Internship Proposal - Real Complexity, Numerical Analysis and Theorem Provers
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Scientific Context

Computation over the Real Numbers

The Blum-Shub-Smale (BSS) model of computation was proposed in 1989 [BSS88, BCSS98] as a model of computation over the field of real numbers, with unit-cost measure for algebraic operations and comparisons, and unbounded computational precision. Classical, structural complexity results have then been extended to this setting: reductions, complexity classes such as P and NP are defined, natural complete problems for these classes are indentified, and the classical question P=NP remains also open over the reals, and seems as challenging as its boolean couterpart.

Numerical Analysis and Condition Numbers

The unbounded computational precision of the BSS model, together with the ability to use any arbitrary real constant in the computation, may seem at first sight contradictory with real world programs in numerical analysis, where round-off errors and finite precision occur. Smooth analysis and condition numbers fill the gap: the condition number expresses the robustness of the algorithm with respect to small pertubations of the input, and is used as a parameter in the complexity analysis (see [BC13] for a comprehensive approach). Many algorithms over the real numbers in the BSS model can then be analysed in a bounded precision setting, see [CKMW12] for an example, where conditionning is used for the problem of counting the set of zeros of a system of polynomial inequalities.

Theorem Provers and Real Numbers

The development of libraries of real numbers and floating numbers in the theorem provers has experienced a significant rise in the last 20 years. Theorem provers such as Coq, HOL, HOL-Light, Isabelle, PVS include this kind of libraries. Formal proofs of numerical analysis problems are also developed, in particularly in Coq [BCF+17]. These developments use the classical standard library of Reals [May01, Coq] as well as Coquelicot [BLM15] library, which should be particularly well adapted to the problem of this internship.

Aim of the Internship

The Internship aims at using the recent advances on the integration of real numbers in Theorem provers for studying, and possibly exhibiting proofs of results in the field of Conditionning in Numerical Analysis. The knowledge of a theorem prover will be appreciated.

Workplace

This internship will be supervised by Micaela Mayero, maître de conférences, and Paulin de Naurois, chargé de recherches CNRS, at the LIPN - UMR 7030 research laboratory of Université Paris 13, as part of a team working on different aspects of logics, complexity and computability theory, programmming theory.

References

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[CKMW12] Felipe Cucker, Teresa Krick, Gregorio Malajovich, and Mario Wschebor. A numerical algorithm for zero counting. III: randomization and condition. *Adv. Appl. Math.*, 48(1):215–248, 2012.

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