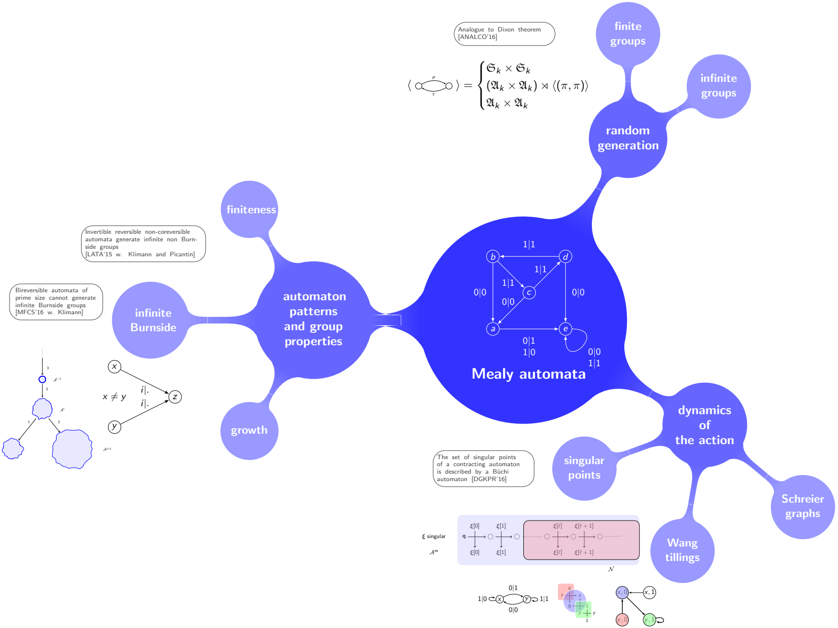
Mealy machines, automaton (semi)groups, decision problems, and random generation

Thibault Godin

Séminaire CALIN Paris 13, October 3, 2017

ANR JCJC 12 JS02 012 01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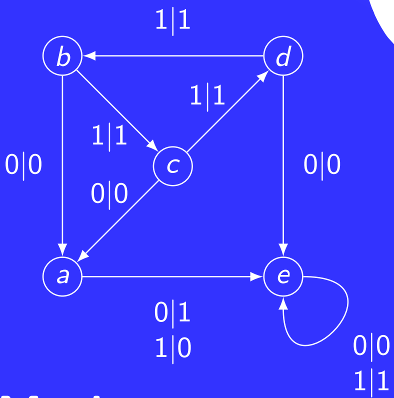

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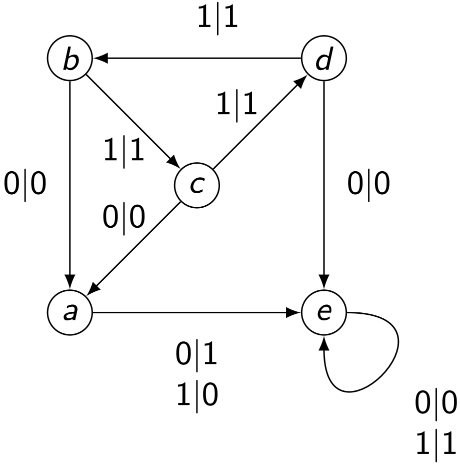
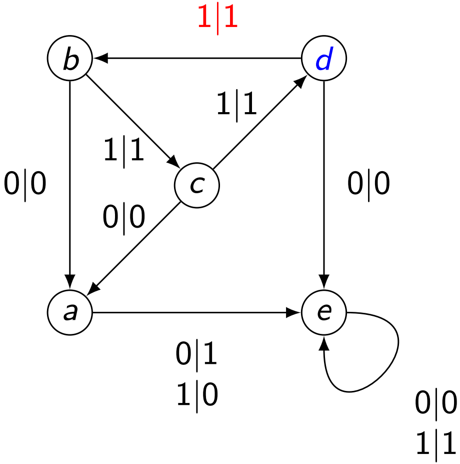


Mealy automaton $\mathcal{G}$

$$\mathcal{A} = (Q, \Sigma, \delta, \rho)$$



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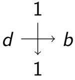

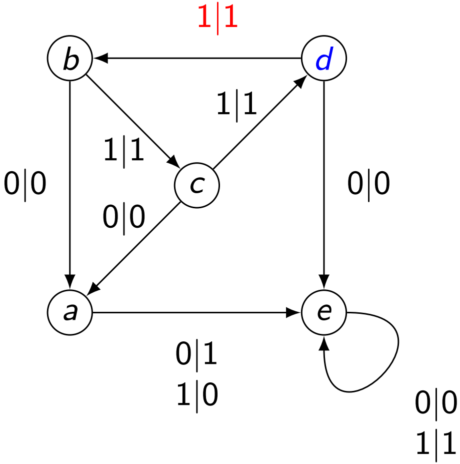

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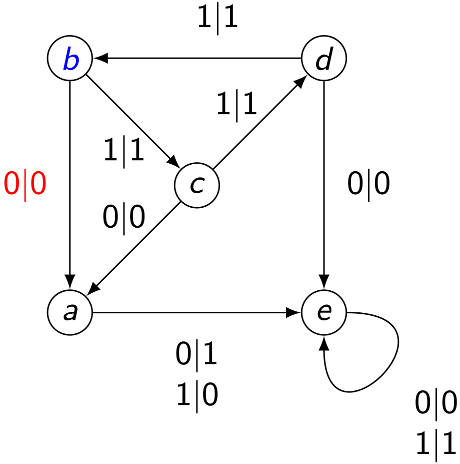

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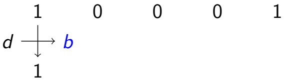

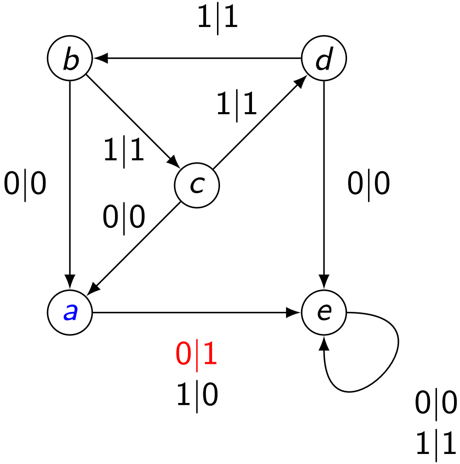

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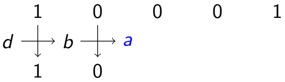

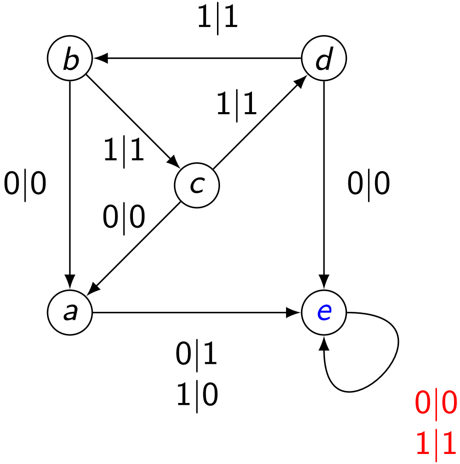

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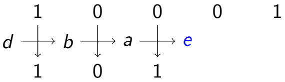

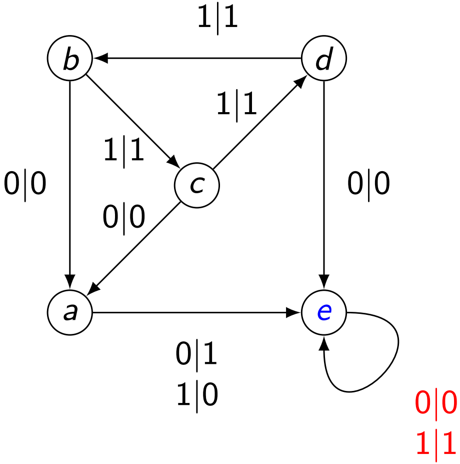

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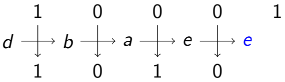

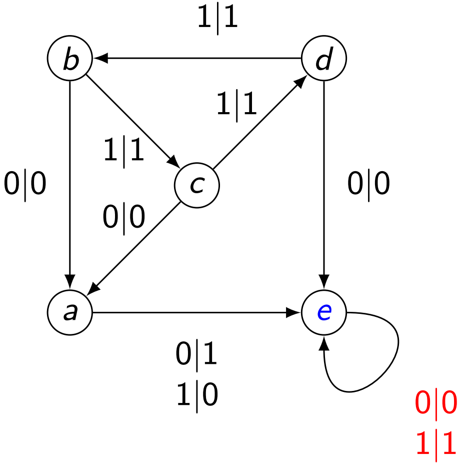

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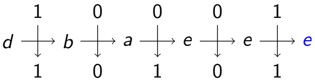

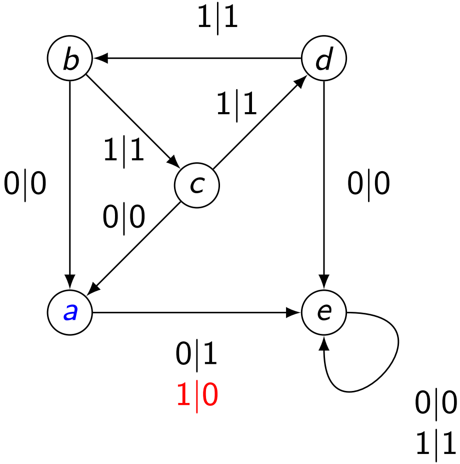

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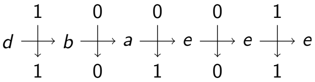

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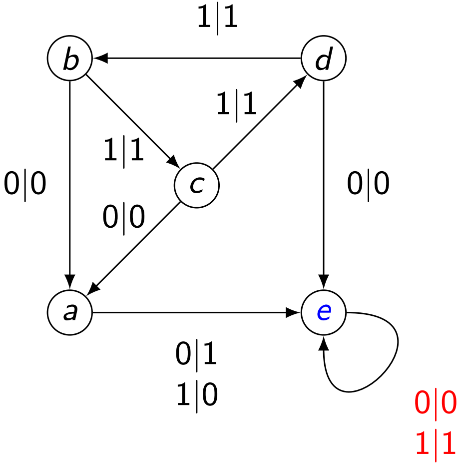

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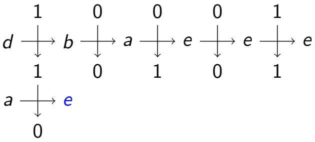

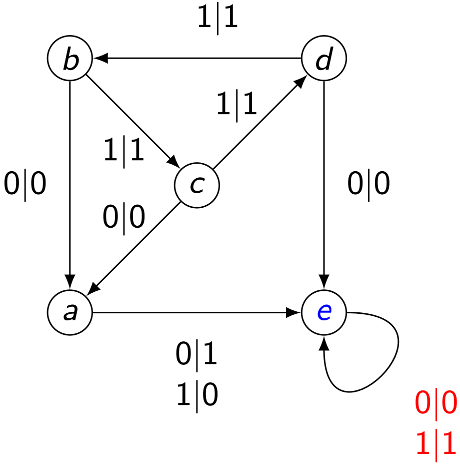

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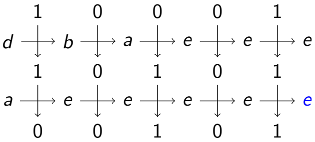

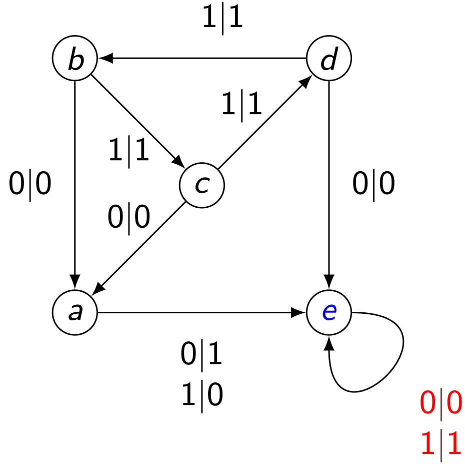

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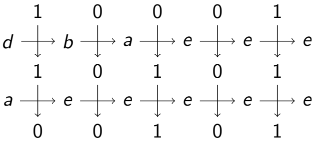

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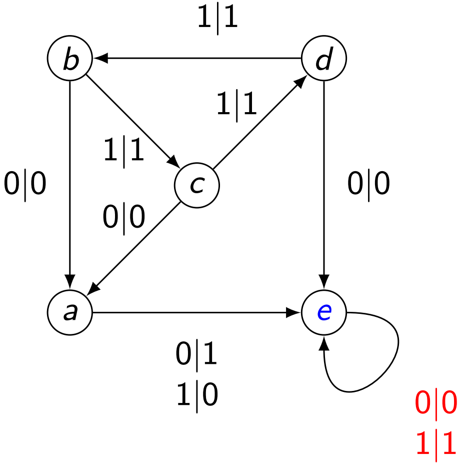

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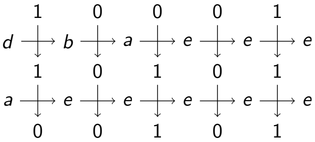

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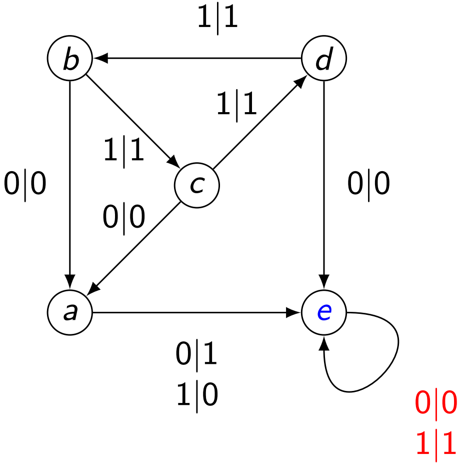

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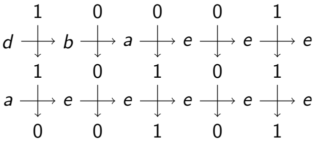

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da is a state of $\mathcal{G}^2$

Growth

Cayley Graph: $\Gamma(G, S)$

$$g \xrightarrow{s} h \qquad g.s = h,\ g, h \in G,\ s \in S$$

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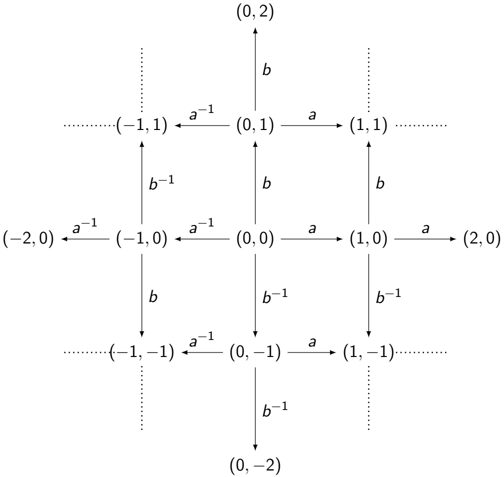

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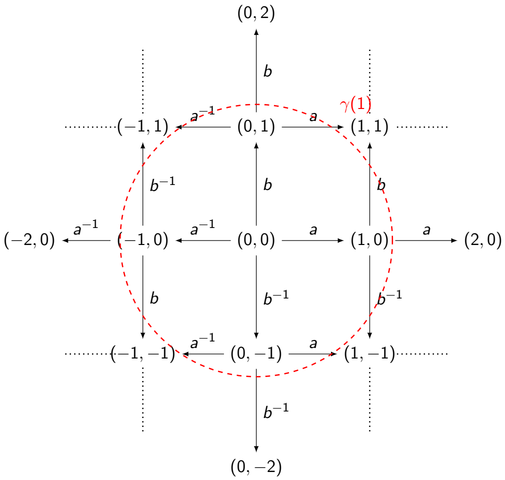

$\gamma(0) = 1$

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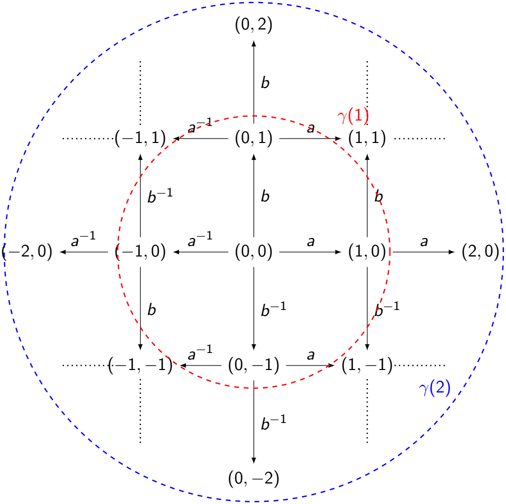

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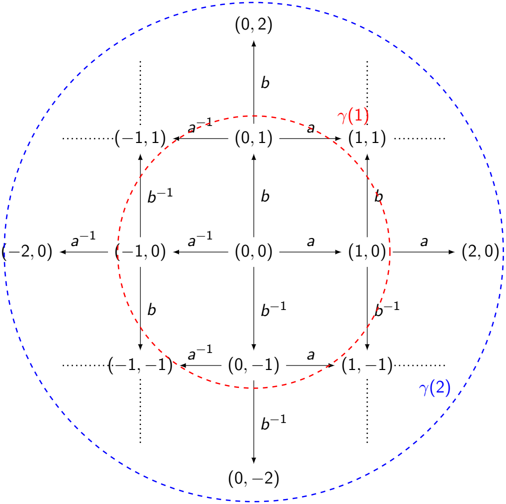

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$\vdots$

$\gamma(n) = 2n^2 + 2n + 1$

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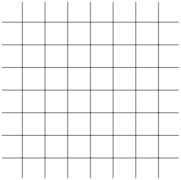

$\Gamma(\mathbb{Z}^2)$

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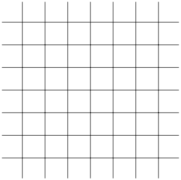

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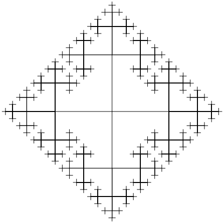

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Milnor's Problem (1968):

Do groups with growth growth between polynomial and exponential exist?

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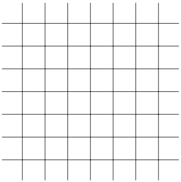
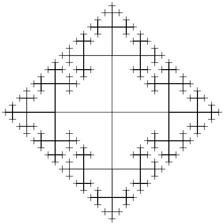


$\Gamma(\mathbb{Z}^2)$ $\Gamma(?)$



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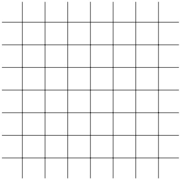

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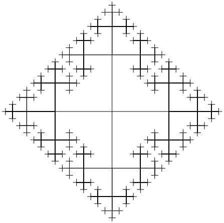

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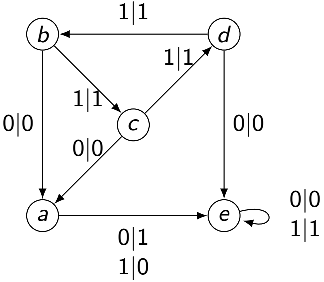

$$e^{n^{0.51}} \leq \gamma(n) \leq e^{n^{0.77}}$$

Order

Order of an element

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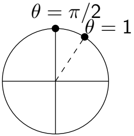
- $\mathbb{Z}/n\mathbb{Z}$: every element has finite order
- $\mathbb{Z}$: 0 is the only element of finite order
- On the circle $\mathbb{R}/2\pi\mathbb{Z}$: $\pi/2$ has finite order, but 1 has infinite order



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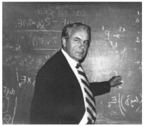

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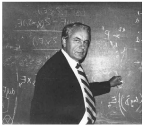

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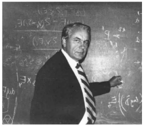

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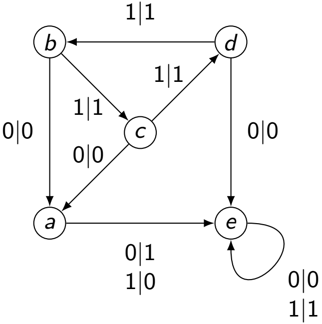

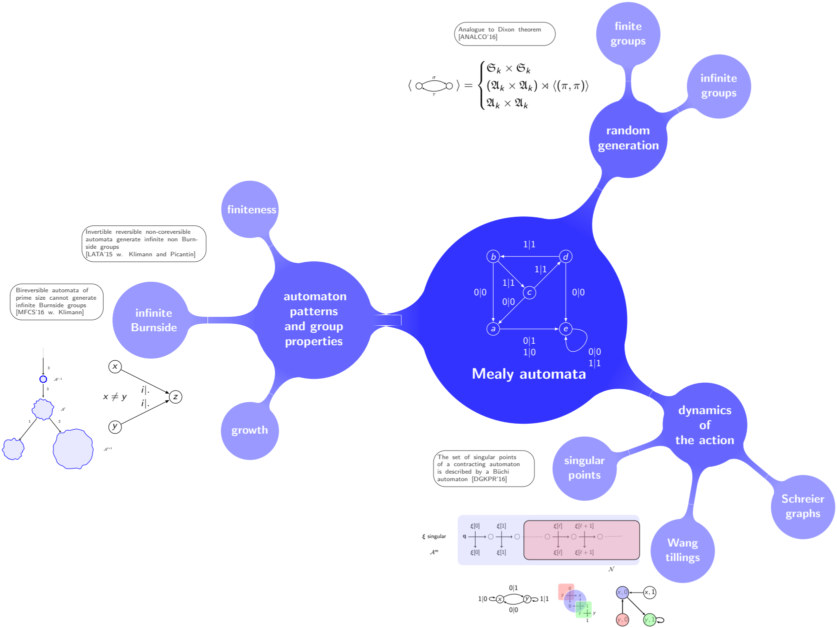

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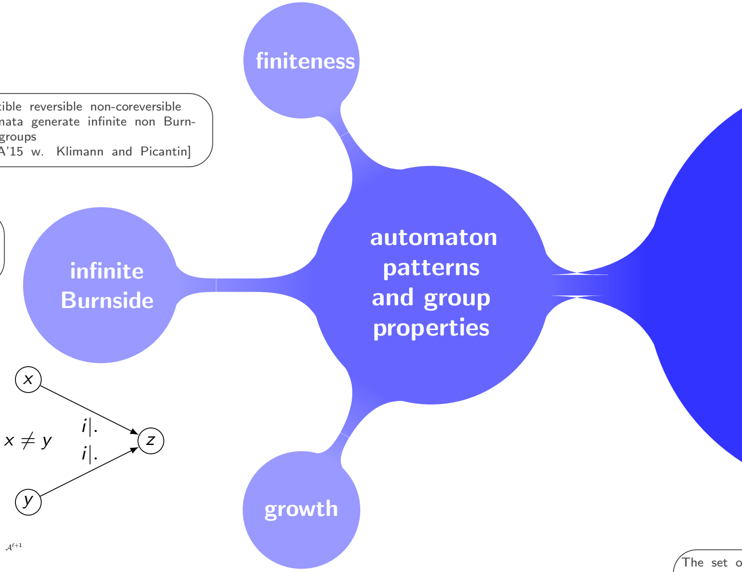

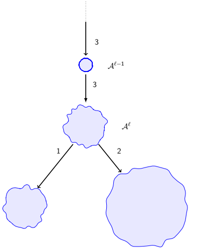

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is describe
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1|0 0|0 1|1

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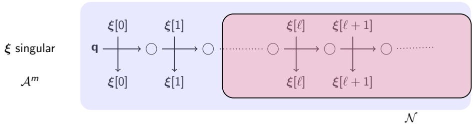

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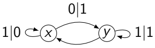
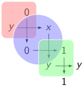
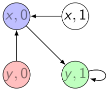
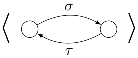


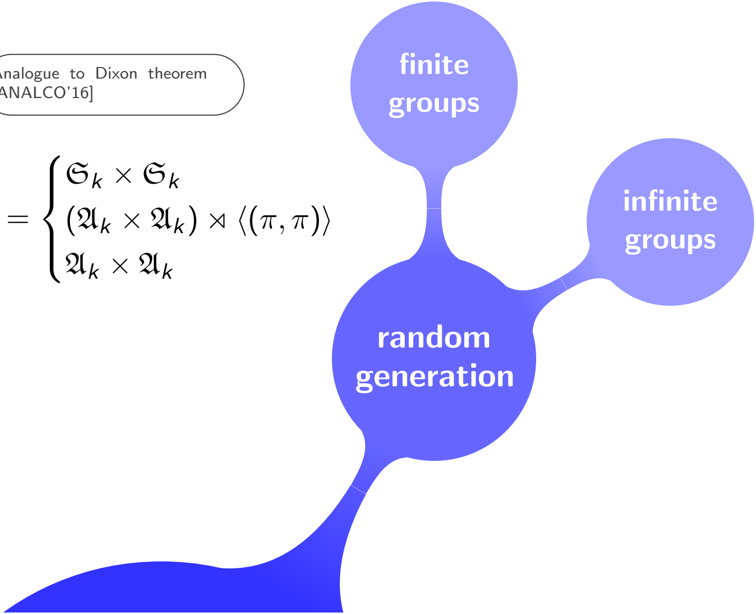




Analogue to Dixon theorem [ANALCO'16]



$$= \begin{cases} \mathfrak{S}_k \times \mathfrak{S}_k \\ (\mathfrak{A}_k \times \mathfrak{A}_k) \rtimes \langle (\pi, \pi) \rangle \\ \mathfrak{A}_k \times \mathfrak{A}_k \end{cases}$$



Finite random groups

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Any finite group G is a subgroup of $\mathfrak{S}_{|G|}$.

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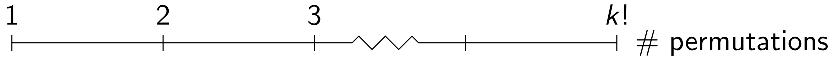

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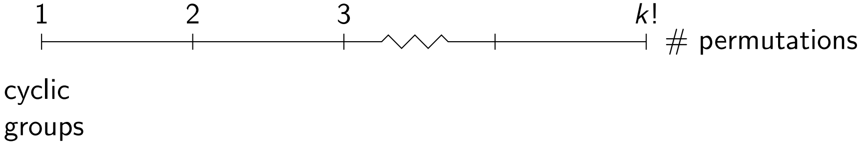

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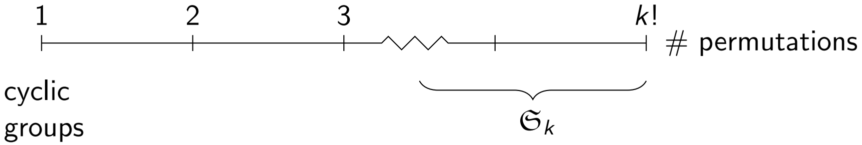

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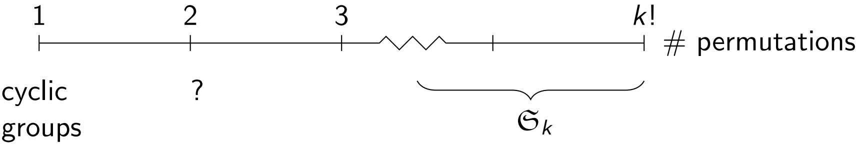
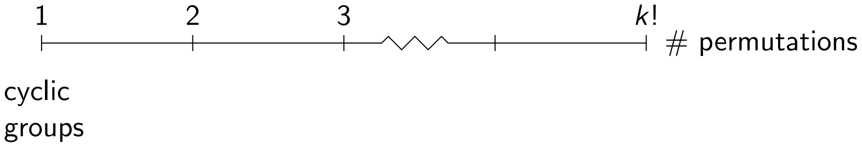


Finite random groups

Theorem (Dixon, 1969)

$$\text{w.g.p. } \langle \sigma, \tau \rangle = \begin{cases} \mathfrak{S}_k \\ \mathfrak{A}_k \end{cases}$$



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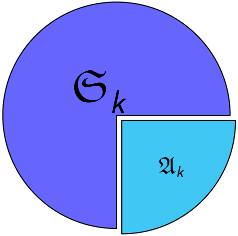

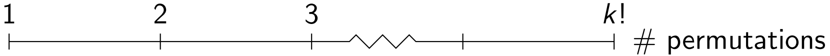

cyclic
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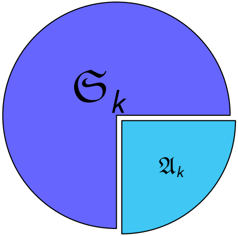
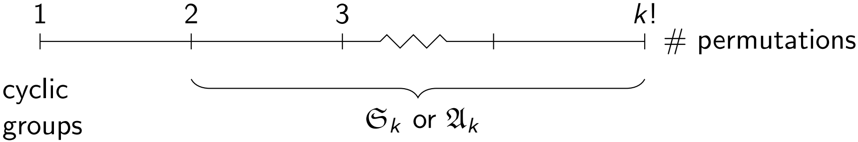
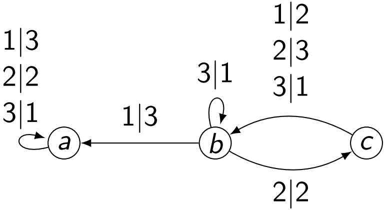


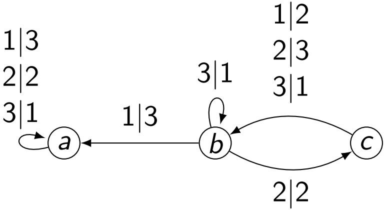


Random automata



Is the generated group finite?

Random automata



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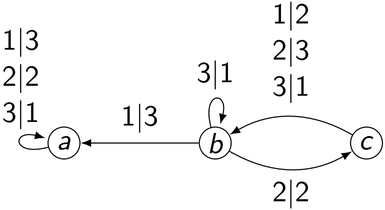

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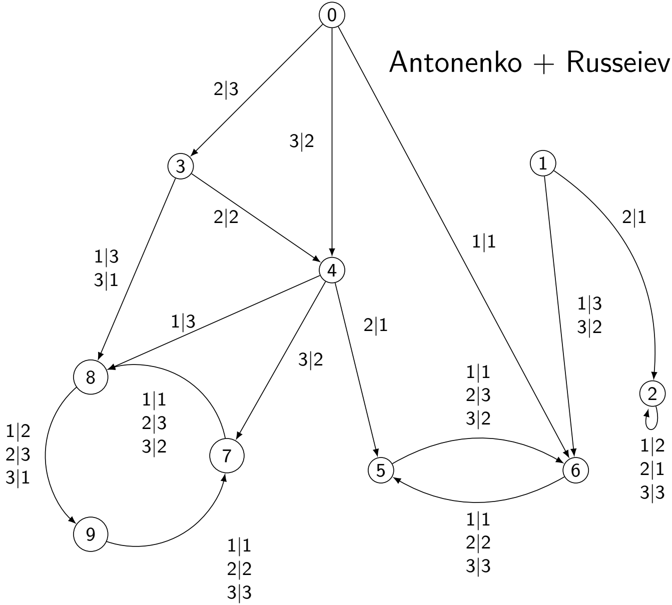
Difficult problem + unefficient rejection sampling.

Random automata

Antonenko + Russeiev



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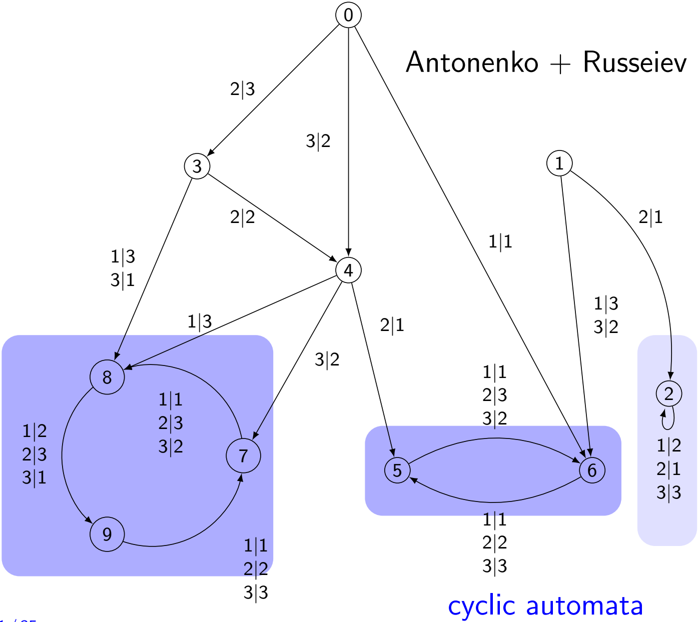
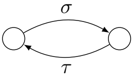


Random 2-state cyclic automata

$\langle$ 

$\rangle = \langle(\sigma,\tau),(\tau,\sigma)\rangle$

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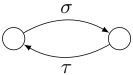
$\langle$ 

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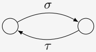
Contribution

$\langle$ 

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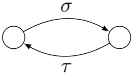
$\langle$ 

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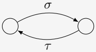
$\langle$ 

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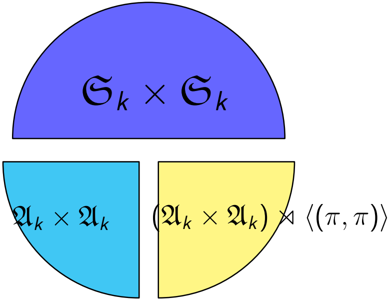

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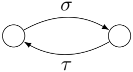
$\langle$  $\rangle = \langle(\sigma,\tau),(\tau,\sigma)\rangle$

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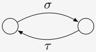
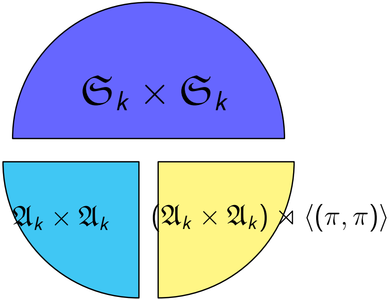
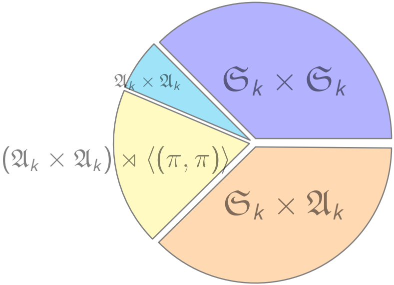
$\langle$  $\rangle = \begin{cases} \mathfrak{S}_k \times \mathfrak{S}_k \\ (\mathfrak{A}_k \times \mathfrak{A}_k) \rtimes \langle(\pi,\pi)\rangle \\ \mathfrak{A}_k \times \mathfrak{A}_k \end{cases}$





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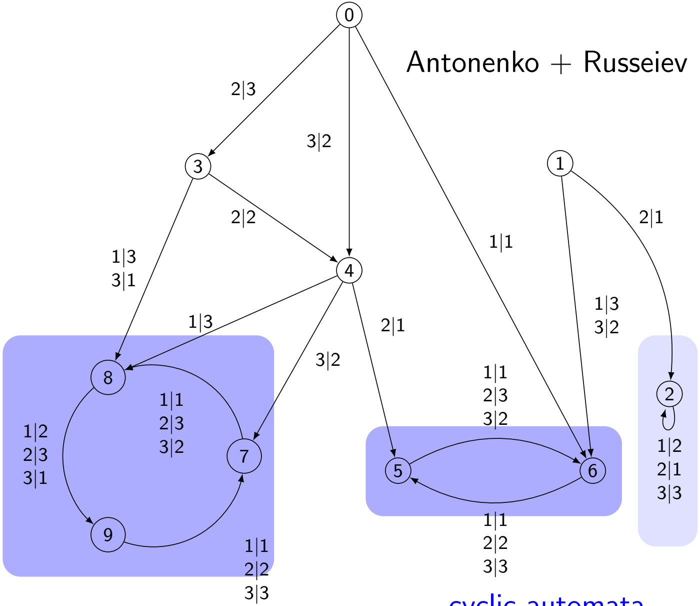

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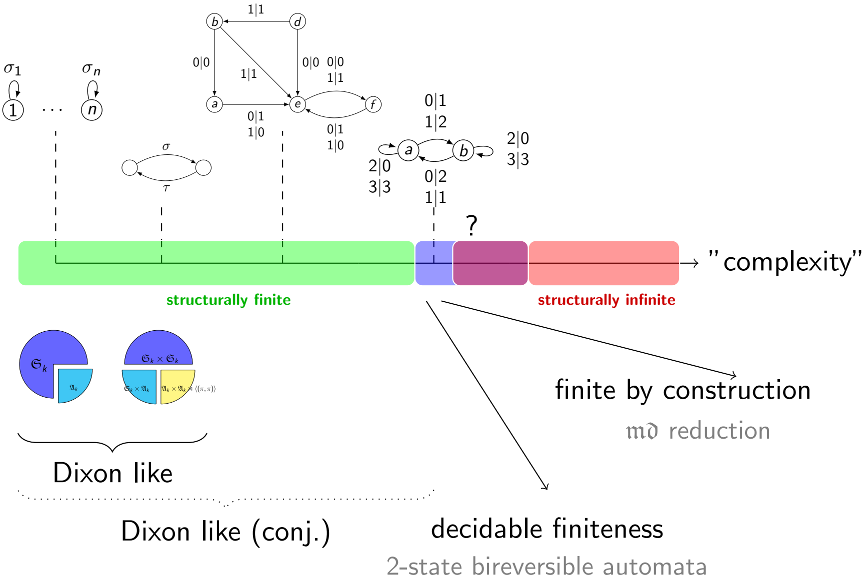
σ_1
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2|0
3|3
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1|1
?
"complexity"
structurally finite
structurally infinite
$\mathfrak{S}_k$
$\mathfrak{A}_k$
$\mathfrak{S}_k \times \mathfrak{S}_k$
$\mathfrak{S}_k \times \mathfrak{A}_k$
$\mathfrak{A}_k \times \mathfrak{A}_k \rtimes \langle(\pi,\pi)\rangle$
Dixon like
Dixon like (conj.)
finite by construction
𝔪𝔡 reduction
decidable finiteness
2-state bireversible automata

Asymptotics

Theorem (Dixon 1969,2005)

$$\mathbb{P}(\langle \sigma, \tau \rangle = \mathfrak{S} \text{ or } \mathfrak{A}) \sim 1 - 1/k - 1/k^2 - 4/k^3 - 23/k^4 - 171/k^5 - \cdots$$

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proof: [Erdős, Turán 1967]

$$\frac{\log|\sigma| - 1/2\log^2 k}{1/\sqrt{3}\log^{3/2} k} \to \mathcal{N}(0,1)$$

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rmq:

$$\mathbb{P}(|\sigma| = p) \sim \frac{1}{\sqrt{p}} k^{k(1-1/p)} \exp\left(-k(1-1/p) + k^{1/p}\right)$$

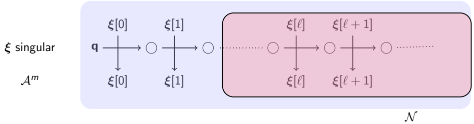
1|0 0|0 1|1

Mealy automata

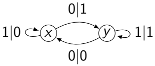
dynamics of the action

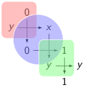
singular points

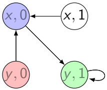
The set of singular points of a contracting automaton is described by a Büchi automaton [DGKPR'16]



Wang tillings







Schreier graphs

Stabilisers and singular points

The stabilisers of an infinite point ξ is $\mathsf{Stab}_{\langle \mathcal{A} \rangle}(\xi) = \{g \in \langle \mathcal{A} \rangle \mid g(\xi) = \xi\}$

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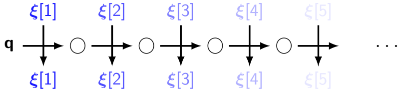

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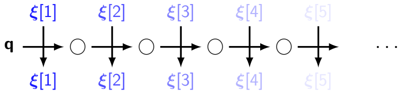
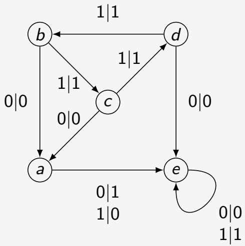


Example



$\rho_e, \rho_b, \rho_c, \rho_d \in \mathrm{Stab}_{\langle\mathcal{G}\rangle}(1^\omega)$
studied by Y. Vorobets

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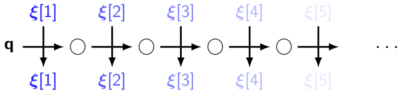

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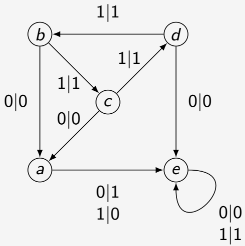

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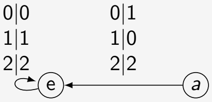
Interesting elements



2^ω is stabilised by ρ_a

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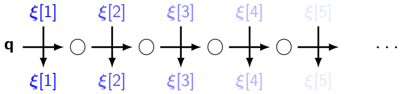

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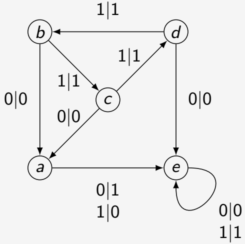

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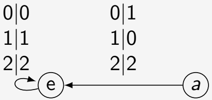

2^ω is stabilised by ρ_a

Singular points

ξ singular if $\exists g$ stabilizing ξ and avoiding ending in e

Contracting automata

$\mathcal{A}$ contracting $\iff \exists$ finite $\mathcal{N}$, $\forall \mathbf{q}, \forall \boldsymbol{\xi}, \exists n,\ \delta_{\boldsymbol{\xi}[:n]}(\mathbf{q}) \in \mathcal{N}$

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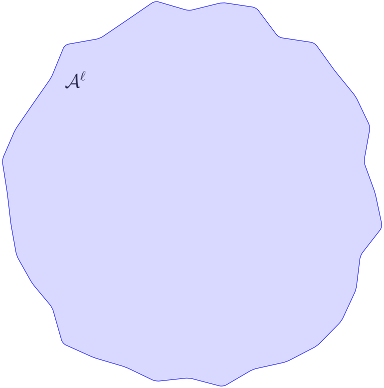

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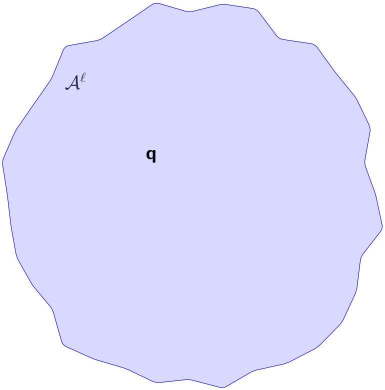

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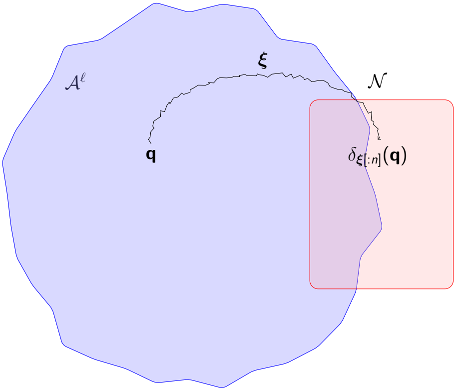

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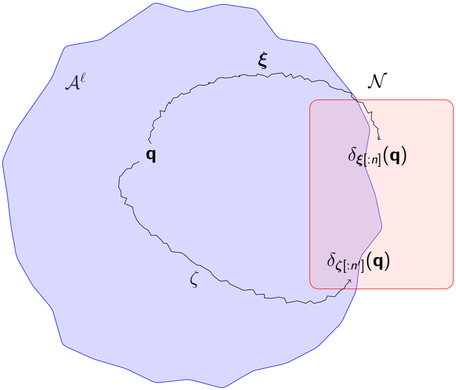

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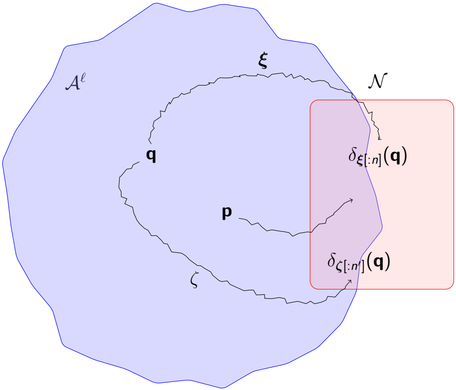

Contracting automata

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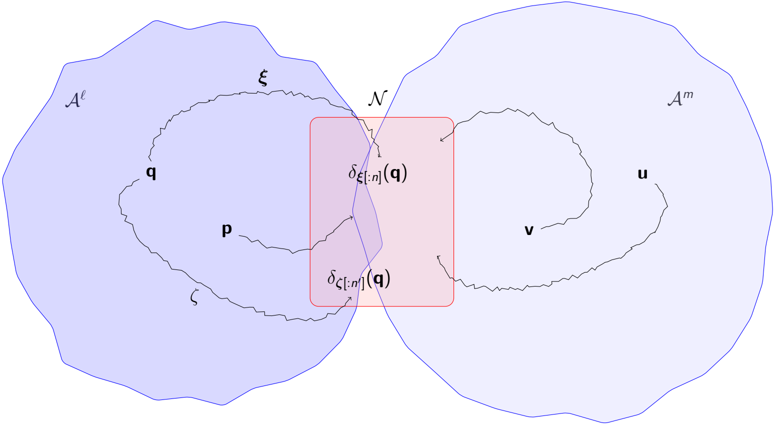

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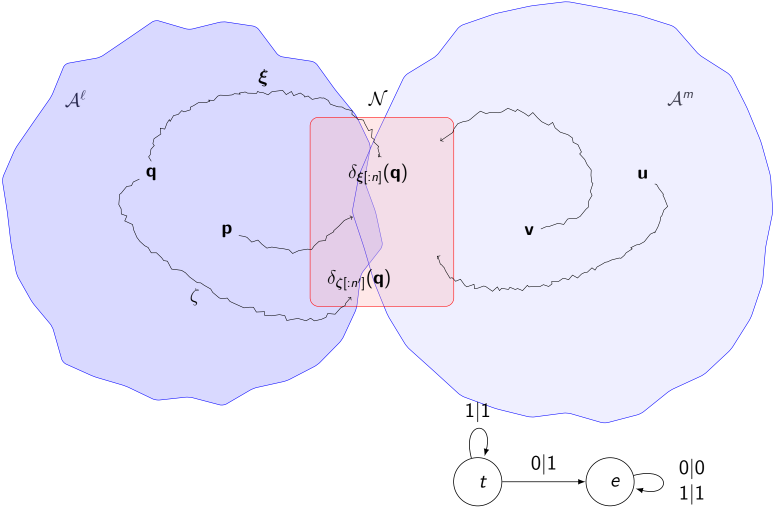

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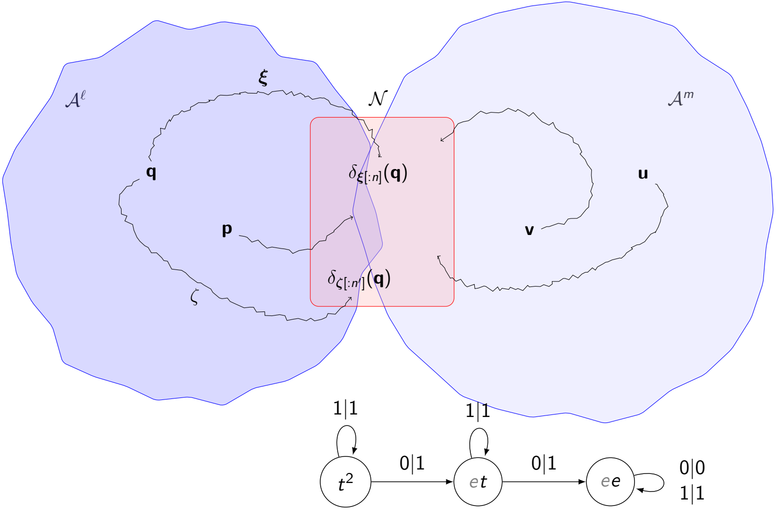

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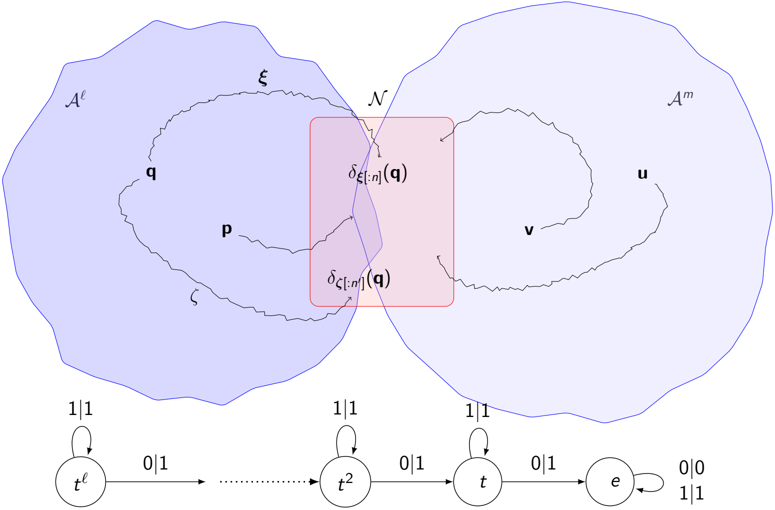
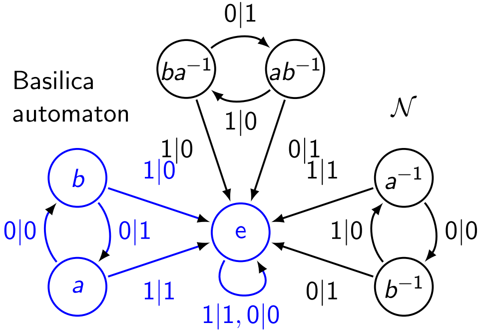


Contracting automata and singular points



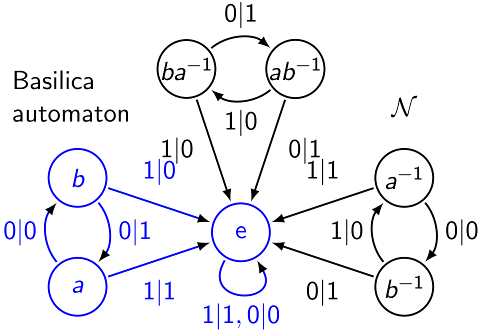
Contracting automata and singular points

0 0

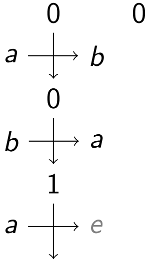
a

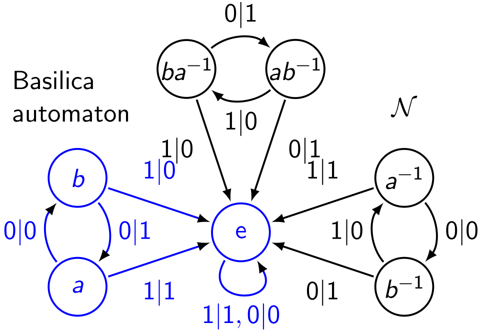
b

a

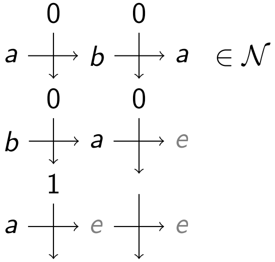


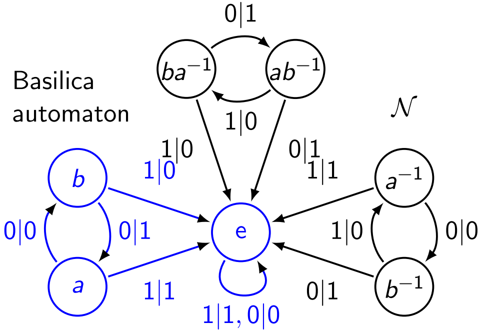
Contracting automata and singular points





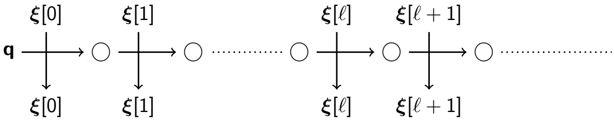
Contracting automata and singular points

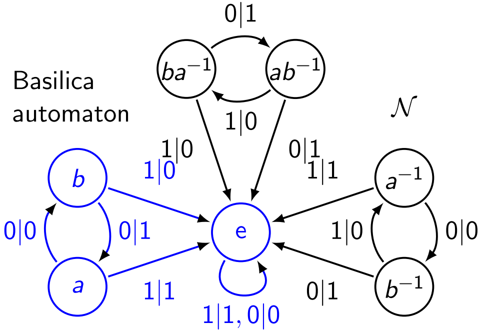




Contracting automata and singular points

ξ singular

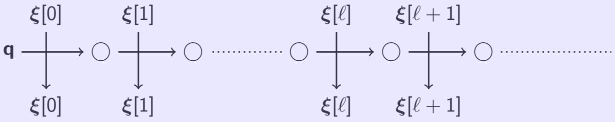


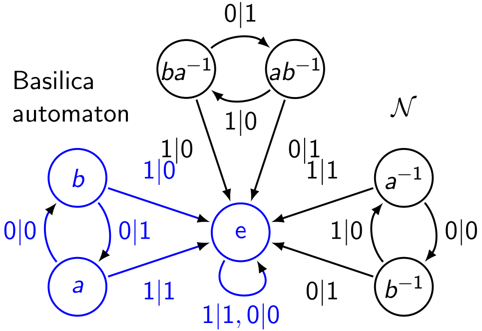


Contracting automata and singular points

ξ singular

$\mathcal{A}^m$

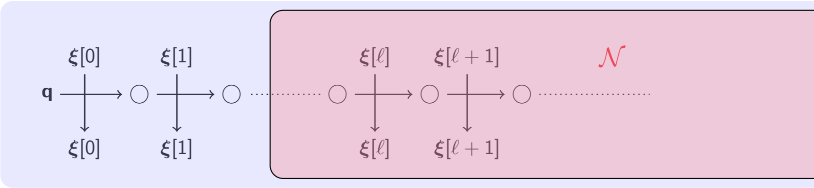


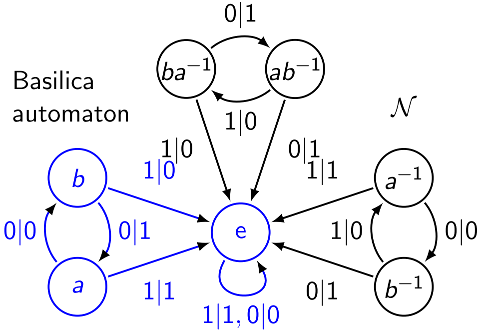


Contracting automata and singular points

ξ singular

$\mathcal{A}^m$





Contracting automata and singular points

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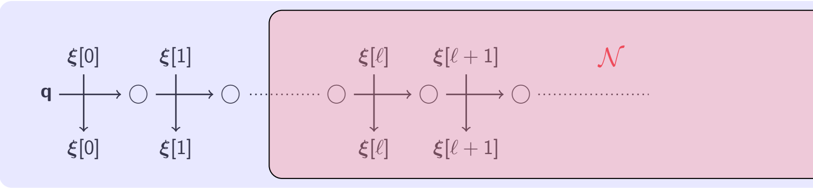
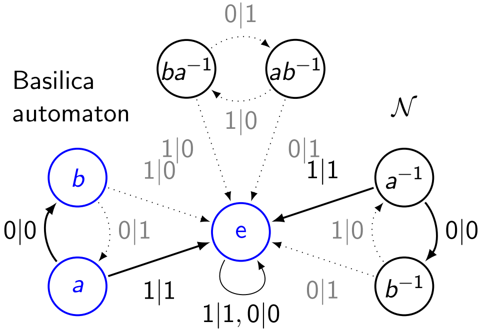
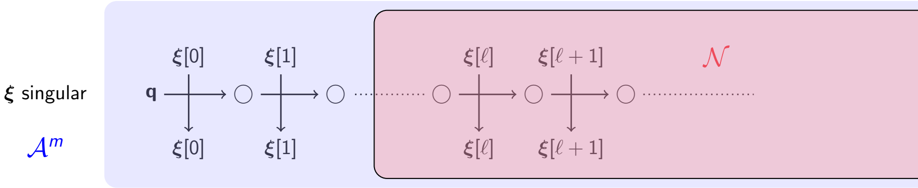
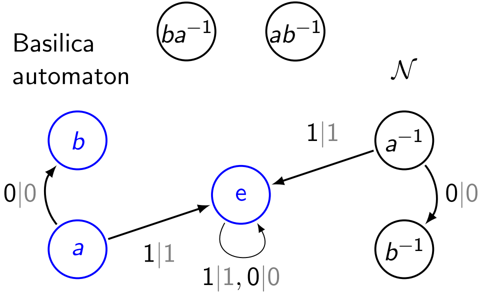


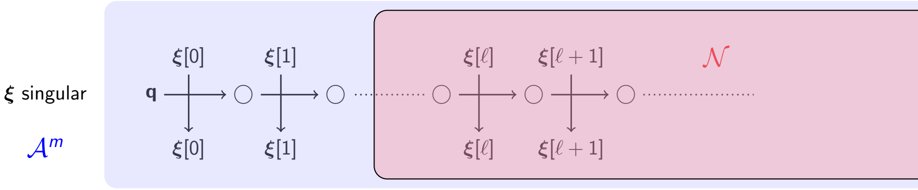


Contracting automata and singular points

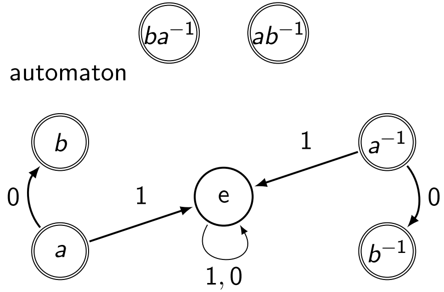




Contracting automata and singular points



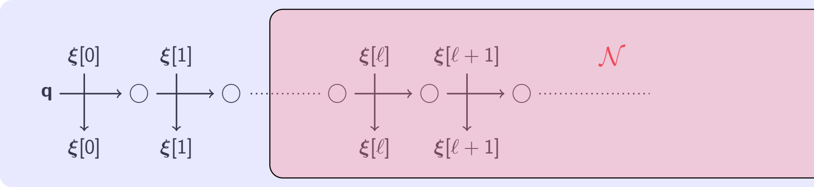
Büchi automaton



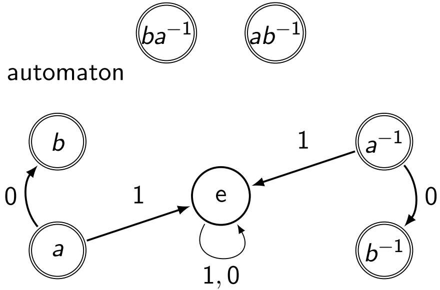
Contracting automata and singular points

ξ singular

$\mathcal{A}^m$



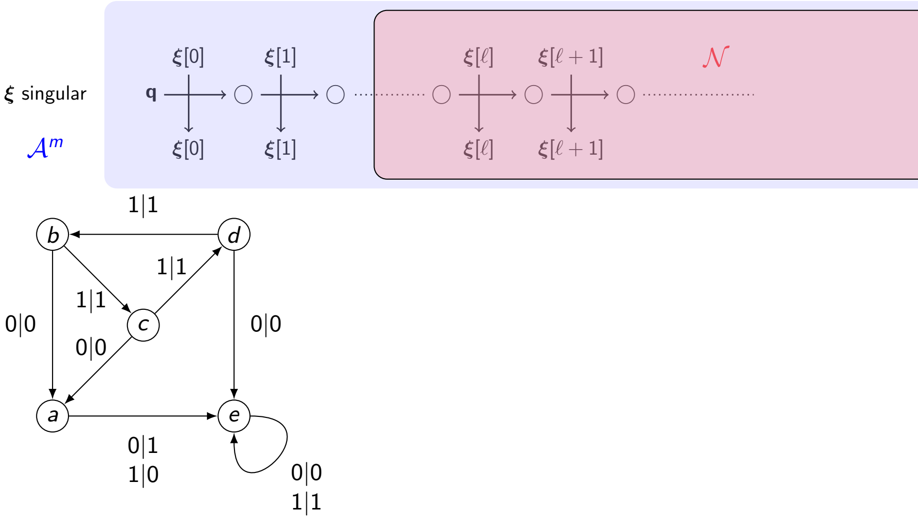
Büchi automaton



Contribution

$\text{Sing}(\mathcal{B}) = \emptyset$.

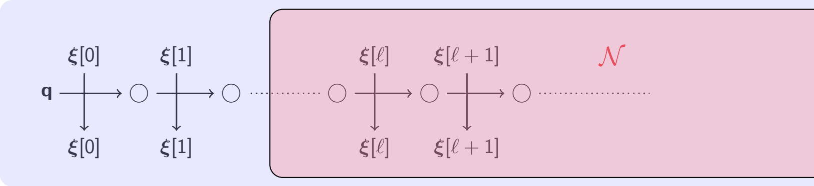
Contracting automata and singular points

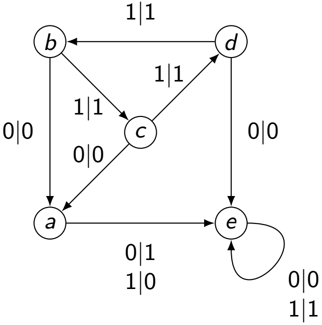


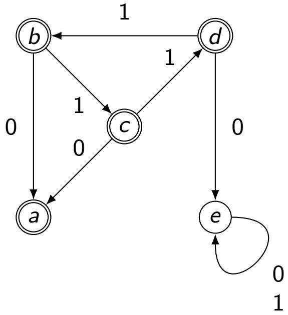
Contracting automata and singular points

ξ singular

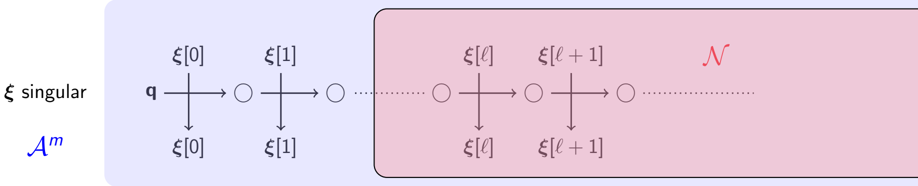
$\mathcal{A}^m$

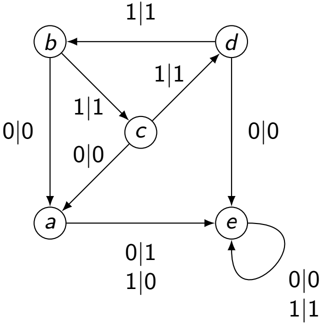


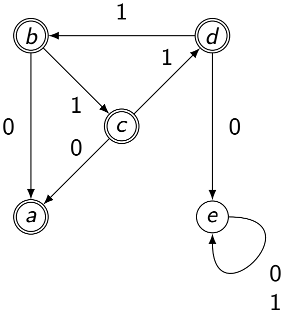




Contracting automata and singular points







Proposition [Vorobets, DGKPR]

$\mathrm{Sing}(\mathcal{G}) = (0+1)^* 1^\omega$.

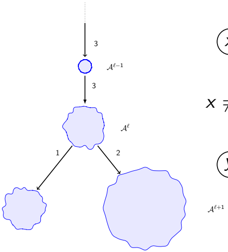
finiteness

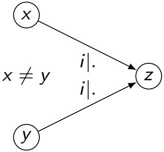
Invertible reversible non-coreversible automata generate infinite non Burnside groups
[LATA'15 w. Klimann and Picantin]

Bireversible automata of prime size cannot generate infinite Burnside groups
[MFCS'16 w. Klimann]

infinite Burnside

automaton patterns and group properties

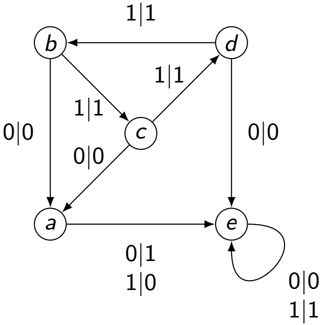




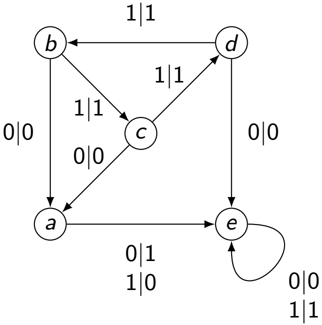
growth

The set o
of a contr
is describe
automaton

About the Grigorchuk automaton



About the Grigorchuk automaton



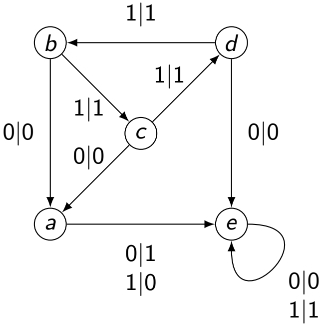
Actions of the states on the letters:

$$\rho_a : 0 \mapsto 1 \mapsto 0$$

$$\rho_b, \rho_c, \rho_d, \rho_e : 0 \mapsto 0; \quad 1 \mapsto 1$$

$\rightarrow$permutations

About the Grigorchuk automaton



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$\rightarrow$**invertible**

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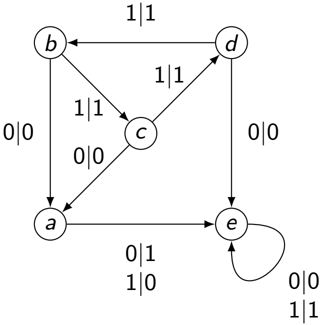

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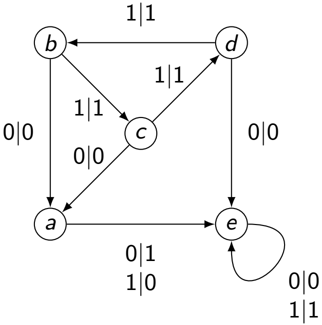
Action of a letter on the states:
$\delta_0 : a, d, e \mapsto e; \quad b, c \mapsto a$

$\rightarrow$not a permutation

About the Grigorchuk automaton



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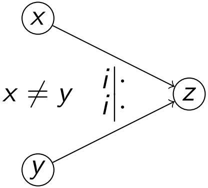
→permutations
→**invertible**

Action of a letter on the states:
$\delta_0 : a, d, e \mapsto e; \quad b, c \mapsto a$

→not a permutation
→**non-reversible**

About the Grigorchuk automaton

Reversibility:
Each input letter permutes the stateset.



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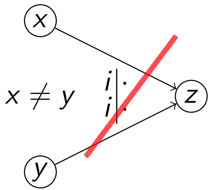

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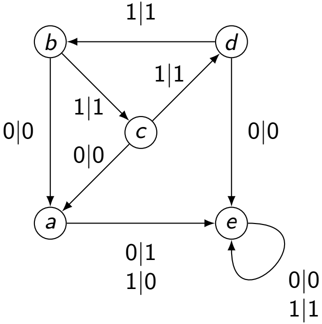
→permutations
→**invertible**

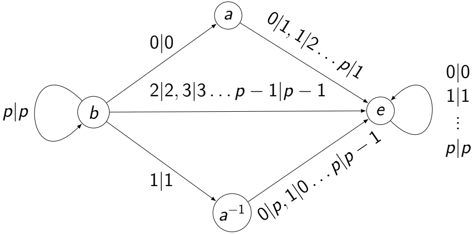
Action of a letter on the states:
$\delta_0 : a, d, e \mapsto e; \quad b, c \mapsto a$

→not a permutation
→**non-reversible**

Observation

Every known automaton generating an infinite Burnside group happens to be non-reversible.





Question

Can a reversible automaton generate an infinite Burnside group?

Theorem(s)

An invertible and reversible automata which is:

cannot generate an infinite Burnside group.

Theorem(s)

An invertible and reversible automata which is:

2-state

[Klimann]

STACS'13

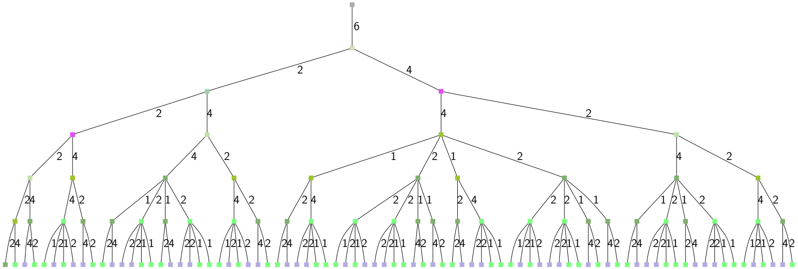
cannot generate an infinite Burnside group.

Theorem(s)

An invertible and reversible automata which is:

2-state	connected 3-state
[Klimann]	[Klimann, Picantin, and Savchuk]
STACS'13	DLT'15

cannot generate an infinite Burnside group.

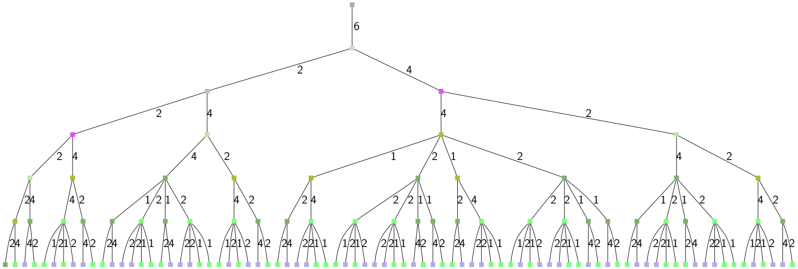


Theorem(s)

An invertible and reversible automata which is:

2-state	connected 3-state	non coreversible
[Klimann]	[Klimann, Picantin, and Savchuk]	[G., Klimann, and Picantin]
STACS'13	DLT'15	LATA'15

cannot generate an infinite Burnside group.



Theorem(s)

An invertible and reversible automata which is:

2-state	connected 3-state	non coreversible	connected with prime size
[Klimann]	[Klimann, Picantin, and Savchuk]	[G., Klimann, and Picantin]	[G. and Klimann]
STACS'13	DLT'15	LATA'15	MFCS'16

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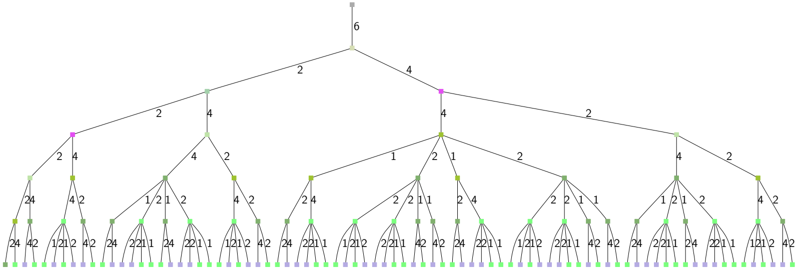
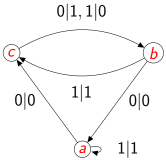


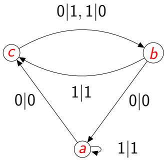
Schreier tree

$\mathcal{A}$

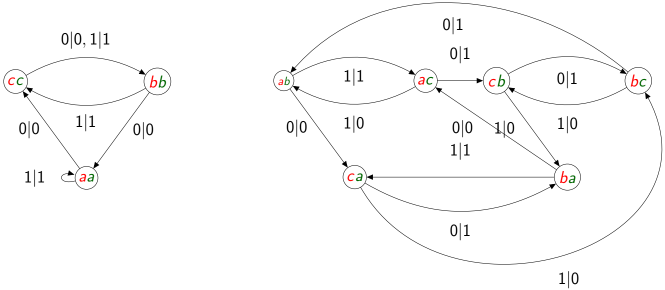


Schreier tree

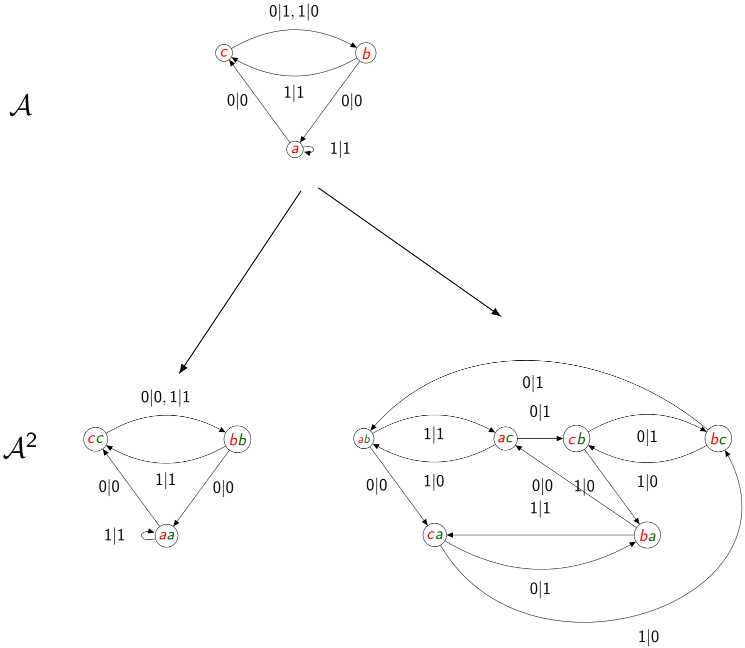
$\mathcal{A}$



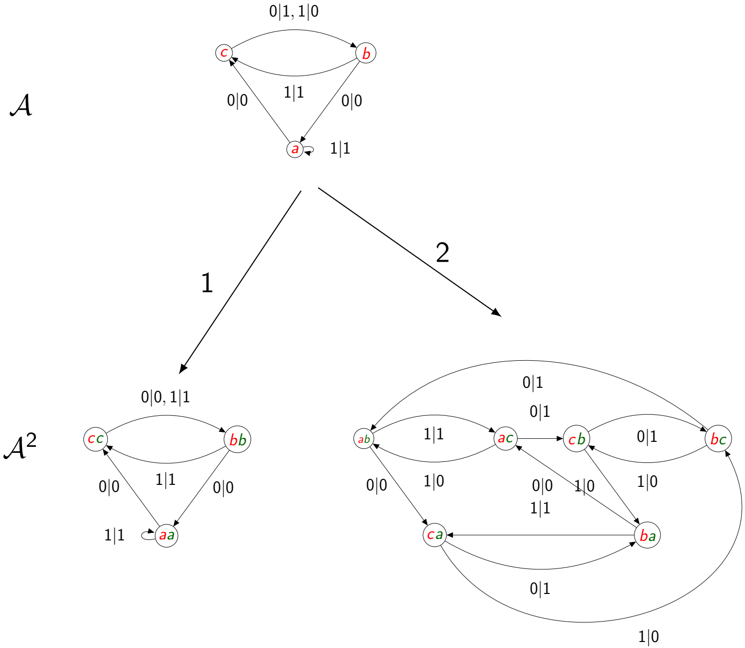
$\mathcal{A}^2$



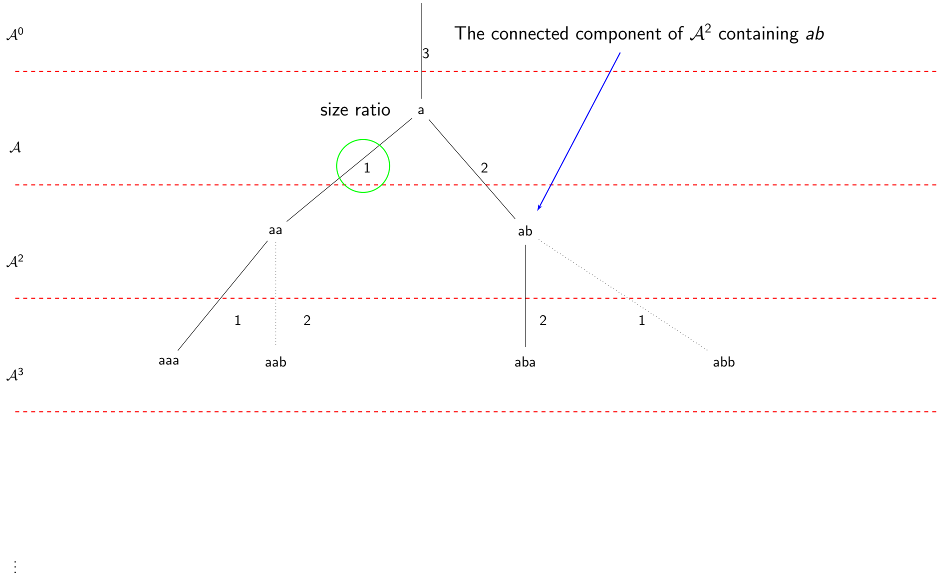
Schreier tree



Schreier tree



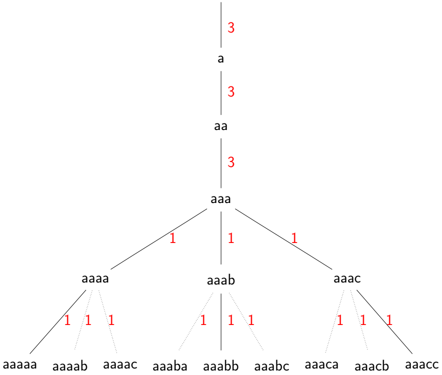
Schreier tree



Boundedness

Proposition

$\langle \mathcal{A} \rangle$ is finite iff the labels of the cc of $(\mathcal{A}^n)_n$ are ultimately 1.



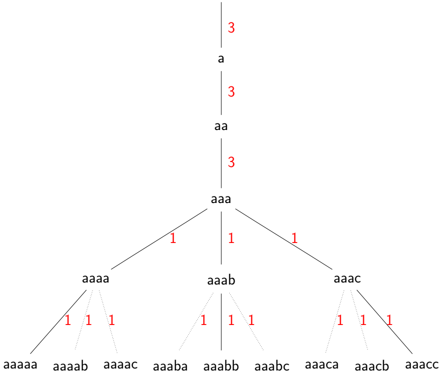
Boundedness

Proposition

$\langle \mathcal{A} \rangle$ is finite iff the labels of the cc of $(\mathcal{A}^n)_n$ are ultimately 1.

Proposition

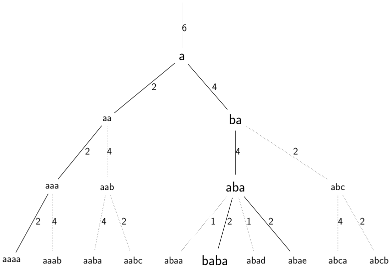
ρ_q has finite order iff the labels of the cc containing q^n are ultimately 1.

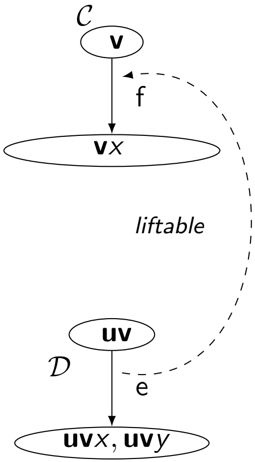


Liftable

Proposition

e liftable to f $\Rightarrow$ label(e) $\leq$ label(f).

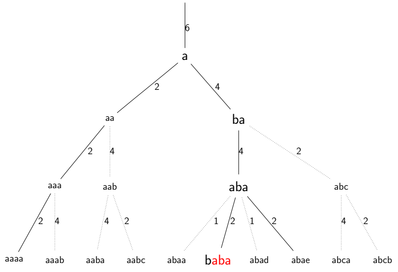


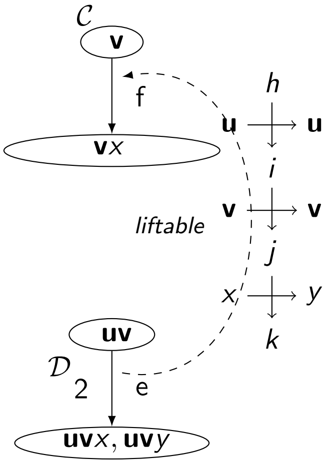


Liftable

Proposition

e liftable to $f \Rightarrow \text{label}(e) \leq \text{label}(f)$.





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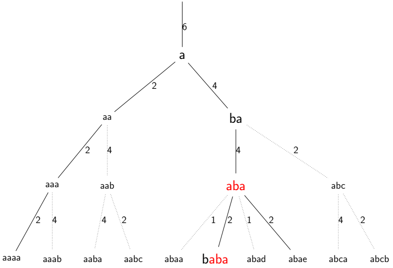

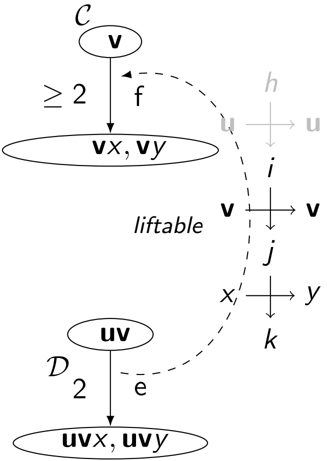

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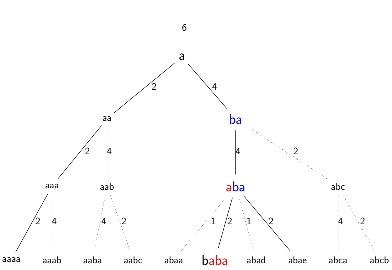
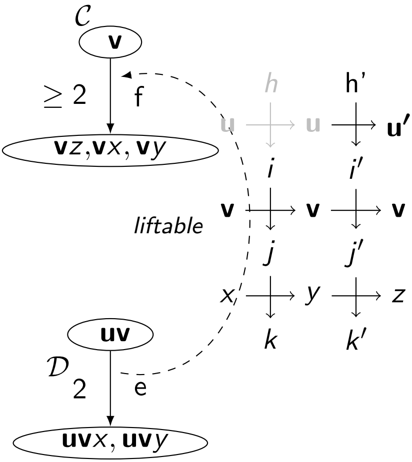
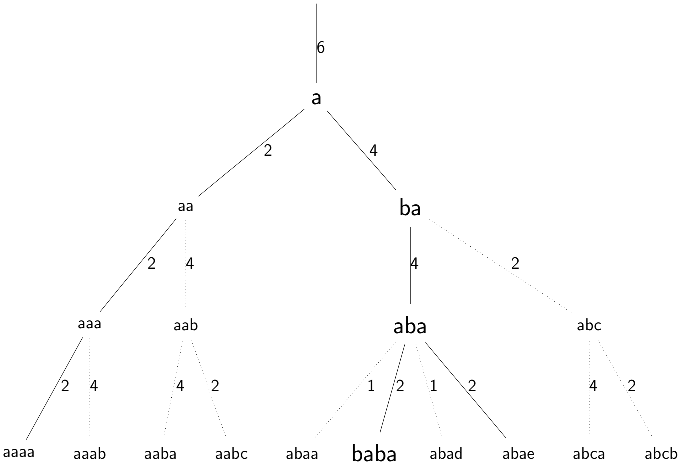
Proposition

e liftable to $f \Rightarrow \text{label}(e) \leq \text{label}(f)$.

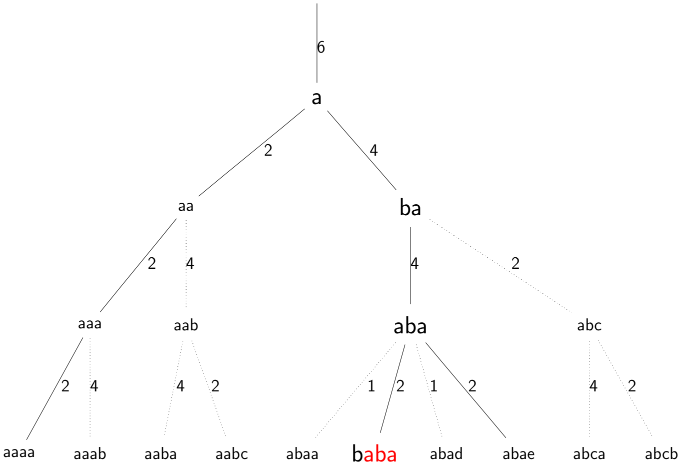




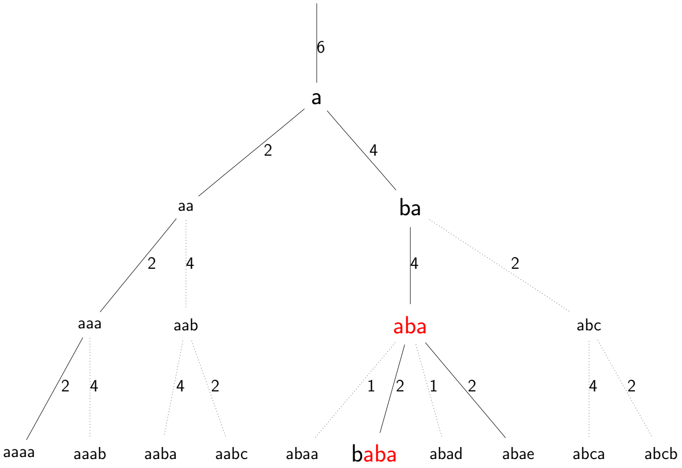
Liftable paths



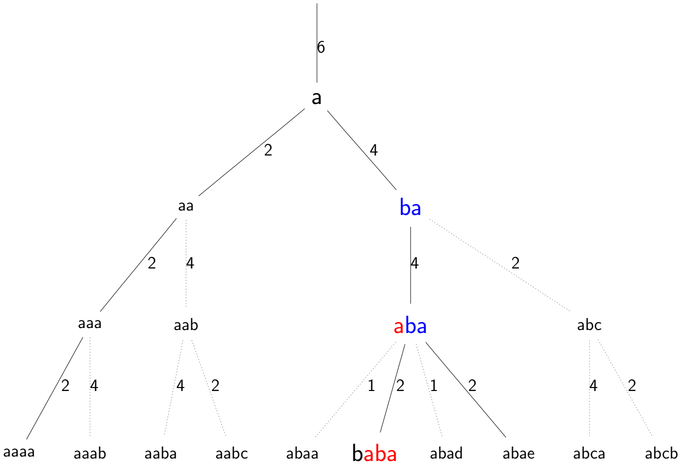
Liftable paths



Liftable paths



Liftable paths



Jungle tree

active $\equiv$ labels not ending with 1^ω.

If active liftable path ✓: not Burnside.

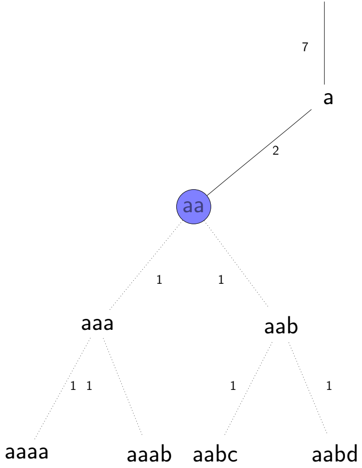
Otherwise :

Jungle tree

active $\equiv$ labels not ending with 1^ω.

If active liftable path ✓: not Burnside.

Otherwise :



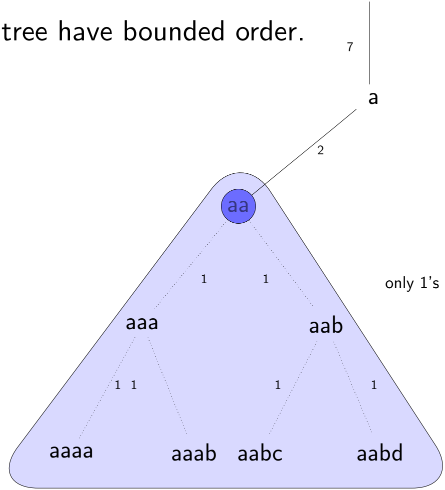
Jungle tree

active ≡ labels not ending with 1^ω.

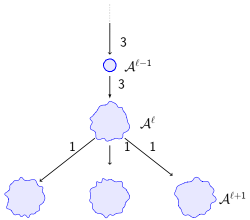
If active liftable path ✓: not Burnside.

Otherwise :

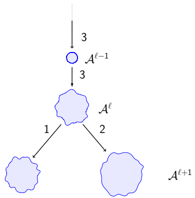
words in the jungle tree have bounded order.



3-state case

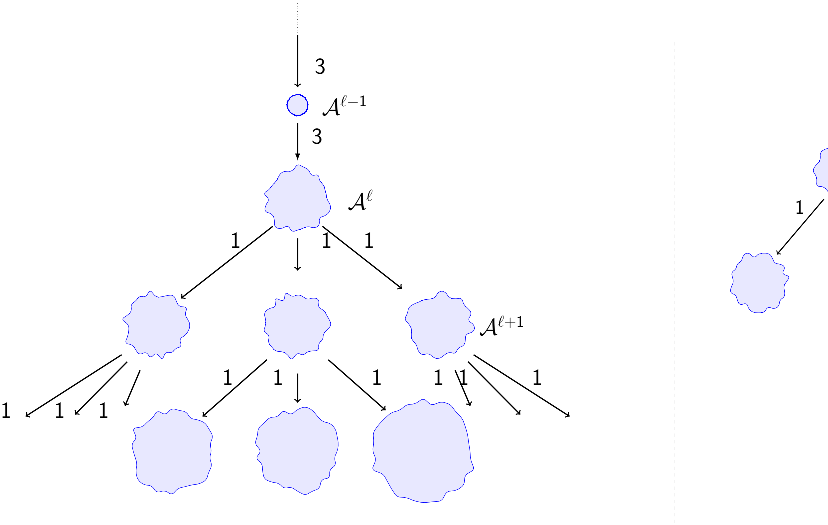


3
$\mathcal{A}^{\ell-1}$
3
$\mathcal{A}^{\ell}$
1
1
1
$\mathcal{A}^{\ell+1}$

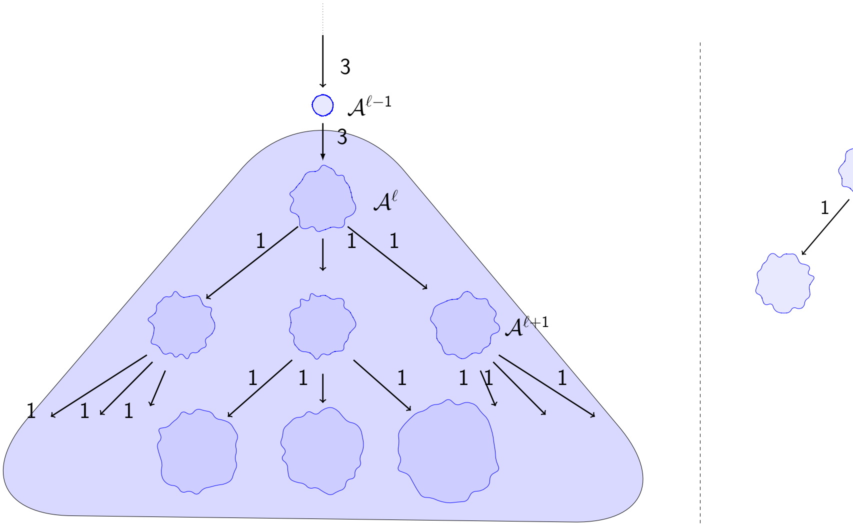


3
$\mathcal{A}^{\ell-1}$
3
$\mathcal{A}^{\ell}$
1
2
$\mathcal{A}^{\ell+1}$

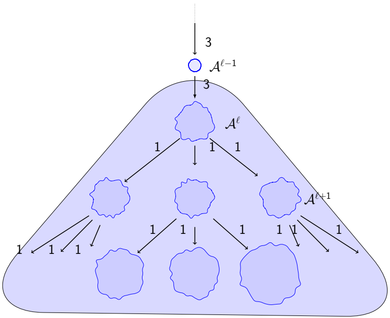
3-state case

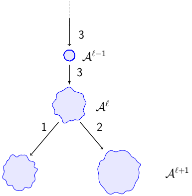

3
$\mathcal{A}^{\ell-1}$
3
$\mathcal{A}^{\ell}$
1
1
1
$\mathcal{A}^{\ell+1}$
1
1
1
1
1
1
1
1
1
1
1

3-state case

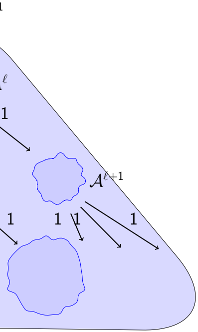

3
$\mathcal{A}^{\ell-1}$
3
$\mathcal{A}^{\ell}$
1 1 1
$\mathcal{A}^{\ell+1}$
1 1 1 1 1 1 1 1 1 1
1

3-state case

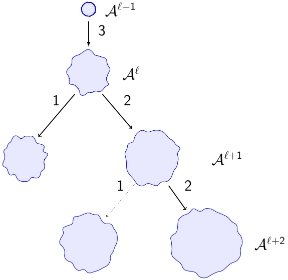

3
$\mathcal{A}^{\ell-1}$
3
$\mathcal{A}^{\ell}$
1 1 1
$\mathcal{A}^{\ell+1}$
1 1 1 1 1 1 1 1 1
1 1 1


3
$\mathcal{A}^{\ell-1}$
3
$\mathcal{A}^{\ell}$
1
2
$\mathcal{A}^{\ell+1}$

3-state case

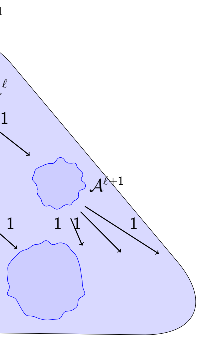


$\mathcal{A}^{\ell}$
1
$\mathcal{A}^{\ell+1}$
1
1
1
1

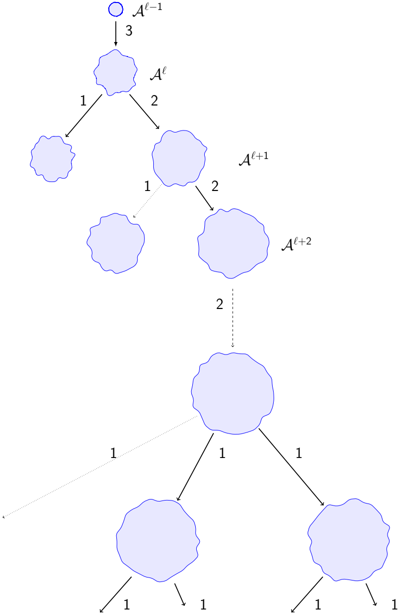


$\mathcal{A}^{\ell-1}$
3
$\mathcal{A}^{\ell}$
1
2
$\mathcal{A}^{\ell+1}$
1
2
$\mathcal{A}^{\ell+2}$

3-state case

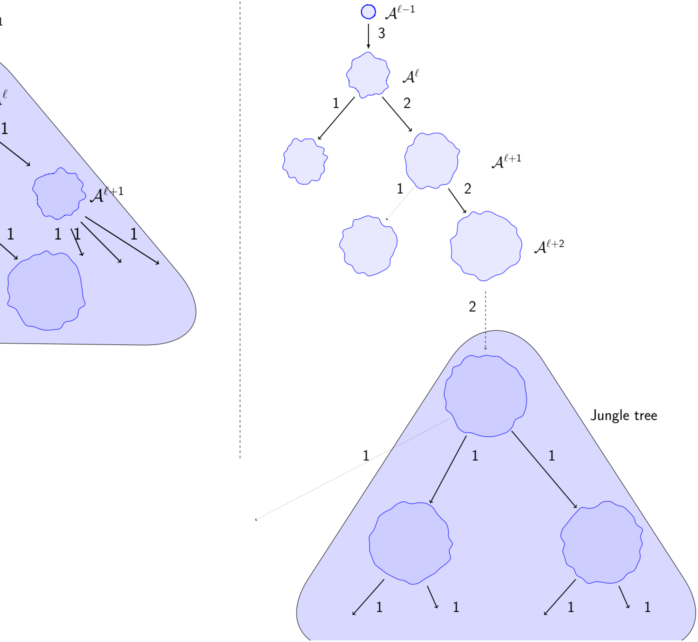


1
1
$\mathcal{A}^{\ell+1}$
1
1
1
1



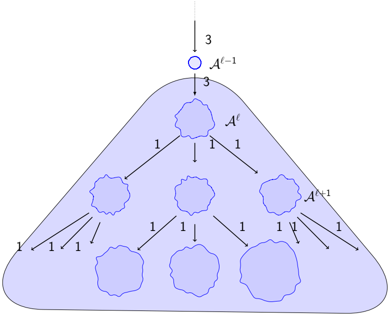
$\mathcal{A}^{\ell-1}$
3
$\mathcal{A}^{\ell}$
1
2
$\mathcal{A}^{\ell+1}$
1
2
$\mathcal{A}^{\ell+2}$
2
1
1
1
1
1
1
1

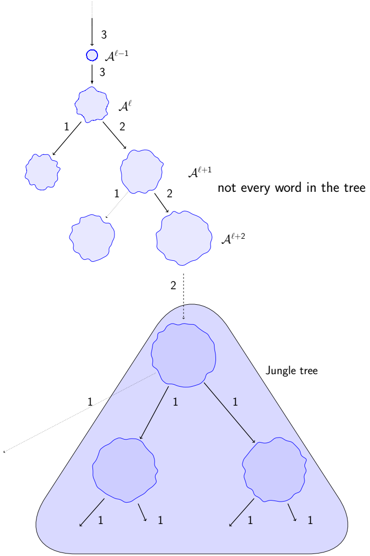
3-state case



$\mathcal{A}^{\ell}$
$\mathcal{A}^{\ell+1}$
1
1
1
1
1
1
$\mathcal{A}^{\ell-1}$
3
$\mathcal{A}^{\ell}$
1
2
$\mathcal{A}^{\ell+1}$
1
2
$\mathcal{A}^{\ell+2}$
2
Jungle tree
1
1
1
1
1
1
1

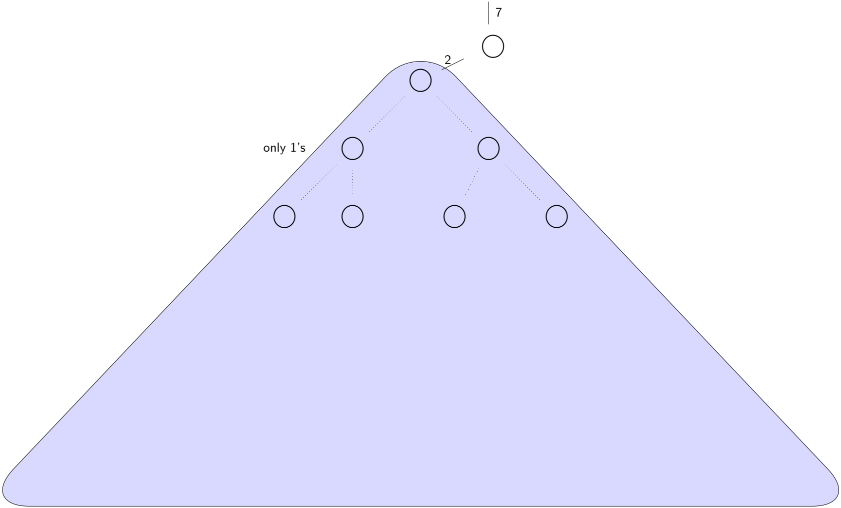
3-state case



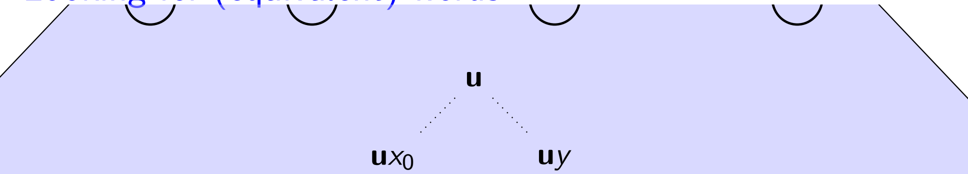


Every word in the tree ✓

Looking for (equivalent) words

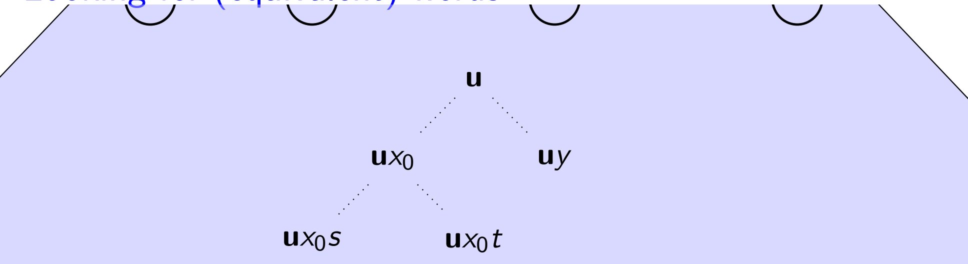


Looking for (equivalent) words



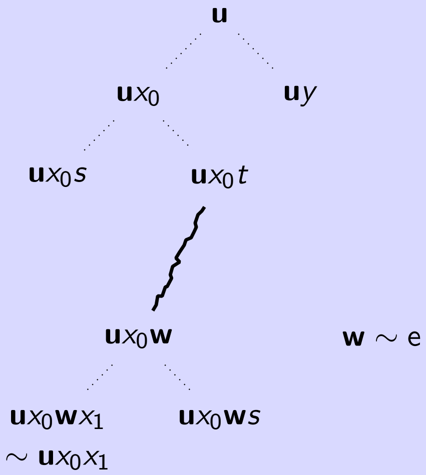
Idea: $\forall x_0 x_1 x_2 \cdots$ find a word with same action in the jungle tree

Looking for (equivalent) words



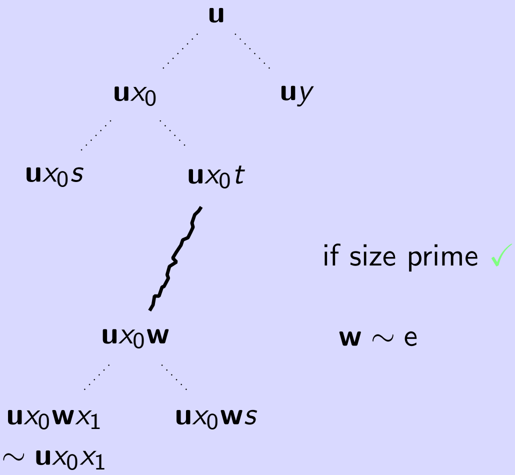
Idea: $\forall x_0 x_1 x_2 \cdots$ find a word with same action in the jungle tree

Looking for (equivalent) words



Idea: $\forall x_0 x_1 x_2 \cdots$ find a word with same action in the jungle tree

Looking for (equivalent) words



Idea: $\forall x_0 x_1 x_2 \cdots$ find a word with same action in the jungle tree

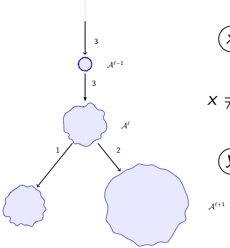
finiteness

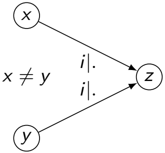
Invertible reversible non-coreversible automata generate infinite non Burnside groups
[LATA'15 w. Klimann and Picantin]

Bireversible automata of prime size cannot generate infinite Burnside groups
[MFCS'16 w. Klimann]

infinite Burnside

automaton patterns and group properties





growth

The set o
of a contr
is describe
automaton

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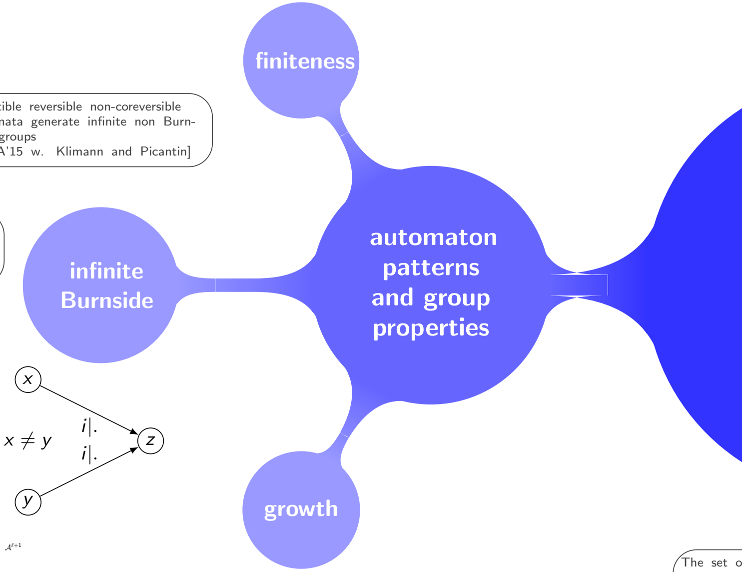
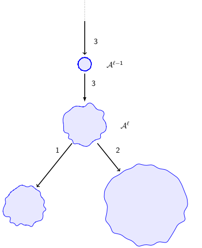




Bireversible automata with an element of infinite order have exponential growth
[Klimann'17+]

The set o
of a contr
is describe
automaton

Structure properties

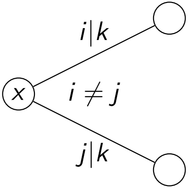
Invertibility:
Each state permutes the alphabet

Reversibility:
Each input letter permutes the stateset

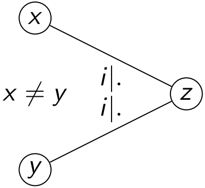
Coreversibility:
Each output letter permutes the stateset

Structure properties

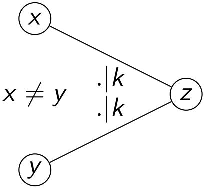
Invertibility:
Each state permutes the alphabet



Reversibility:
Each input letter permutes the stateset



Coreversibility:
Each output letter permutes the stateset



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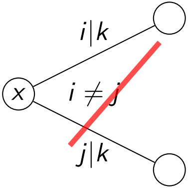
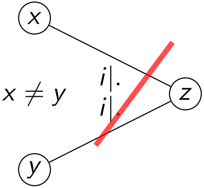
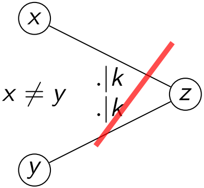


Reversibility:
Each input letter permutes the stateset



Coreversibility:
Each output letter permutes the stateset



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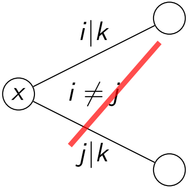
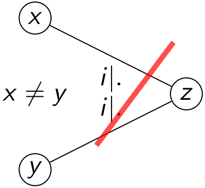
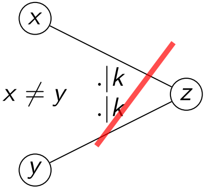


Reversibility:
Each input letter permutes the stateset



Coreversibility:
Each output letter permutes the stateset



Question

How to enumerate or/and (randomly) generate bireversible Mealy automata?

Invertible reversible non-coreversible automata generate infinite non Burnside groups
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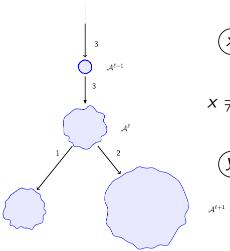

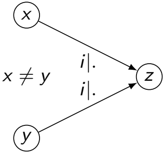

growth

The set o
of a contr
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automaton

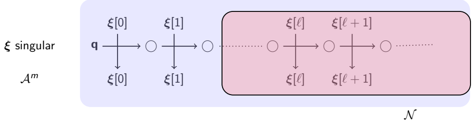
Bireversible automata with an element of infinite order have exponential growth
[Klimann'17+]

Mealy automata

dynamics of the action

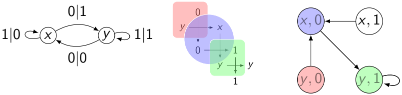
singular points

The set of singular points of a contracting automaton is described by a Büchi automaton [DGKPR'16]

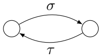


Wang tillings

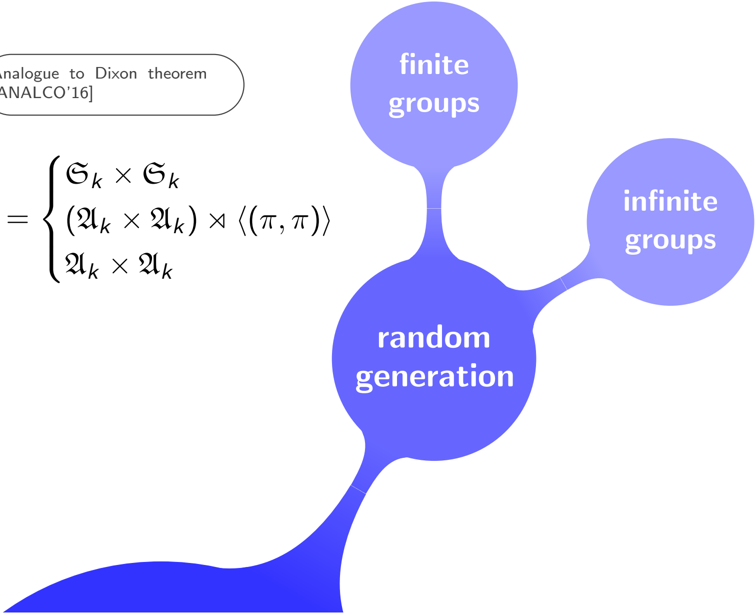
Schreier graphs



Analogue to Dixon theorem [ANALCO'16]



$$\langle \; \cdot \; \rangle = \begin{cases} \mathfrak{S}_k \times \mathfrak{S}_k \\ (\mathfrak{A}_k \times \mathfrak{A}_k) \rtimes \langle (\pi, \pi) \rangle \\ \mathfrak{A}_k \times \mathfrak{A}_k \end{cases}$$



Thanks!