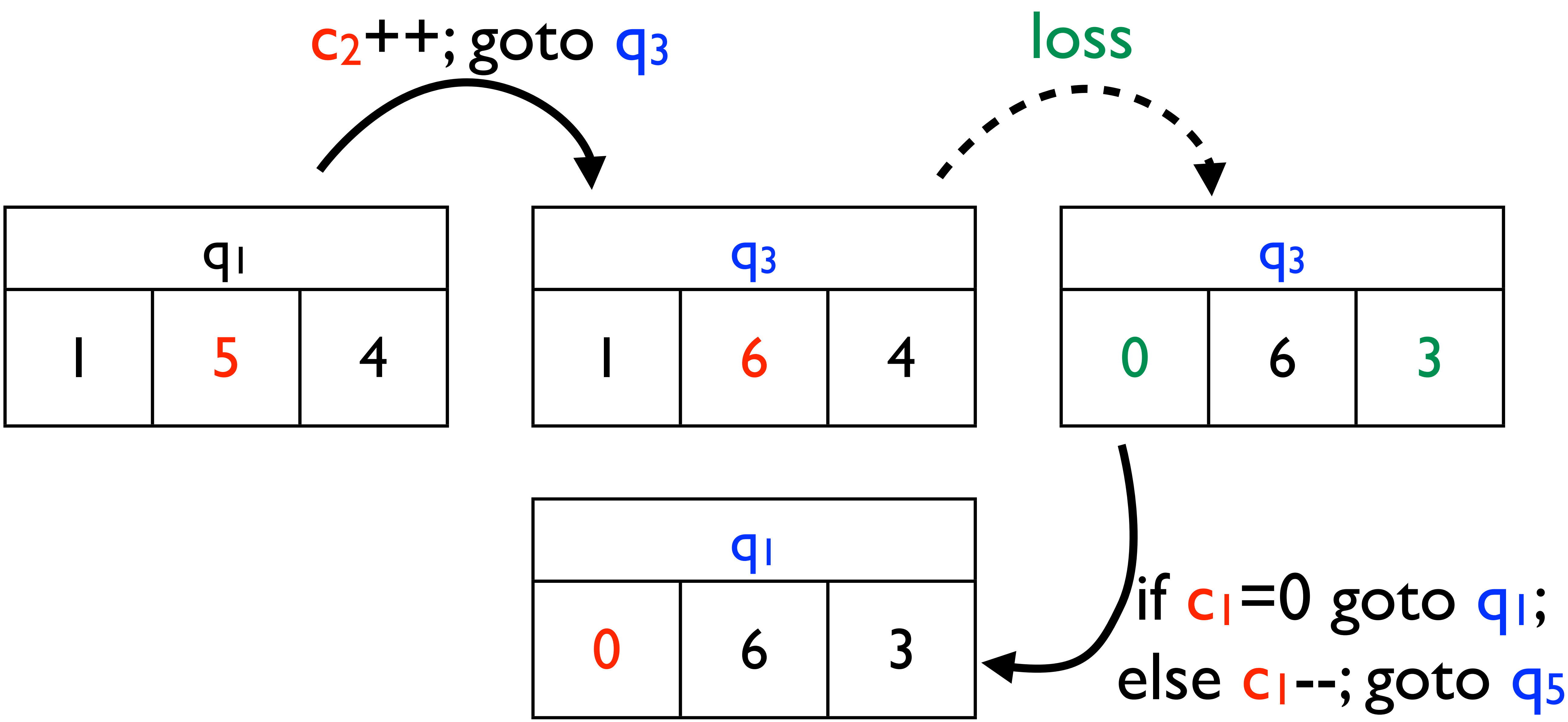

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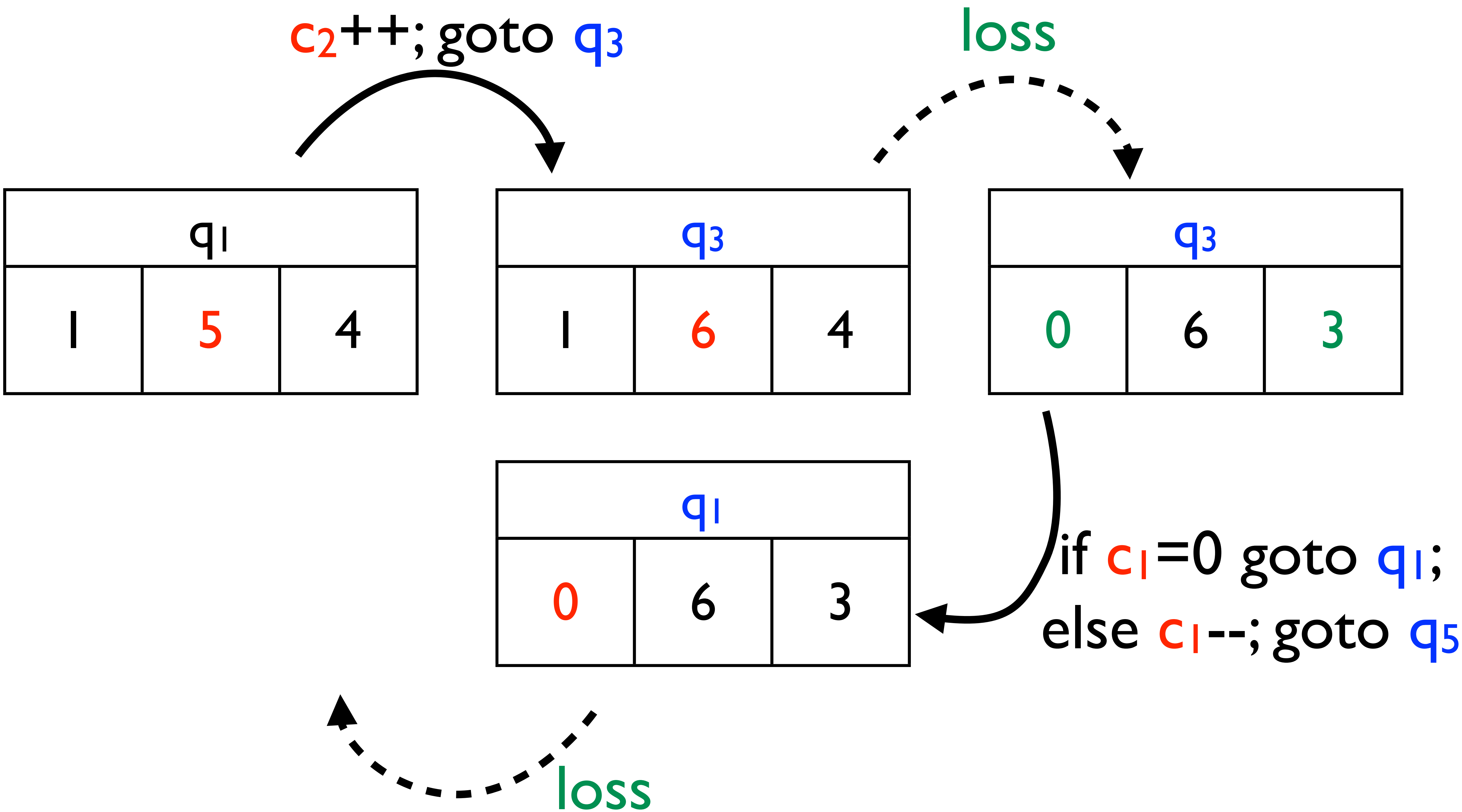

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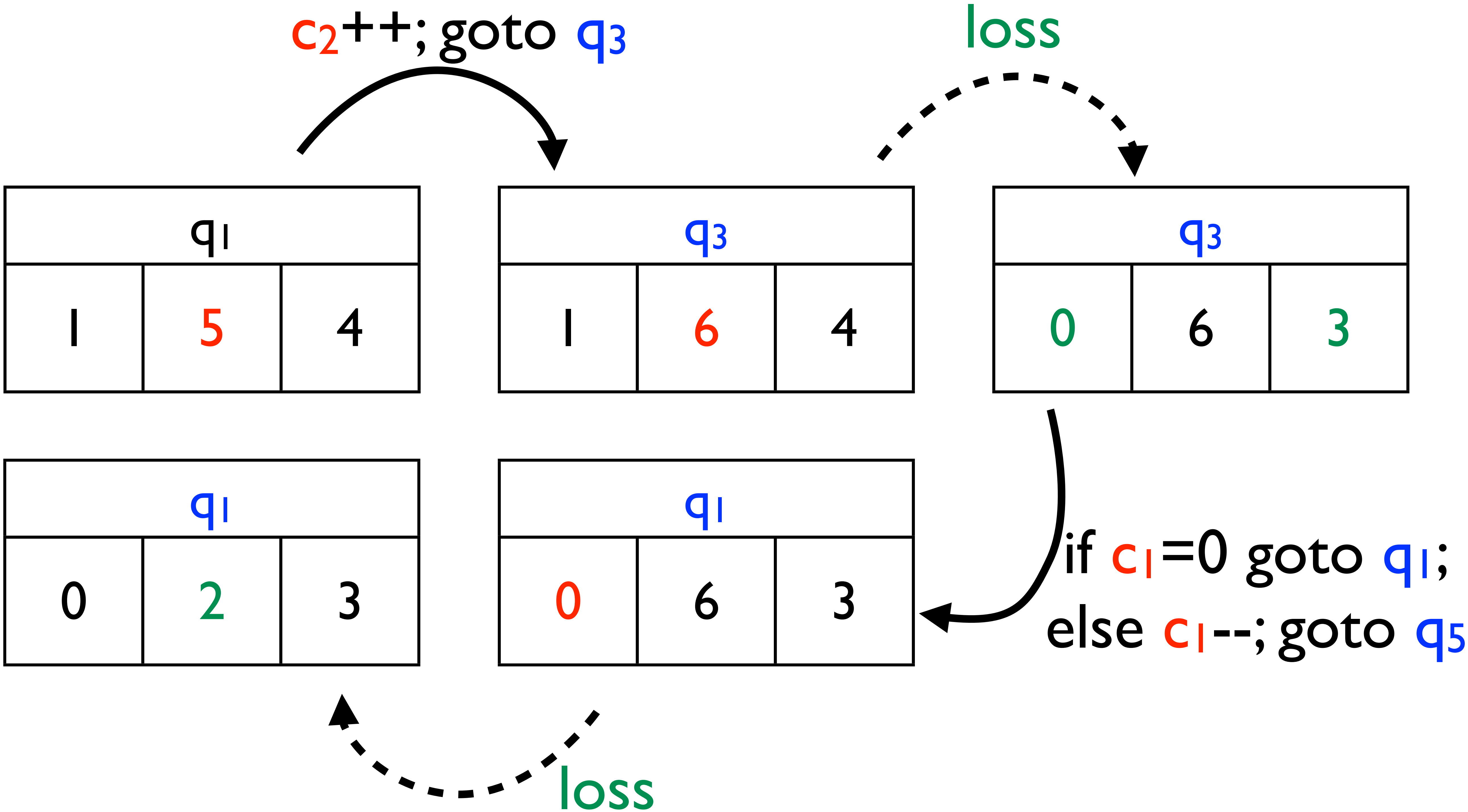

Encoding runs

q		
1	3	0

Encoding runs

q		
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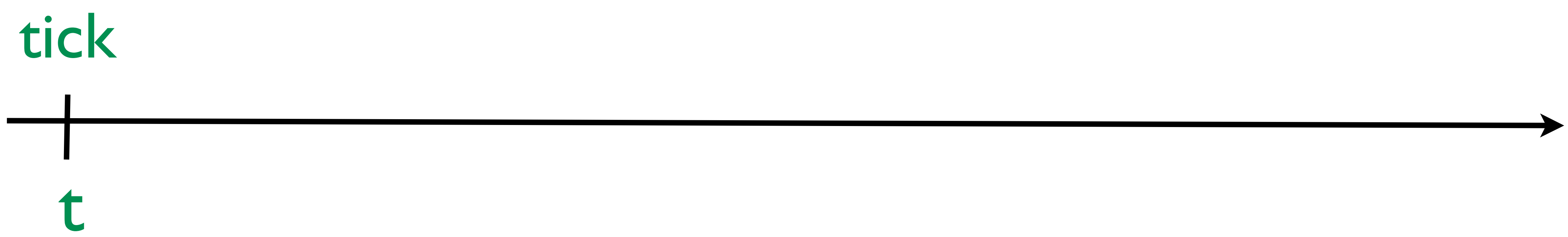

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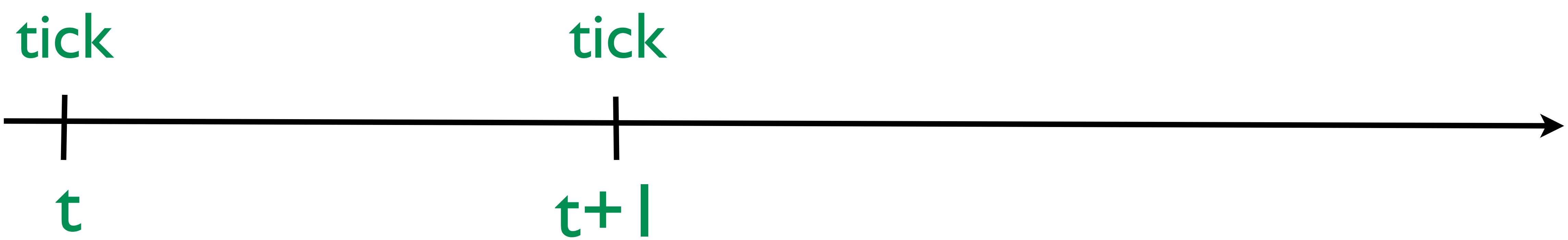

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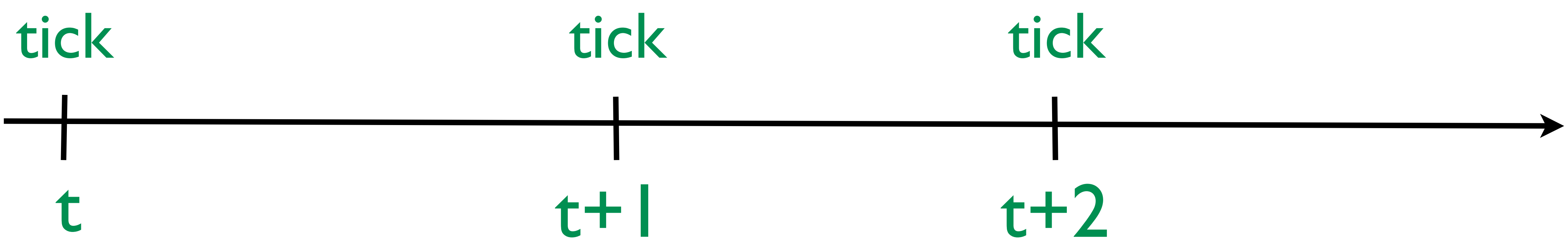

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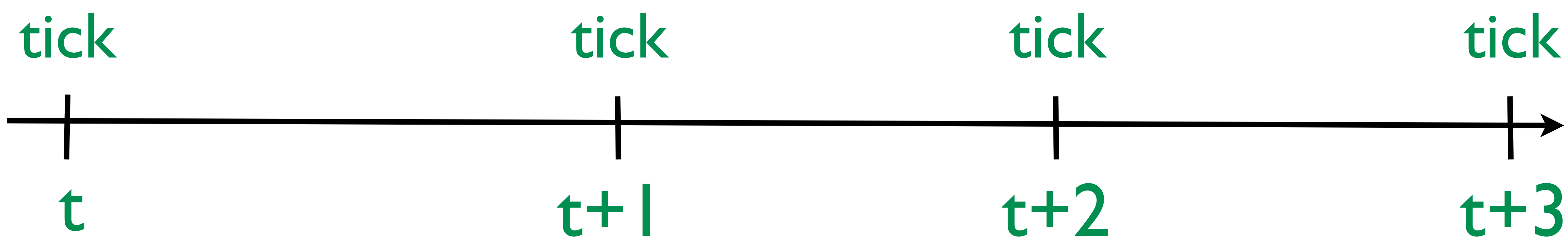

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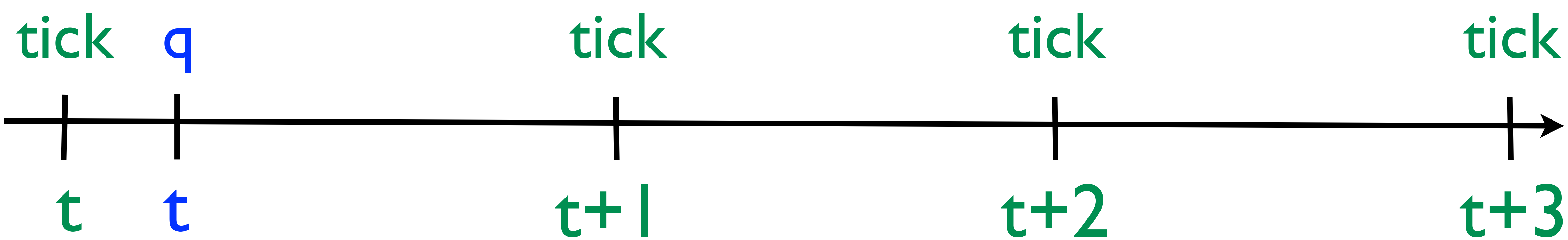

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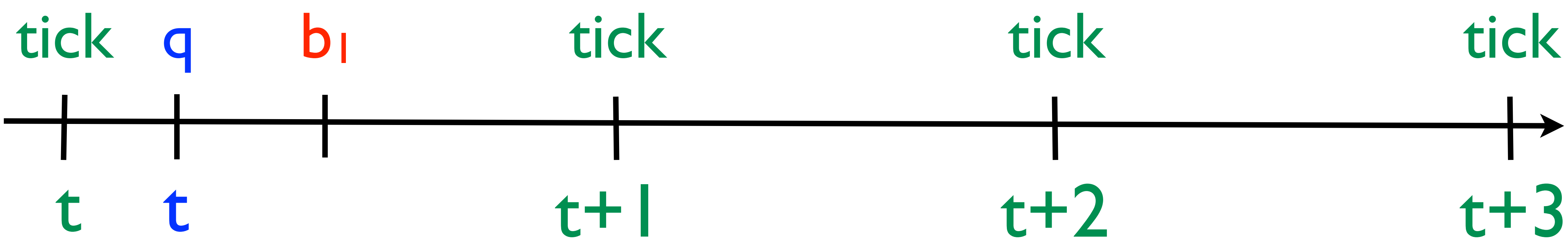

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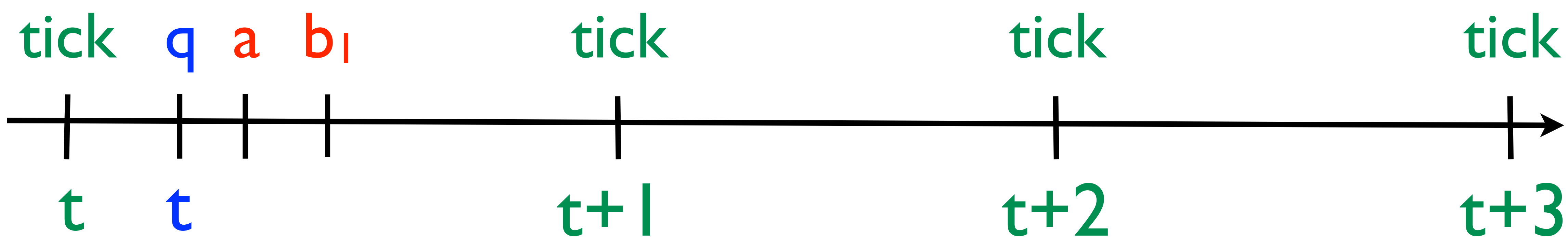

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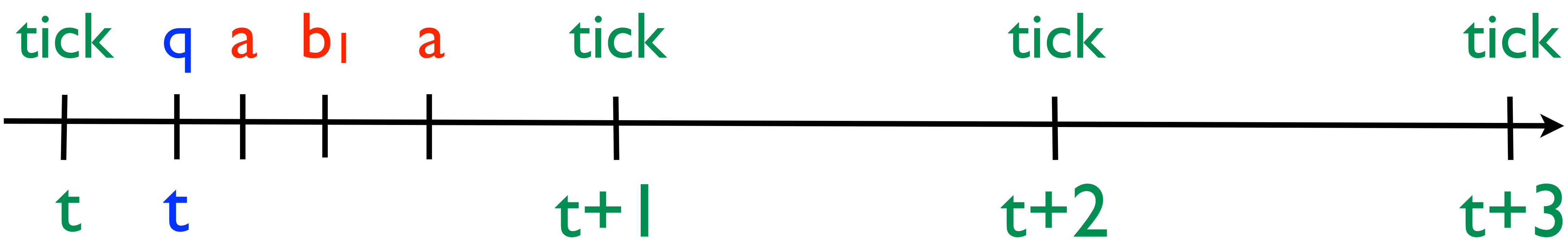

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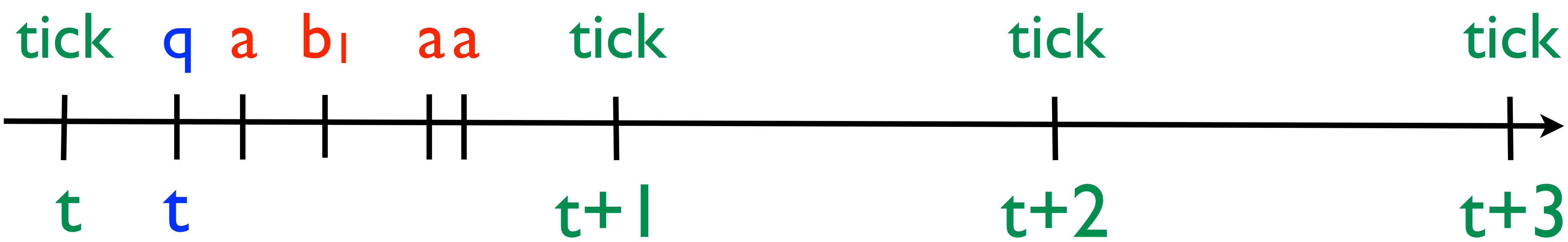

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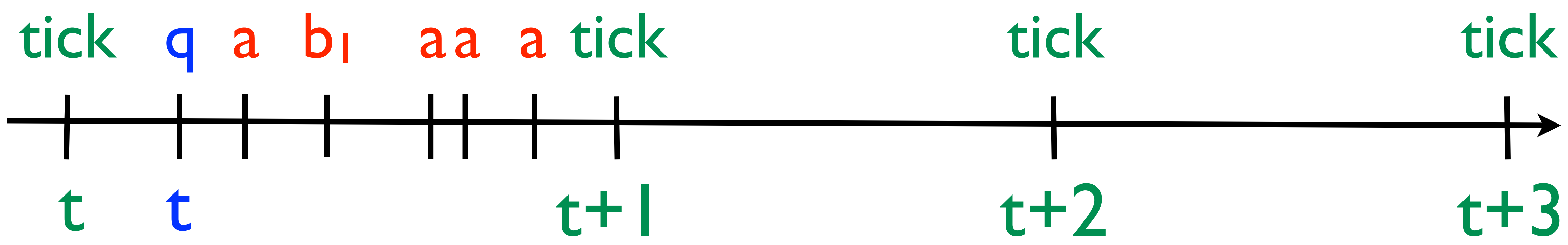

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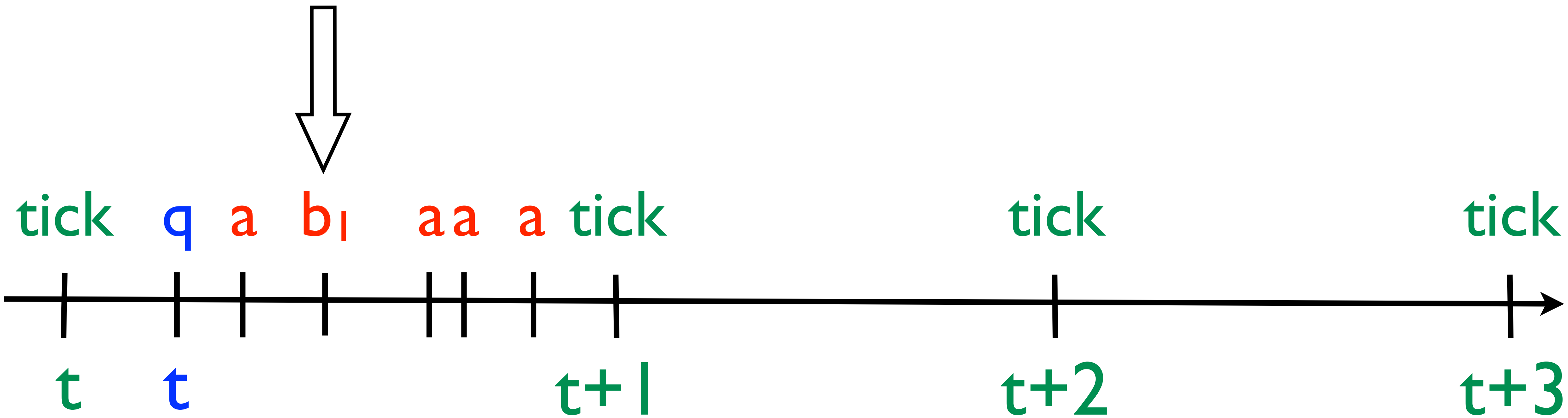

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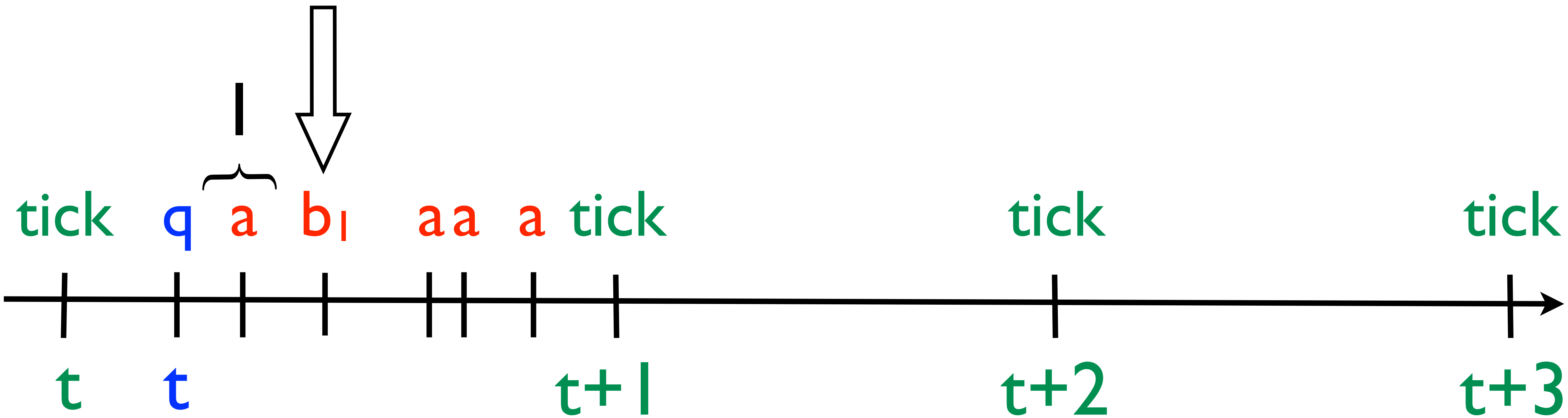

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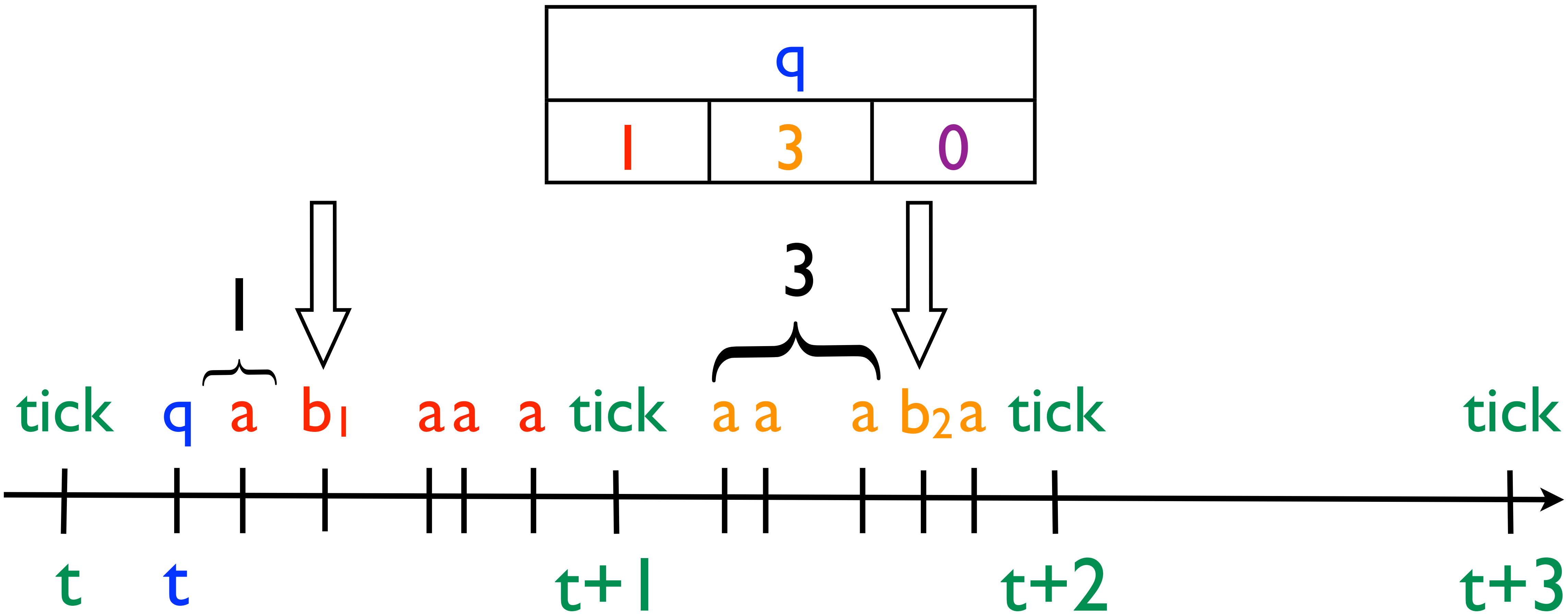

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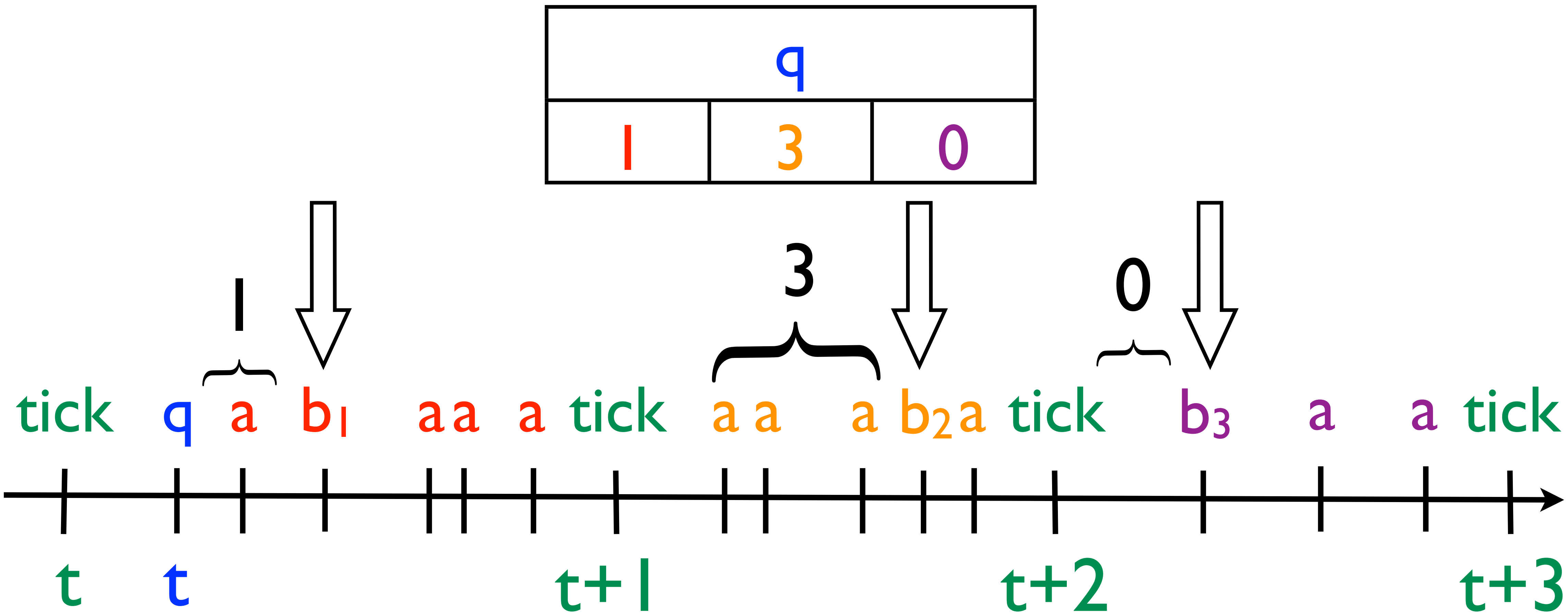

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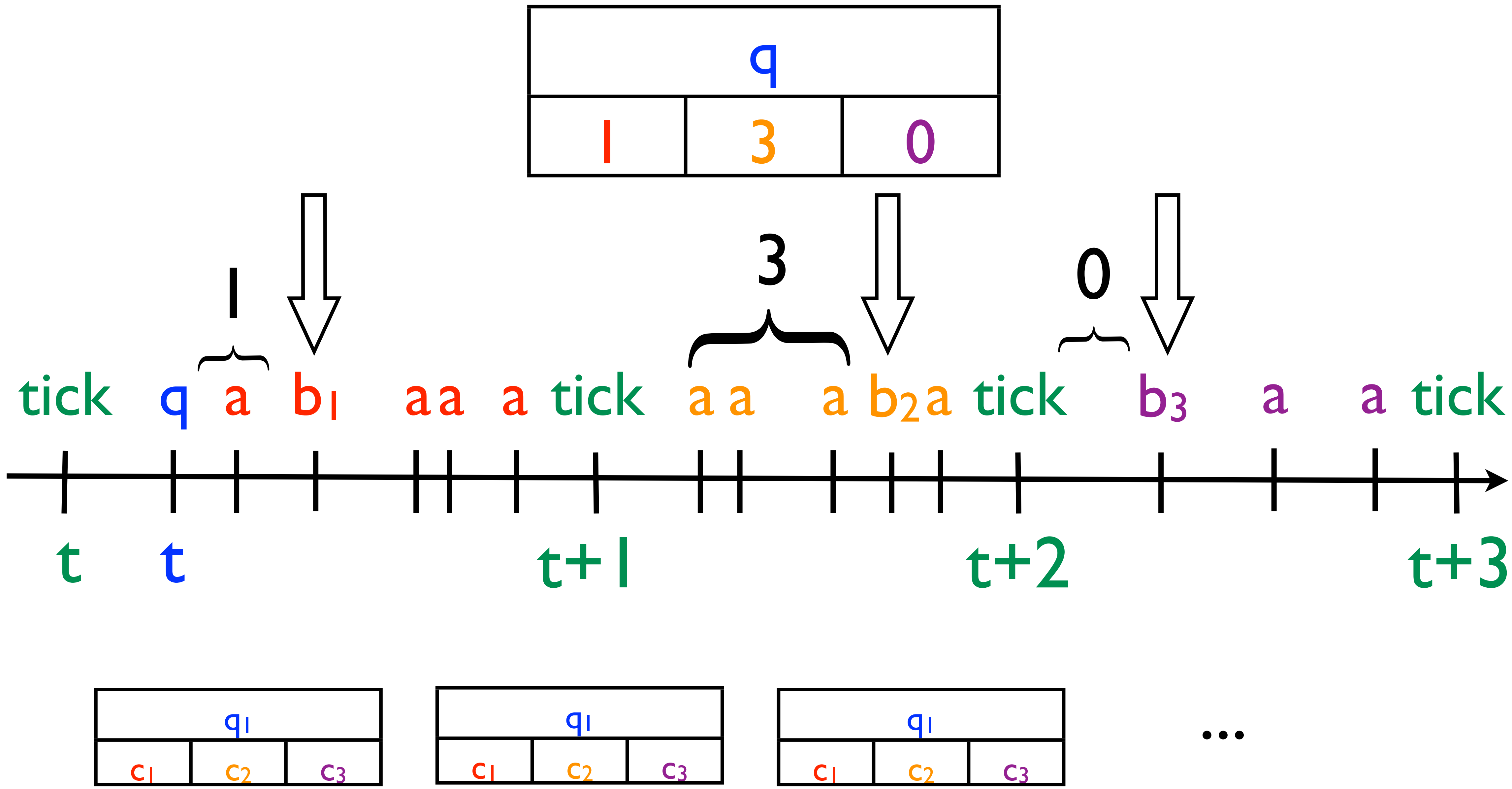
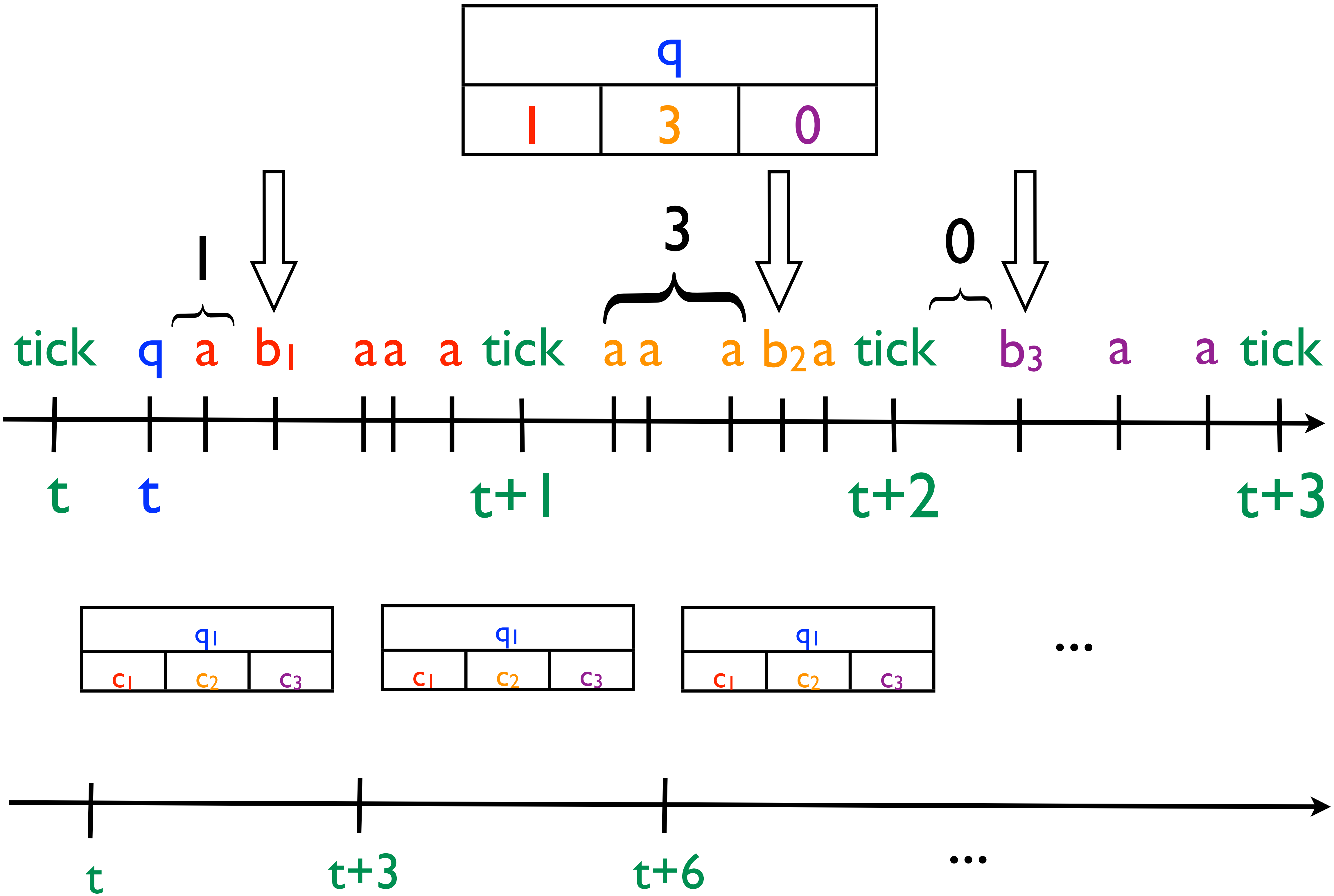


Encoding runs



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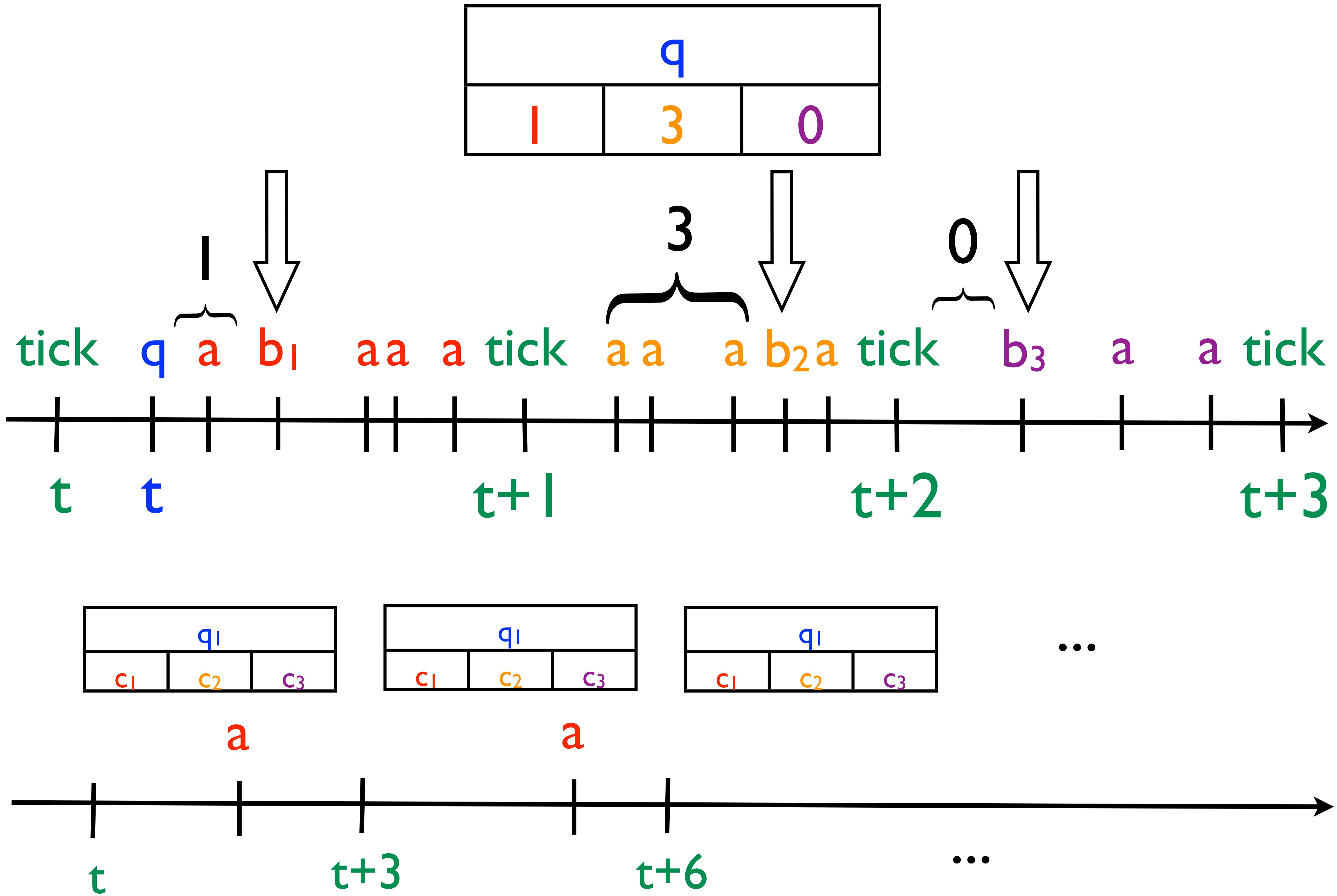

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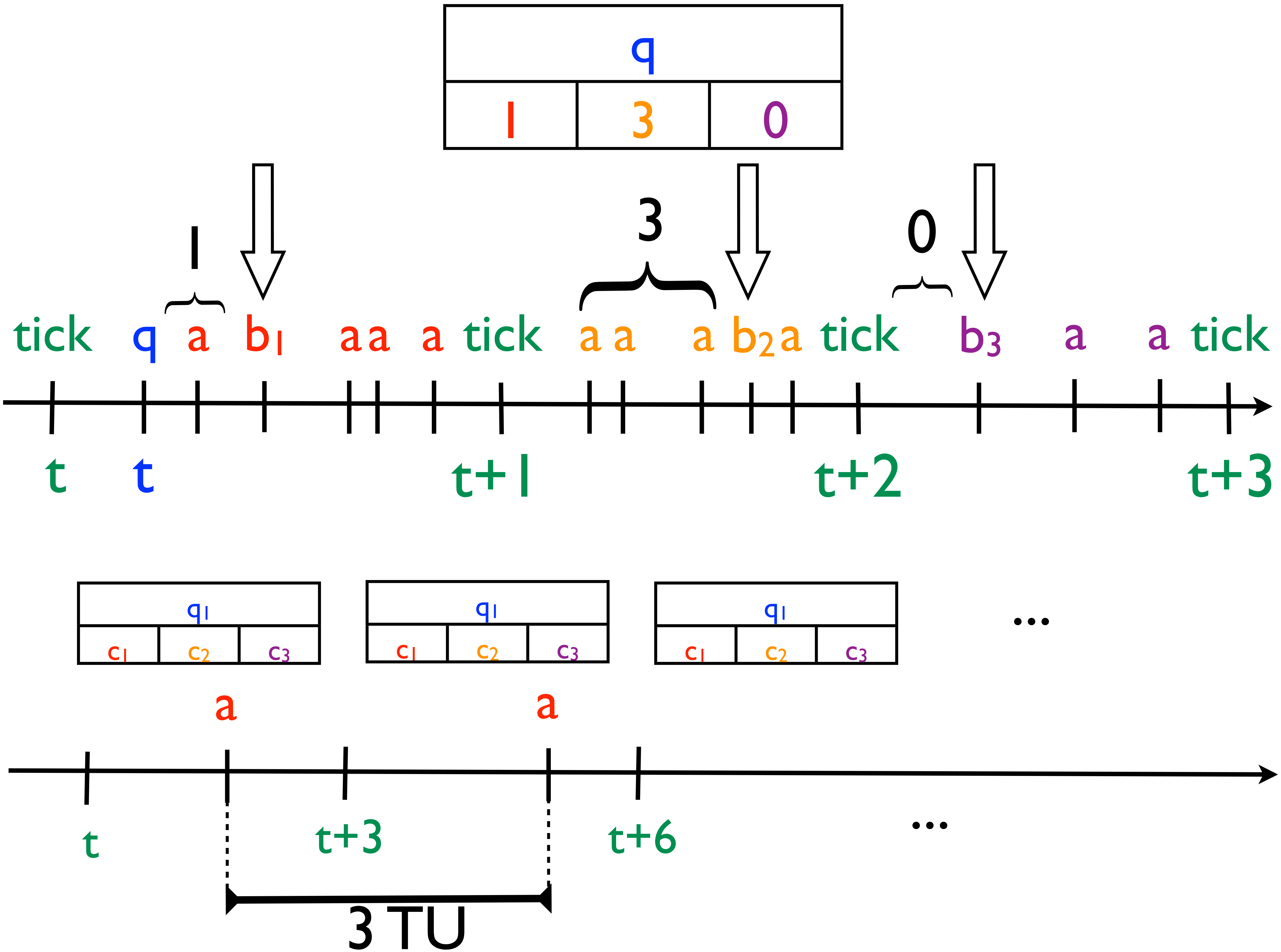
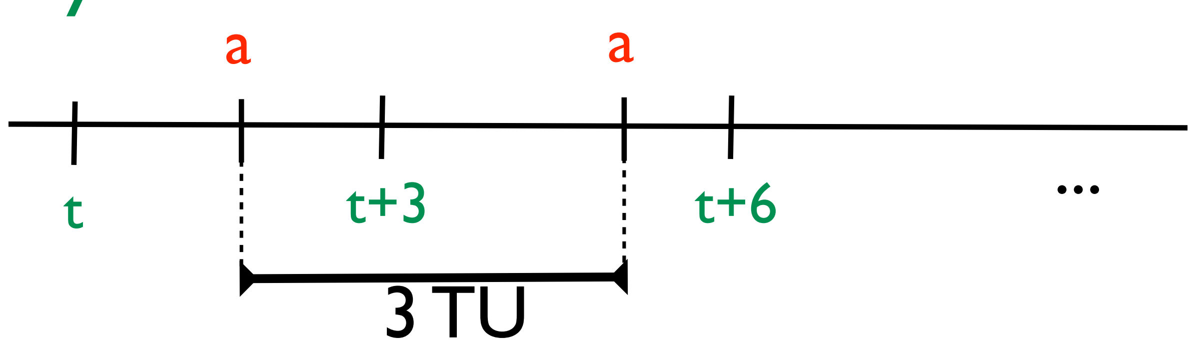


Reduction

- Given a 3CM M
  - Can we devise an ECL formula φ_M s.t.

> φM is satisfiable
> iff
> M admits an infinite bounded run ?

- **NO** !
  - Otherwise ECL satisfiability would be undecidable
  - We can't use ECL to specify that «every a or b should be preceded by an a or b 3 T.U. before» requirement

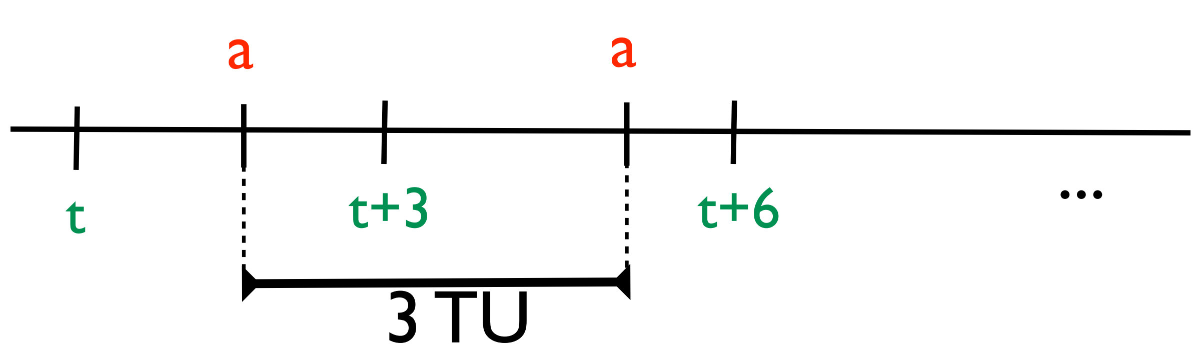


Reduction

- Given a 3CM M
  - Can we devise a timed game $\langle\Sigma_1,\Sigma_2,[\![\varphi_M]\!]\rangle$, where φ_M is an ECL formula s.t.

Player 1 has a winning strategy
iff
M admits an infinite bounded run ?

  - **YES** !
  - Player 1 controls the encoding symbols
  - We use Player 2 as an arbiter to check that Player 1 respects:



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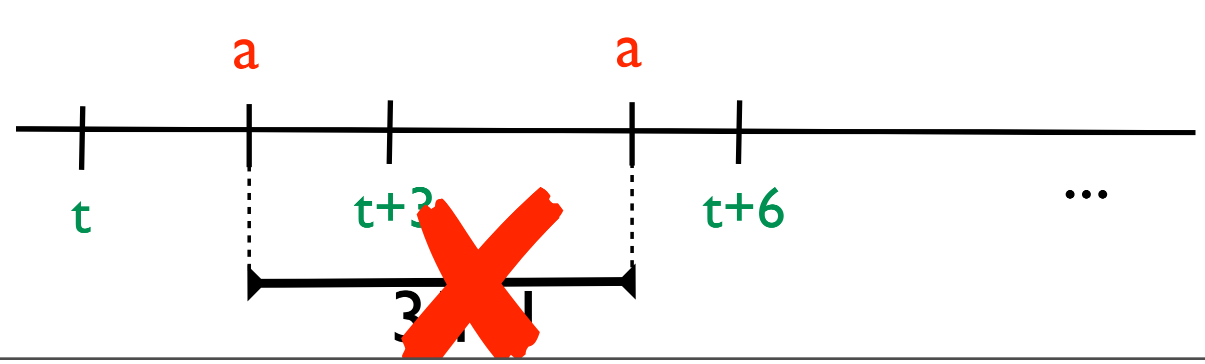

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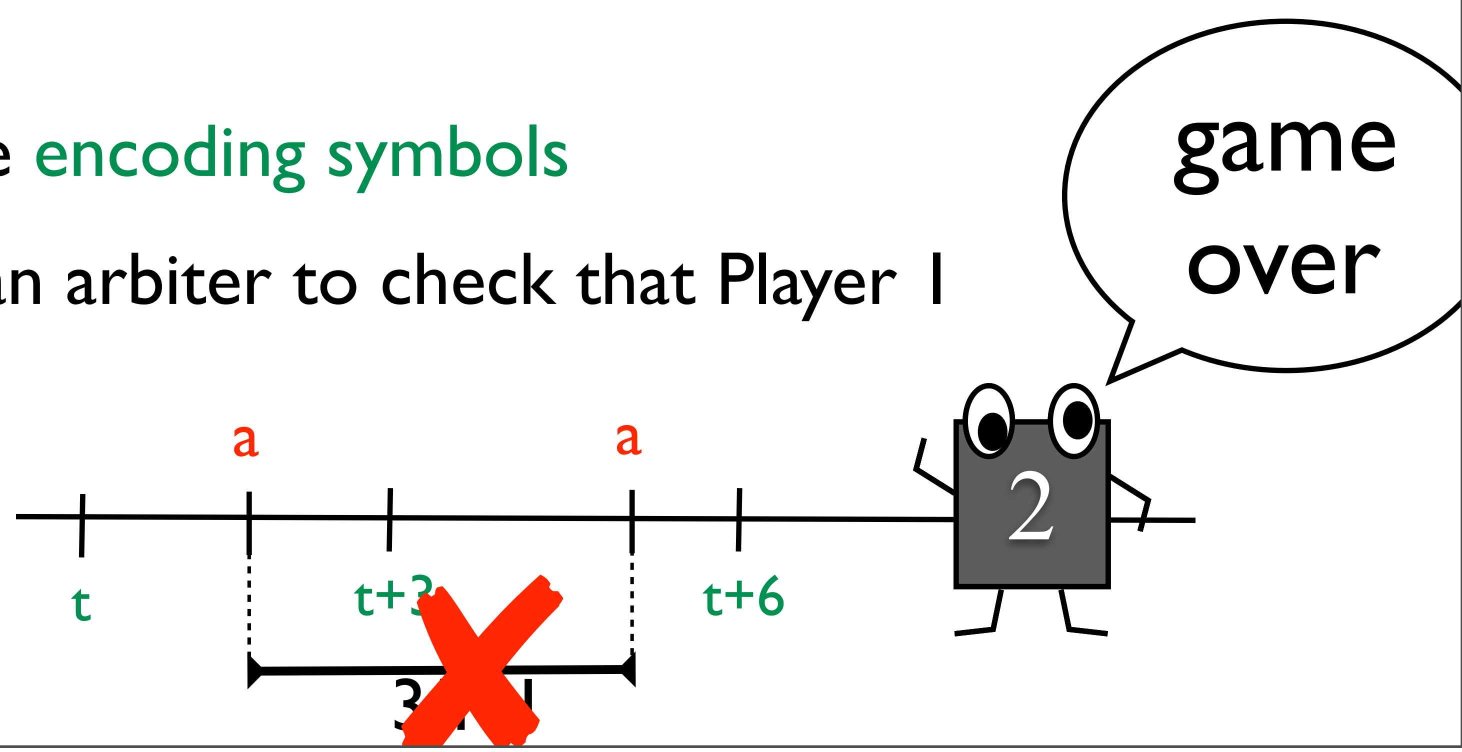

Deterministic ?

$\Sigma=\Sigma_1\cup\Sigma_2$

Spec.
Φ
$LTL_\triangleleft$

$?(\Sigma_1) \,||\, Env(\Sigma_2) \models \Phi$

$\exists\lambda_1 \bullet \forall\lambda_2 \bullet$ $\exists$run r of A_Φ $\bullet$ r accepts Outcome(λ_1,λ_2)

Remove second alternation by **determinization** of A_Φ.

$\exists\lambda_1 \bullet \forall\lambda_2 \bullet$ unique r of A^d on Outcome(λ_1,λ_2) is accepting

Universal co-Büchi ERA

- Instead of considering classical Büchi condition, we will consider Universal co-Büchi condition
  - Büchi = ∃ a run on w that visits accepting states infinitely often
  - co-Büchi = all runs on w visit accepting states finitely often
- These conditions are dual !

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φ

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$$\varphi \longrightarrow$$

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$$\varphi \longrightarrow \neg\varphi$$

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«Büchi construction»

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«Büchi construction»

Then: $L_{UcoB}(A_{\neg\varphi}) = [\![\varphi]\!]$