Flip Graphs and Matroids / A Mini-Course

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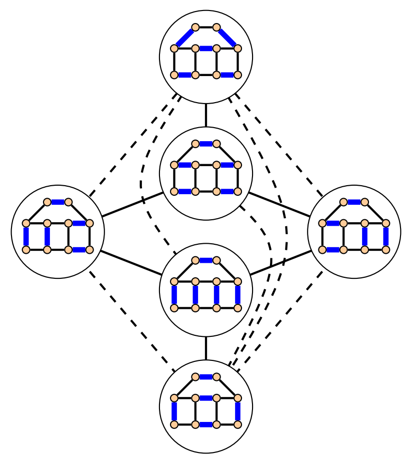
The term “flip graph” has originally been used for studying flips in triangulations, but has since been defined for a variety of combinatorial structures, where a flip can be defined as an elementary, local, reversible operation. Flip graphs are defined on sets of combinatorial structures (typically all structures of a given fixed size n), and two vertices are adjacent whenever they differ by exactly one flip.

In order to contribute to a unified theory of flip graphs and the associated combinatorial and computational problems, we will adopt the viewpoint of matroid theory. Matroids are well-studied set systems that obey axioms capturing an abstract notion of independence generalizing linear independence. There are many fruitful connections to be established between polytopal flip graphs described in the algebraic combinatorics literature (Postnikov) and matroid structures developed decades ago in the context of combinatorial optimization (Edmonds). Through these connections, we can cast several fundamental computational issues related to geodesics, diameters, and Hamiltonian walks in a larger context, and formulate new ones.



The tentative plan for three lectures, each of roughly 90 minutes, is as follows.

1. **Flip graphs from matroids.** Introduction to Matroids. Rank functions. Matroid bases polytopes. Graphical matroids. Lattice paths. Diameter. Hamiltonicity.

2. **Flip graphs from polymatroids.** Polymatroids and generalized permutohedra. Hypergraphic polytopes. Graphical zonotopes. Loday’s associahedra. Graph associahedra. Acyclic orientations of hypergraphs. Hamiltonicity.

3. **Flip graphs from matroid intersections.** Matroid intersection theorems. Common bases polytopes. Frank-Tardos Theorem. Perfect matchings and α-orientations. Complexity of flip distances. Linear programming.

Time allowing, flip graphs from **greedoids** and arborescences (Korte, Bondy, and Lovász) could also be discussed.

The mini-course could be of interest to a varied audience, including researchers and students in algorithms, discrete geometry, combinatorial optimization, graph theory, and combinatorics.