

# An Algebraic Semantics for Contract-Based Software Components

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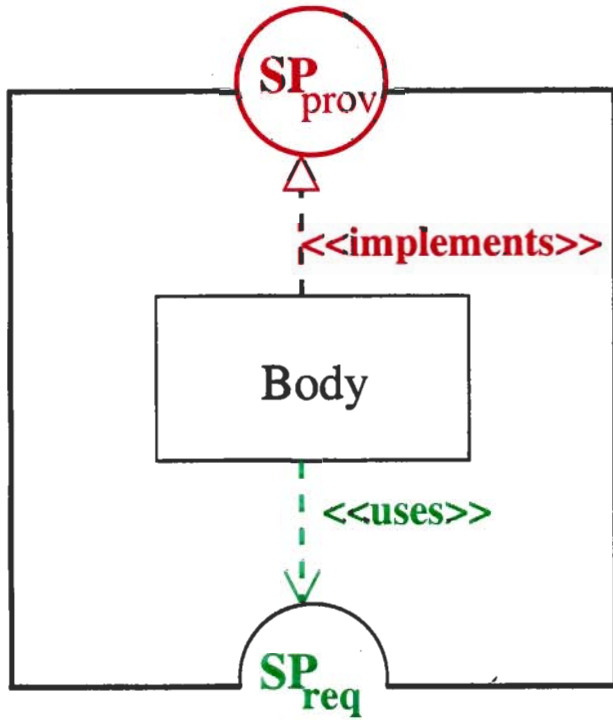
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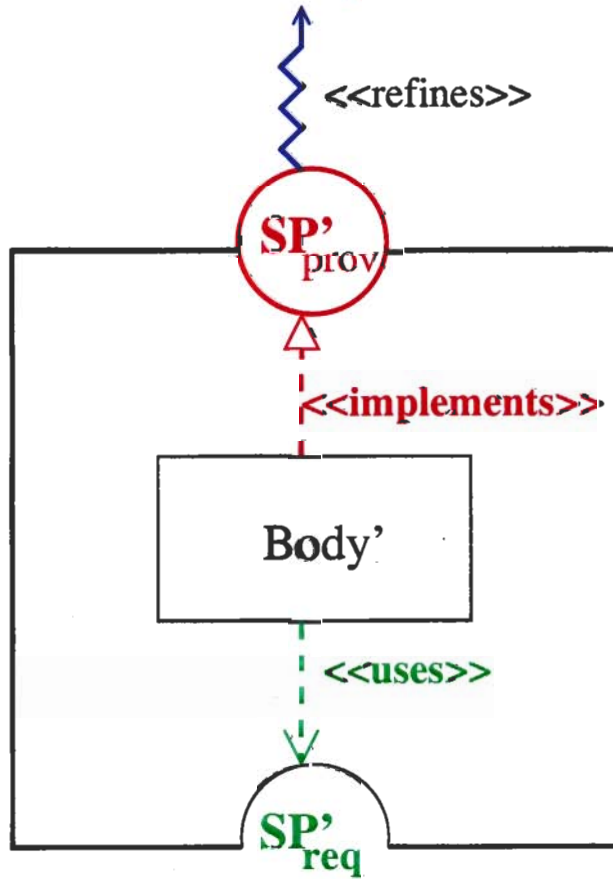
Scope: Assertion-based (sequential)  
 └ user SW components  
 └ implementor



provided interface specification

implementation of the provided operations

required interface specification



Literature:

ACT TWO  
 LASL architect. specs  
 Com Unity

+ Cos

OO: Eiffel

ZML  
 Spec#

ADLs: Wright

Darwin

...

Here :

## Model-theoretic approach

- signatures, sentences
- models, satisfaction relation

$$M \models \varphi$$

- loose semantics for
  - \* implementation relations

$$\text{Body} \dashrightarrow SP_{\text{prov}}$$

- \* refinement relations

$$SP'_{\text{prov}} \rightsquigarrow SP_{\text{req}}$$

Abstract framework to be applicable to

- concrete specification formalisms
- concrete implementation languages

Concrete enough to support

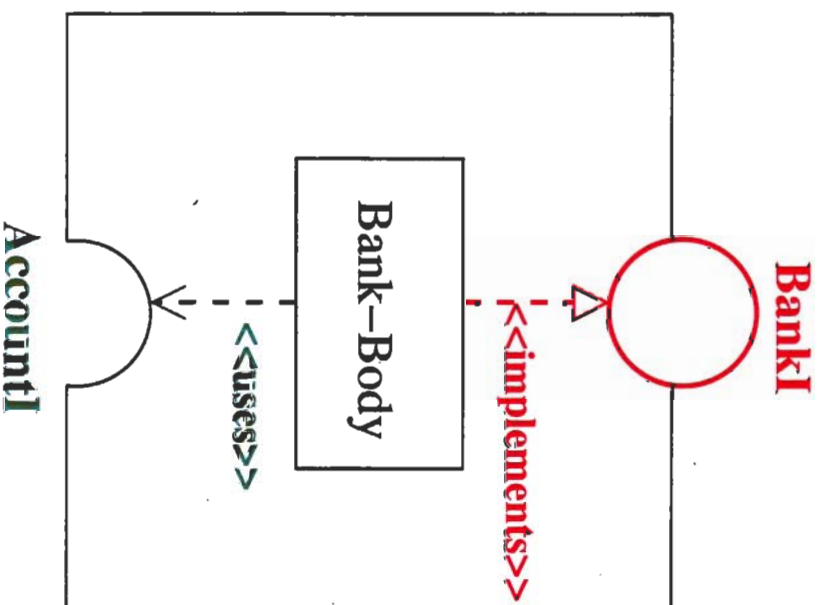
- definition of **states** (abstract + concrete)
- **dynamic evolution** of component instances and configurations
- **strong encapsulation** of states
- **contract principle**

Background: observ. obj. specs, architect. specs  
OO-specs

## Plan of the Talk

1. Interface specifications
2. Component bodies and how they use required interface specs
3. Component bodies and how they **implement** provided interface specs
4. Semantics of components
5. Component composition

## Example: The Bank Component



# 1. Interface Specifications $SP_I = (\Sigma_I, Ax)$

interface spec Account1 =

```

primitive types int, ... } observer signature
observer getBal: -> int; }  $\Sigma_{obs} = (\Sigma_{prim}, obs)$ 
operations
credit(int i: i >= 0); } domain constraint
withdraw(int i: i >= 0 and getBal >= i);
inv getBal >= 0; } state predicate
effects
credit : getBal = getBal@pre + i; } transition
withdraw : getBal = getBal@pre - i; } predicates
    
```

Semantics: "class of all correct realizations" (Hoare)

$$\llbracket SP_I \rrbracket = \text{class of all } \Sigma_I\text{-models satisfying } Ax$$

$\Sigma_{obs}$  - abstraktion

$\Sigma_I$  - model  $M_{AccountI}$

int = "integers"  
getBal = -20

$\neq$  getBal  $\geq 0$

int = "integers"  
getBal = 0

$\models$  getBal  $\geq 0$

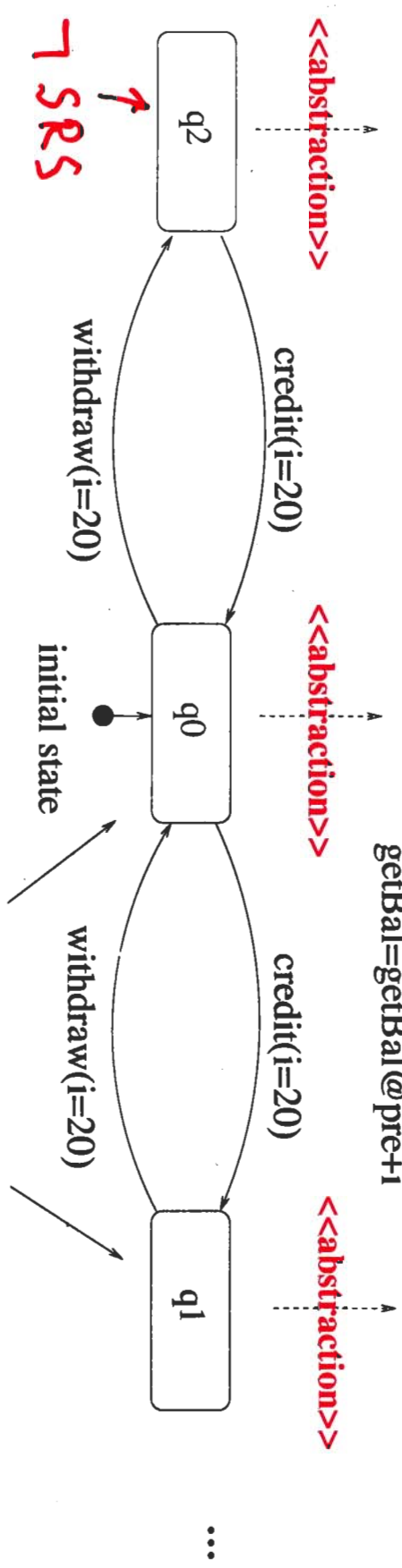
int = "integers"  
getBal = 20

$\models$  getBal  $\geq 0$

<<abstraction>>

<<abstraction>>

<<abstraction>>



"strongly reachable states (SRS)"

$M_{AccountI} \models_I \text{inv } \text{getBal} \geq 0$  i.e. "getBal  $\geq 0$  holds in all (abstracted) SRS "

$M_{AccountI} \models_I \text{credit: } \text{getBal} = \text{getBal}@pre + i$   
i.e. "getBal=getBal@pre+i holds for all transitions on SRS "

How  $\Sigma_I$ -models reflect the "contract principle"

Responsibility of the user:

Any system run goes only through strongly reachable states!

Responsibility of the implementor:

Any system run through strongly reachable states respects the given axioms!



## 2. Component bodies and how they use required interface specifications

body Bank-Body =

*required type*

```
let Account : Account!;
private Map<nat, Account> accounts;
```

*attribute sig.*  
 $\Sigma_{Att}$

```
public int balAcc(nat no)
```

```
{ Account acc = accounts.get(no);
```

```
return acc.getBal();} }
```

*body sig.*  
 $\Sigma_{Body} = (\Sigma_{Att}, Op)$

```
public void transfer(nat from, to; int i)
```

```
{ Account source = accounts.get(from); source.withdraw(i);
```

```
Account target = accounts.get(to); target.credit(i);} }
```

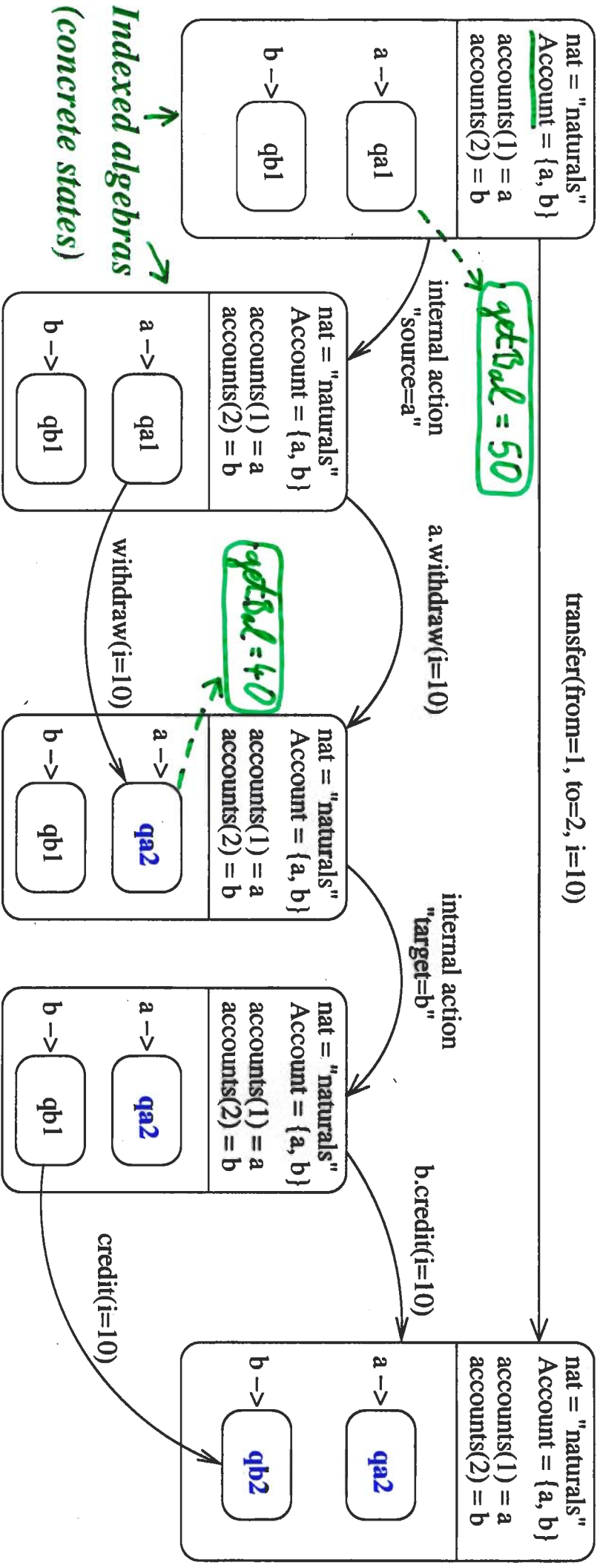
*Op - Impl*

*WTE assume*

$\llbracket Op - Impl \rrbracket : \llbracket SP_{req} \rrbracket \rightarrow \text{class of } \Sigma_{Body} \text{ - models}$

$M_{req} \xrightarrow{w} \Sigma_{Body} \text{ - model } M_{over} M_{req}$

# Σ Body - model M<sub>Bank</sub> over M<sub>Account</sub>T



## !!! Requirements (strong encapsulation)

- Internal actions do not modify the state of instances of a required type!
- Calling a required operation on an instance o can only modify the state of o!

Σ<sub>Body</sub> - model : Transition system with indexed algebras on states satisfying the above requirements

## Theorem

Let  $M_{req} \in \llbracket SP_{req} \rrbracket$ , let  $M$  be a  $\Sigma_{body}$ -model over  $M_{req}$ .

If  $M$  is a "correct user" of  $M_{req}$  then:

$$1. M_{req} \models_I \text{inv } \varphi \stackrel{\Rightarrow}{\Leftarrow} \text{iff } M \models \text{inv } \forall x: \text{rt } x: \varphi$$

$\downarrow$   
req. type

e.g.

$$M_{\text{AccountI}} \models_I \text{inv } \text{getBal} \geq 0 \text{ iff } M_{\text{Bank}} \models$$

$$\forall x: \text{Account } x. \text{getBal} \geq 0$$

$$2. M_{req} \models_I \text{op}_{req} : \Pi \text{ iff } M \models \text{op}_{req} ! \forall x: \text{rt } x: \Pi$$

e.g.

$$M_{\text{AccountI}} \models_I \text{withdraws} : \text{getBal} = \text{getBal}_{\text{opre}} - i \text{ iff}$$

$$M_{\text{Bank}} \models \text{withdraws} : \forall x: \text{Account } x. \text{getBal} = x. \text{getBal} - i$$

### 3. Component bodies and how they implement provided interface specifications

provided interface spec Bank1 =

primitive types int, nat, ...  
observers

balAcc: nat -> int;

operations

transfer(nat from, to; int i: *i >= 0 and balAcc(from) >= i*);

...

inv balAcc(no) >= 0;

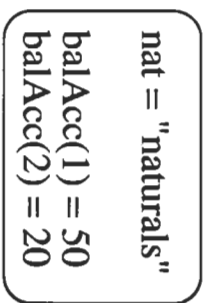
effect

transfer : balAcc(from) = balAcc(from)@pre - i and

balAcc(to) = balAcc(to)@pre + i;

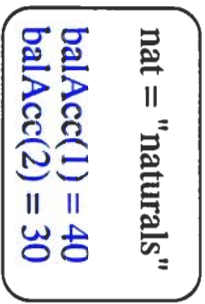
*domain constraint*

# How to construct interface models out of body models



$\models \text{balAcc}(\text{from}) \geq i$

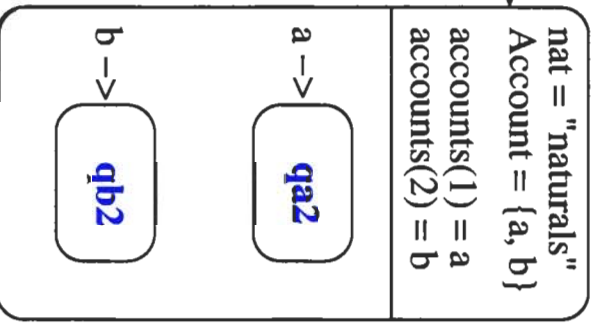
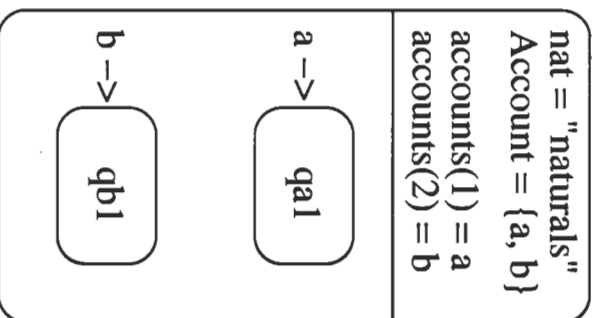
$\text{balAcc}(\text{from}) = \text{balAcc}(\text{from})@pre-i$   
 and  $\text{balAcc}(\text{to}) = \text{balAcc}(\text{to})@pre+i$



**abstraction**

**abstraction**

$\text{transfer}(\text{from}=1, \text{to}=2, i=10)$



*Correctness*

Restrict sig(BankI) ( [Bank-Body] (M<sub>AccountI</sub>) ) ∈ [BankI]

# Modular Verification

nat = "naturals"  
 balAcc(1) = 50  
 balAcc(2) = 20

$\models$  balAcc(from)  $\geq i$

nat = "naturals"  
 balAcc(1) = 40  
 balAcc(2) = 30

$\models$  balAcc(from) = balAcc(from) @ pre-i  
 and balAcc(to) = balAcc(to) @ pre+i

$\wedge$

$\wedge$

nat = "naturals"  
 Account = {a, b}  
 accounts(1) = a  
 accounts(2) = b

a  $\rightarrow$  qa1  
 $\models$  getBal  $\geq i$

b  $\rightarrow$  qb1

transfer(from=1, to=2, i=10)

$\rightarrow$  getBal = 50

internal action  
 "source=a"

a.withdraw(i=10)

internal action  
 "target=b"

b.credit(i=10)

nat = "naturals"  
 Account = {a, b}  
 accounts(1) = a  
 accounts(2) = b

a  $\rightarrow$  qa1  
 $\models$  getBal  $\geq i$

b  $\rightarrow$  qb1

$\rightarrow$  getBal = 40

withdraw(i=10)

getBal = getBal @ pre-i

nat = "naturals"  
 Account = {a, b}  
 accounts(1) = a  
 accounts(2) = b

a  $\rightarrow$  qa2

b  $\rightarrow$  qb1

nat = "naturals"  
 Account = {a, b}  
 accounts(1) = a  
 accounts(2) = b

a  $\rightarrow$  qa2

b  $\rightarrow$  qb1

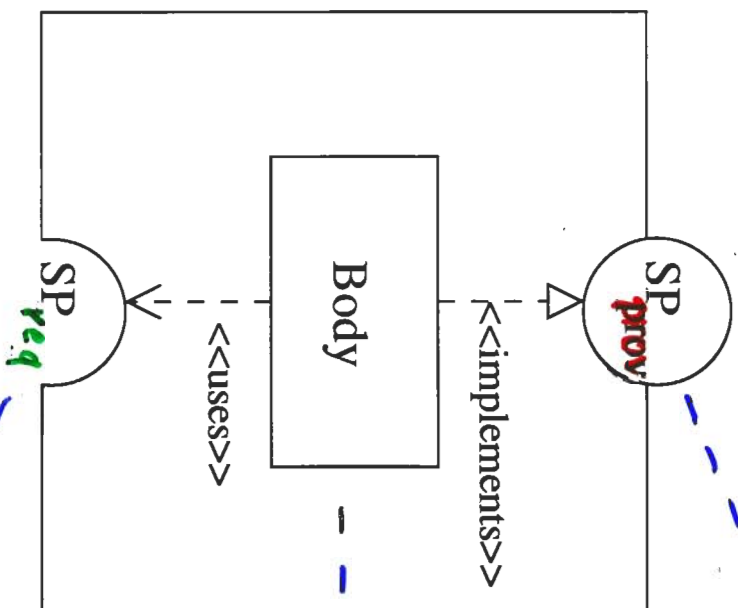
credit(i=10)

nat = "naturals"  
 Account = {a, b}  
 accounts(1) = a  
 accounts(2) = b

a  $\rightarrow$  qa2

b  $\rightarrow$  qb2

# 4. Components



$$(\Sigma_{prov}^I, Ax_{prov}^I)$$

$$(\Sigma_{obs}^{obs}, Op_{prov}, dom_{prov})$$

$$(\Sigma_{prim}, Obs_{prov})$$

$$(\Sigma_{Body}, \gamma_{obs} : \llbracket SP_{req} \rrbracket \rightarrow Mod(\Sigma_{Body}))$$

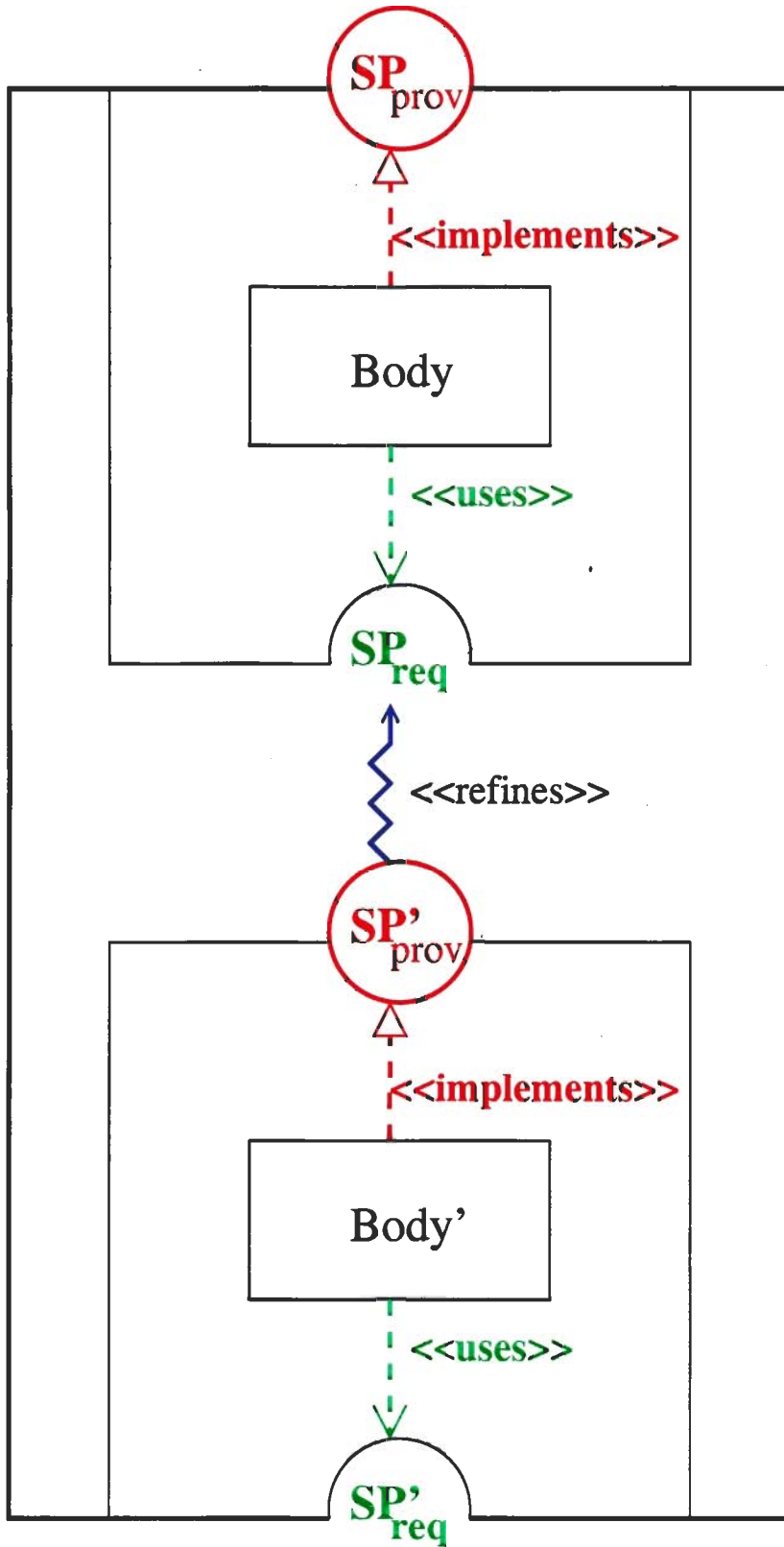
$$(\Sigma_{Att}, op)$$

$$(nt : SP_{req}, \Sigma_{prim} + nt, Att)$$

$$(\Sigma_{req}^I, Ax_{req}^I)$$

Correctness:

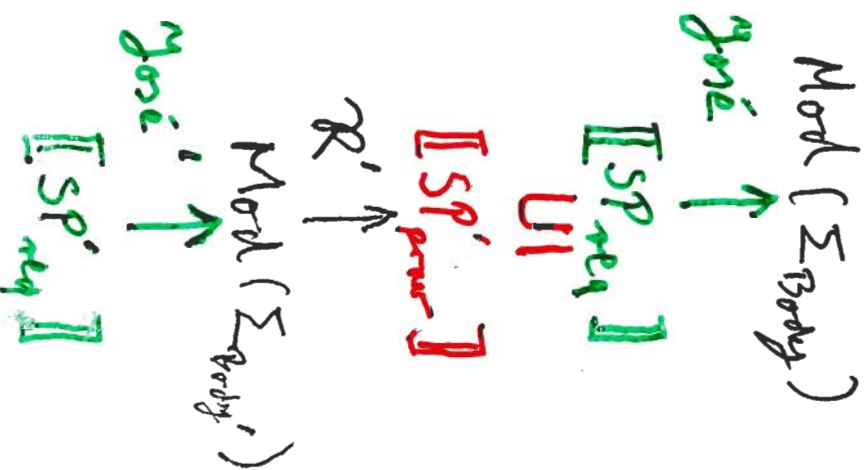
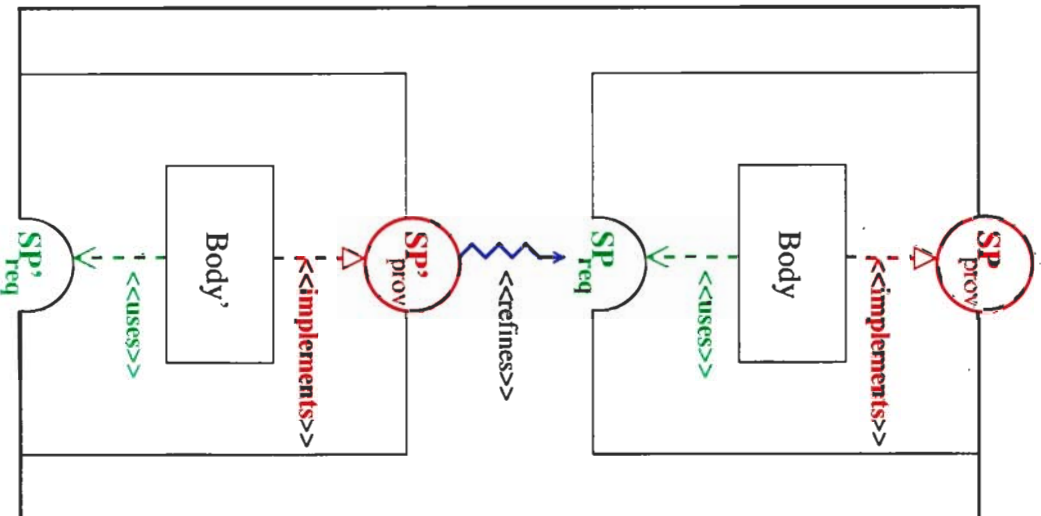
$$R_{\Sigma_{prov}^I}(\gamma_{obs}(\llbracket SP_{req} \rrbracket)) \subseteq \llbracket SP_{prov} \rrbracket$$



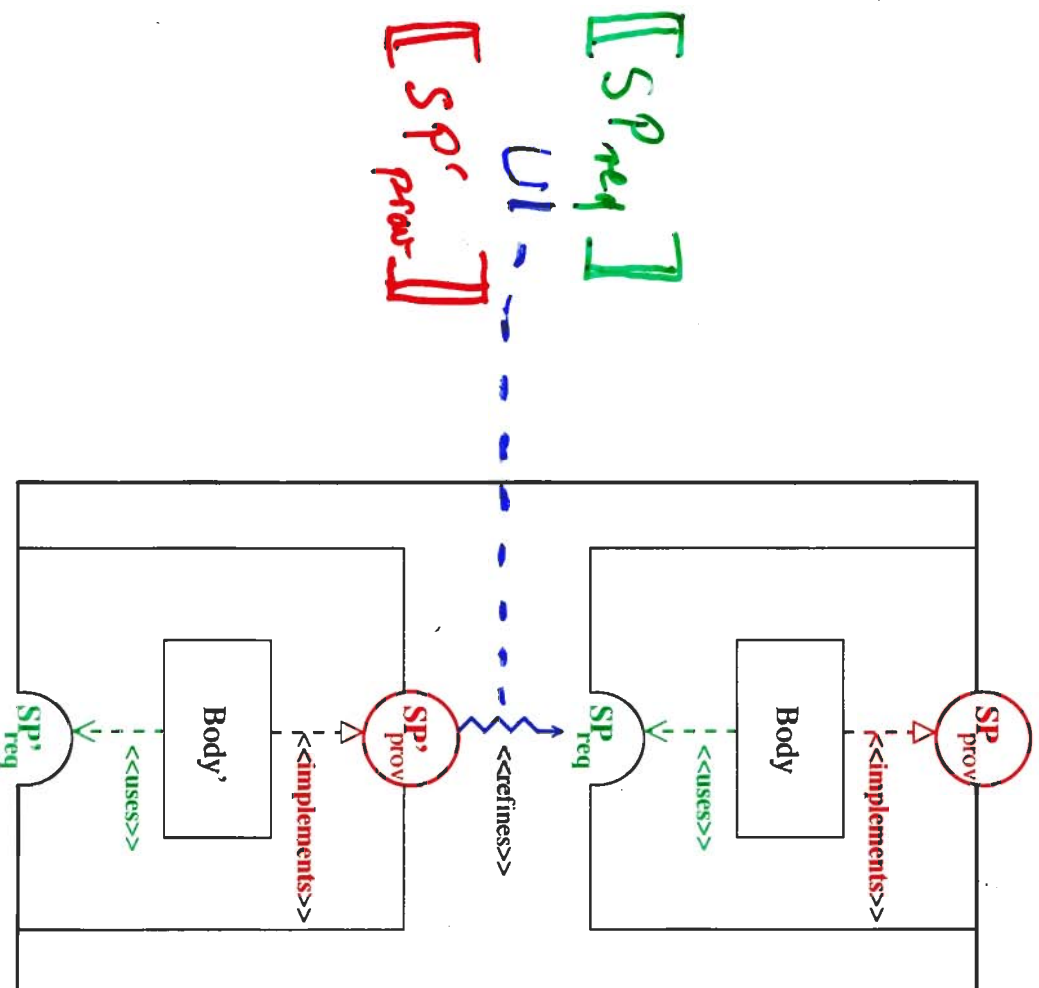
$Mod(\Sigma_{Body})$   
 José  $\uparrow$   
 $[SP_{req}]$   
 UI  
 $[SP'_{prov}]$   
 $R' \uparrow$   
 $Mod(\Sigma_{Body'})$   
 José'  $\uparrow$   
 $[SP'_{req}]$



# 5. Component composition

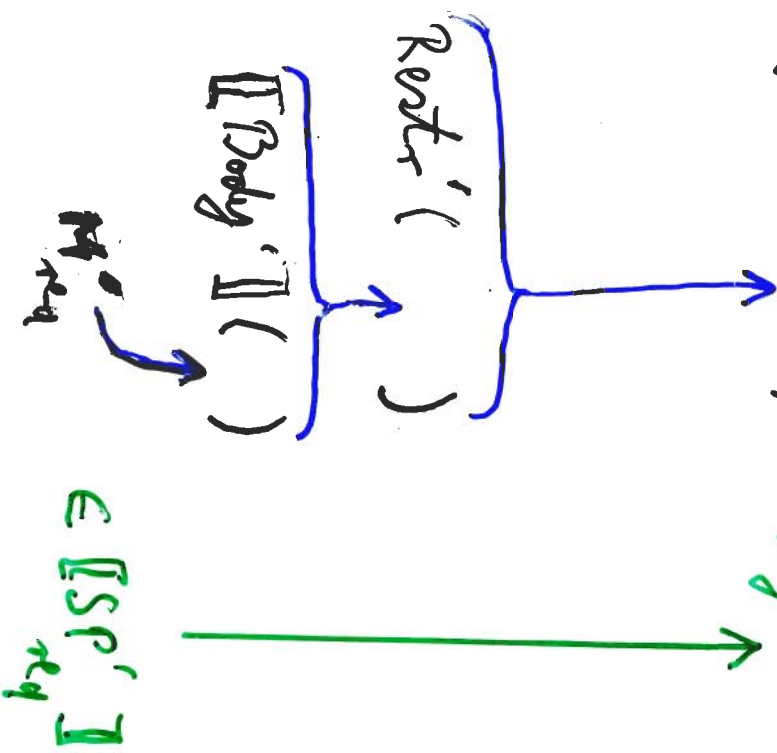


## 5. Component composition



$[[SP_{req}]]$   
 UI  
 $[[SP'_{prov}]]$

$[[Body]] ( ) \in \Sigma_{Body} - models$



Fact: Corectness of sub-components propagates to CC

## Conclusion

- Rigorous model - theoretic foundation of (sequential) component - based systems supporting
  - contract - based system developments
  - strong encapsulations of states
  - dynamic evaluation based on "indexed algebras"
- Future work
  - addition of behaviour protocols
  - extension to concurrent, distributed components