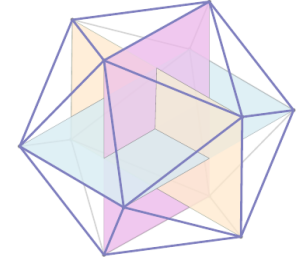




Survol d'algorithmes



Patrice Ossona de Mendez

Centre d'Analyse et de Mathématique Sociales (UMR 8557 CNRS/EHESS)

et

Computer Science Institute of Charles University (IUUK)

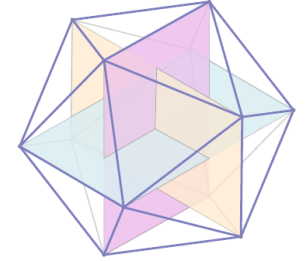
et

Laboratoire International Associé STRUCO

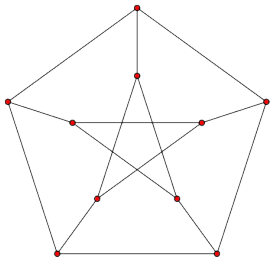




Qu'est-ce qu'un graphe?



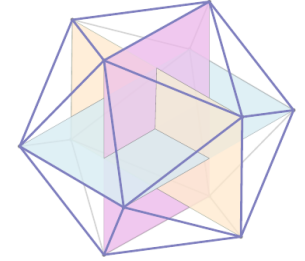
- Incidence entre $\|G\|$ arêtes et $|G|$ sommets
 - Relation binaire (adjacence)



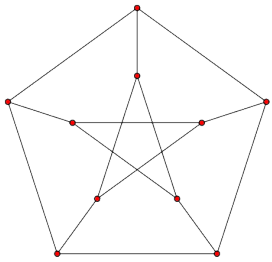
$$R = \left\{ \{1,2\}, \{2,3\}, \{3,4\}, \{4,5\}, \{5,1\}, \right. \\ \left. \{1,6\}, \{2,7\}, \{3,8\}, \{4,9\}, \{5,10\}, \right. \\ \left. \{6,8\}, \{7,9\}, \{8,10\}, \{9,6\}, \{10,7\} \right\}$$



Qu'est-ce qu'un graphe?



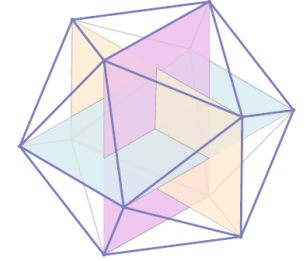
- Incidence entre $\|G\|$ arêtes et $|G|$ sommets
 - Relation binaire (adjacence)
 - Matrice symétrique de 0 et 1



$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$



Qu'est-ce qu'un graphe?

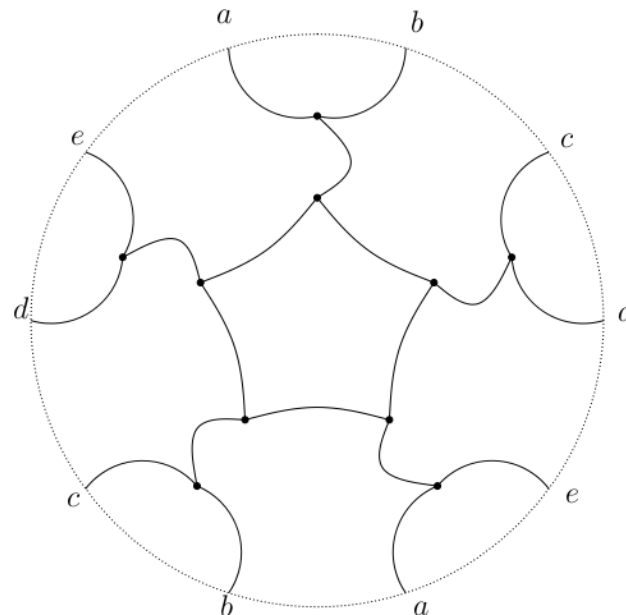
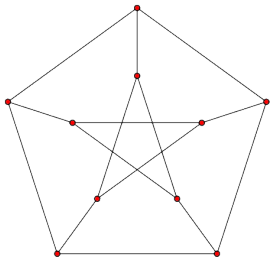


- Incidence entre $\|G\|$ arêtes et $|G|$ sommets

– Relation binaire (adjacence)

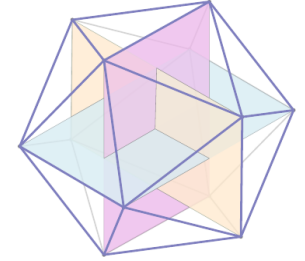
– Matrice symétrique de 0 et 1

– Des arcs joignant des points (sur une surface)



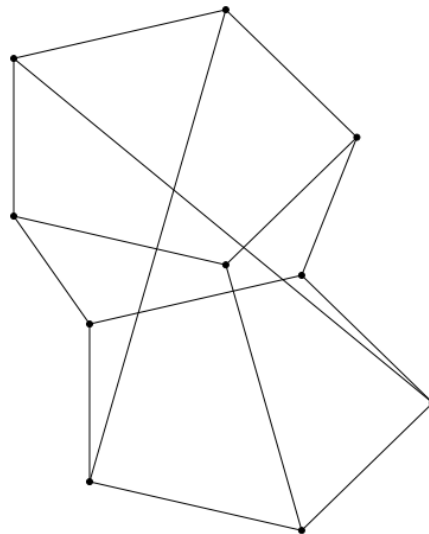
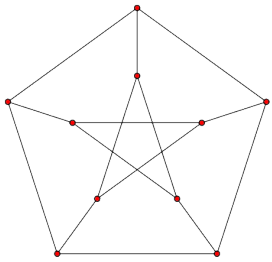


Qu'est-ce qu'un graphe?



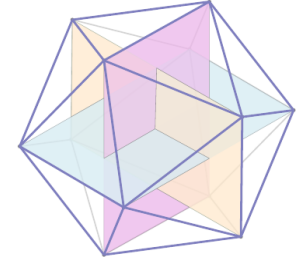
- Incidence entre $\|G\|$ arêtes et $|G|$ sommets

- Relation binaire (adjacence)
- Matrice symétrique de 0 et 1
- Des arcs joignant des points (sur une surface)
- **1-complexe simplicial**

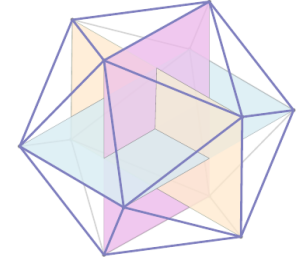




Quelques approches

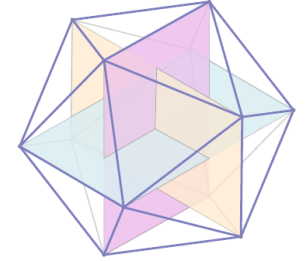


- Approche algébrique
 - Analyse spectrale
- Approche topologique
 - Parcours
- Approche structurelle
 - Orientations





Analyse spectrale



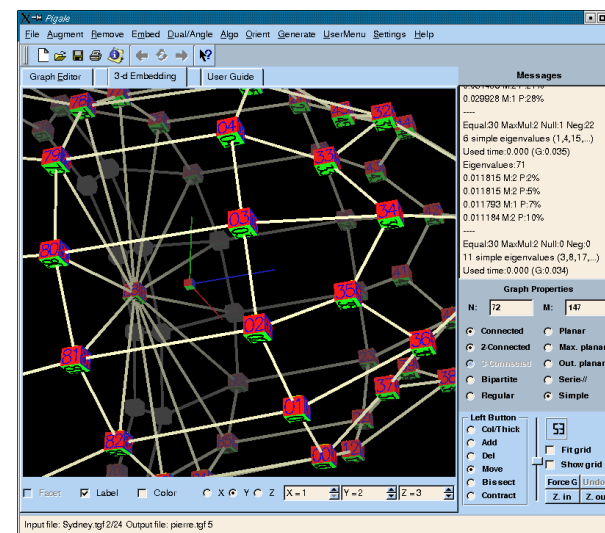
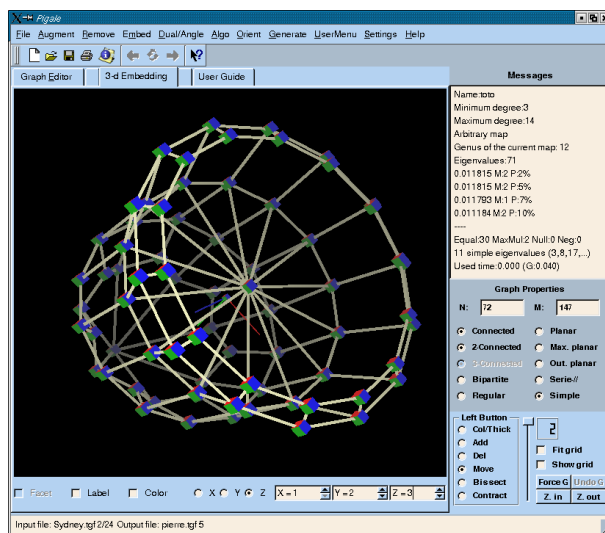
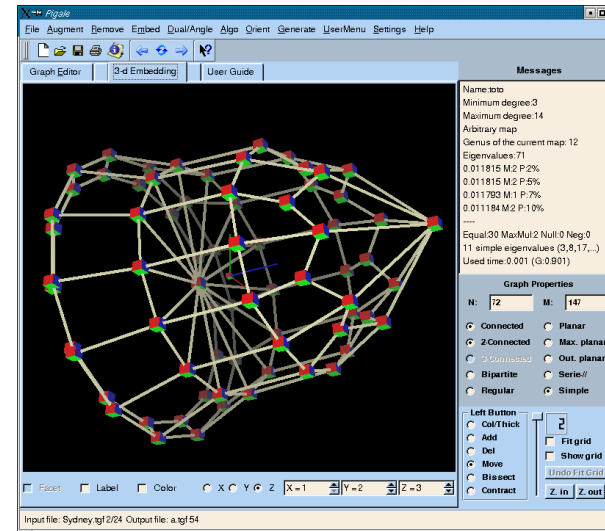
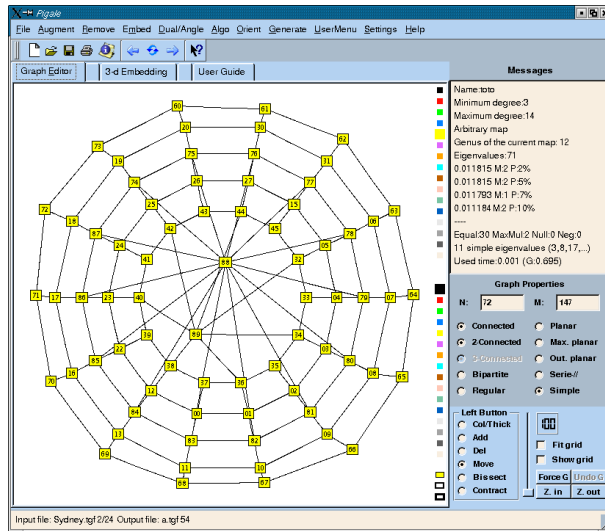
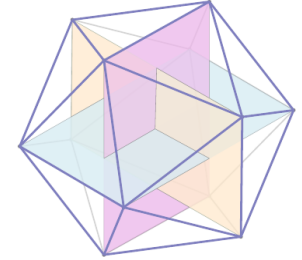
- Définition d'une distance intrinsèque à partir d'une prédistance D
 - Adjacence,
 - Distance dans le graphe,
 - Distance de Czekanovski-Dice bissectée,
 - *Laplacien*,
 - ...

$$D^* = D - 2\lambda_r(J - I)$$

$$B^* = -\frac{1}{2} \left(I - \frac{1}{n} J \right) D^* \left(I - \frac{1}{n} J \right) \quad (\text{PSD})$$

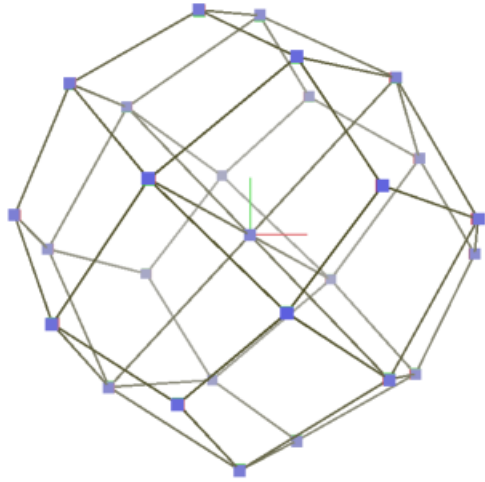
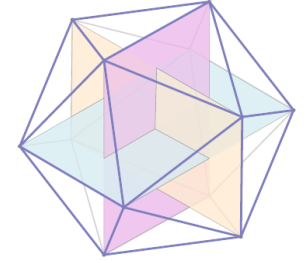


Plongement Euclidien



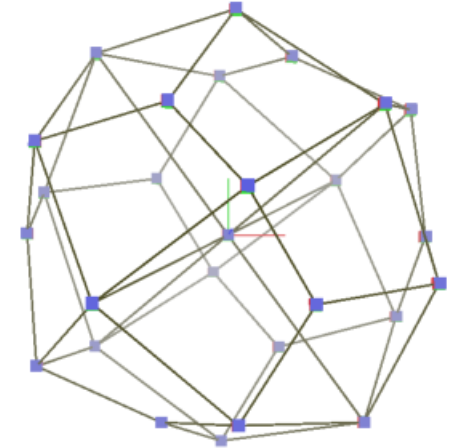


Distances

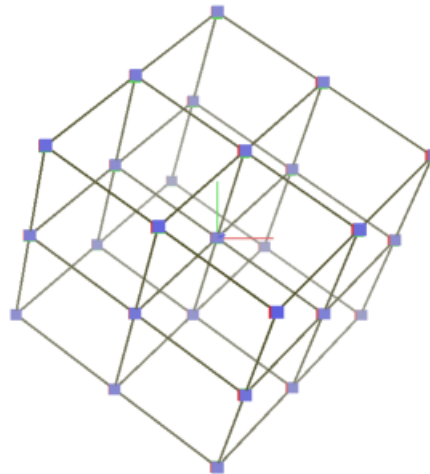


Czekanovski-Dice

Laplacien

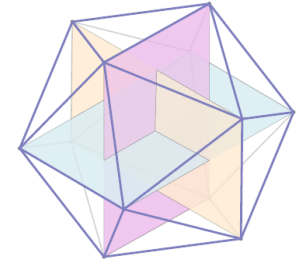


Adjacence translatée





Automorphismes

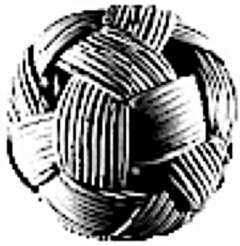


Caractérisation des automorphismes

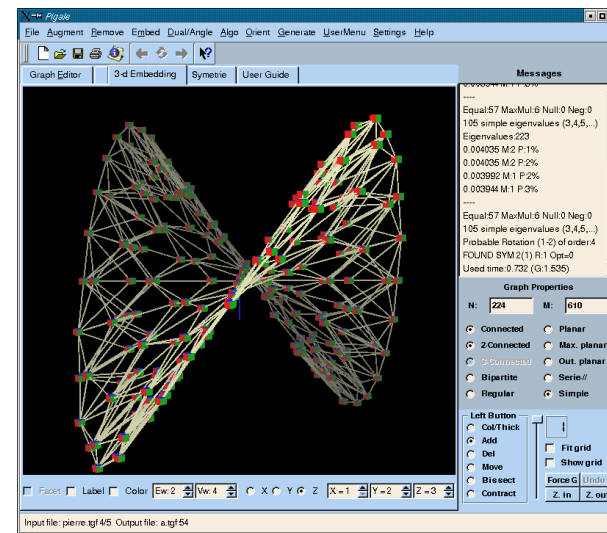
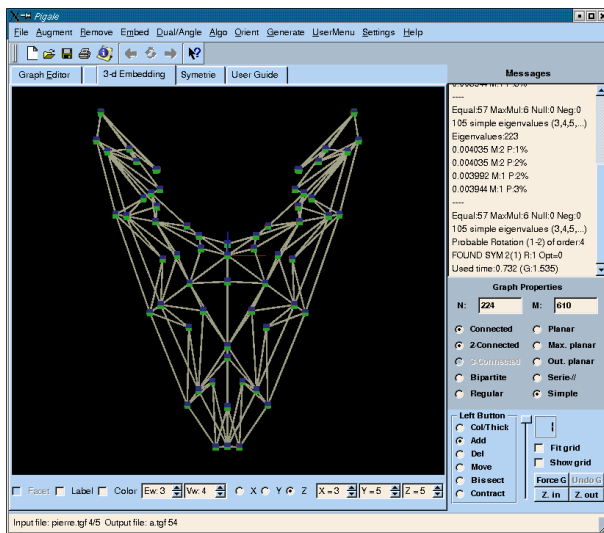
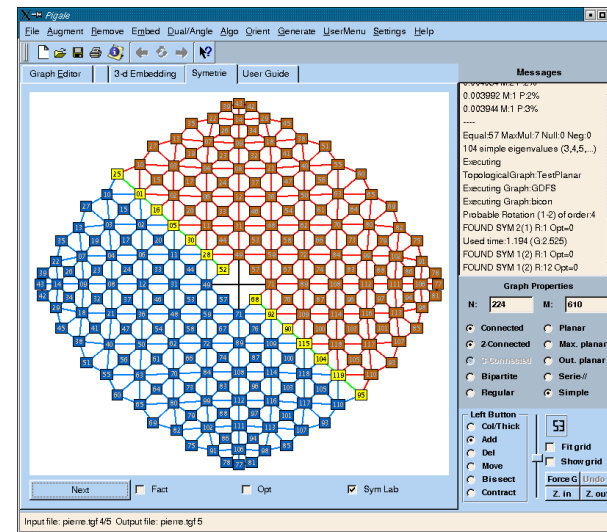
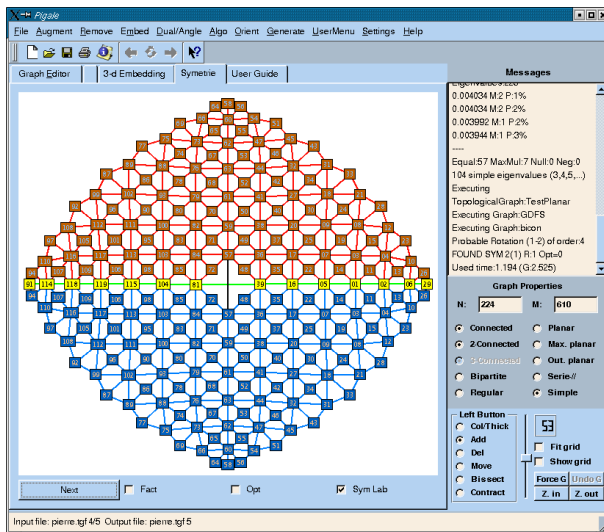
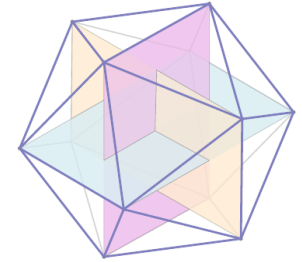
(de Fraysseix, POM)

Automorphismes du graphe = Isométries
laissant les sommets globalement invariants

$$\text{Aut}(G) \cong \omega(m_2, P_2) \oplus \omega(m_3, P_3) \oplus \cdots \oplus \omega(m_{s-1}, P_{s-1})$$

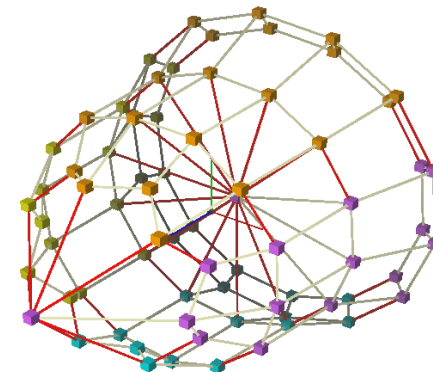
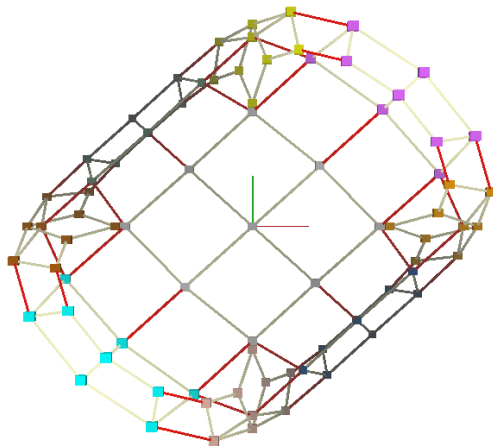
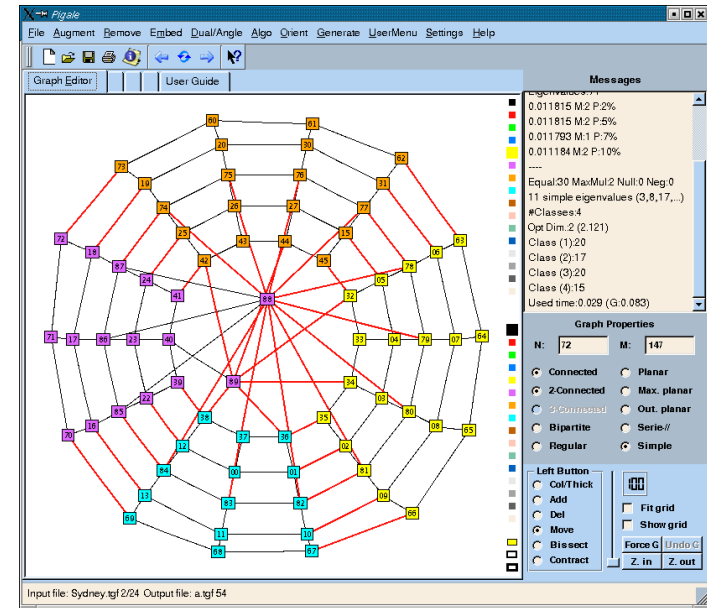
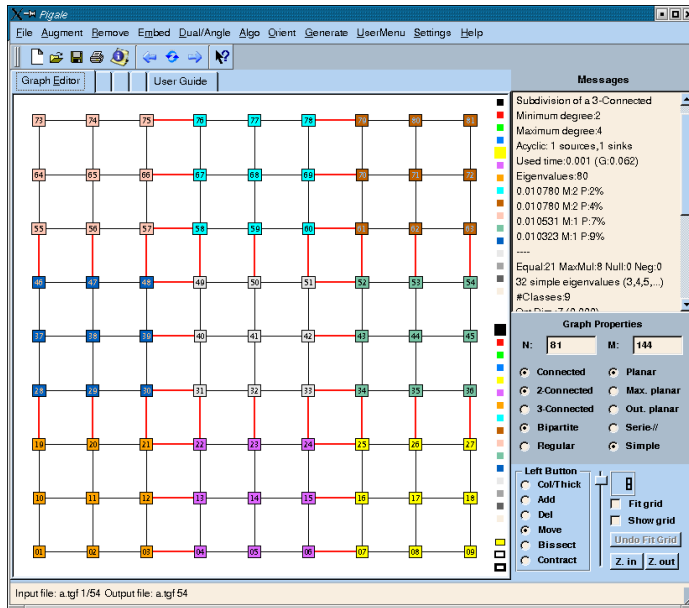
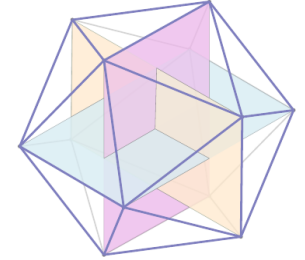


Symétries



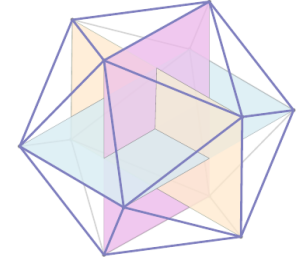


Partitionement



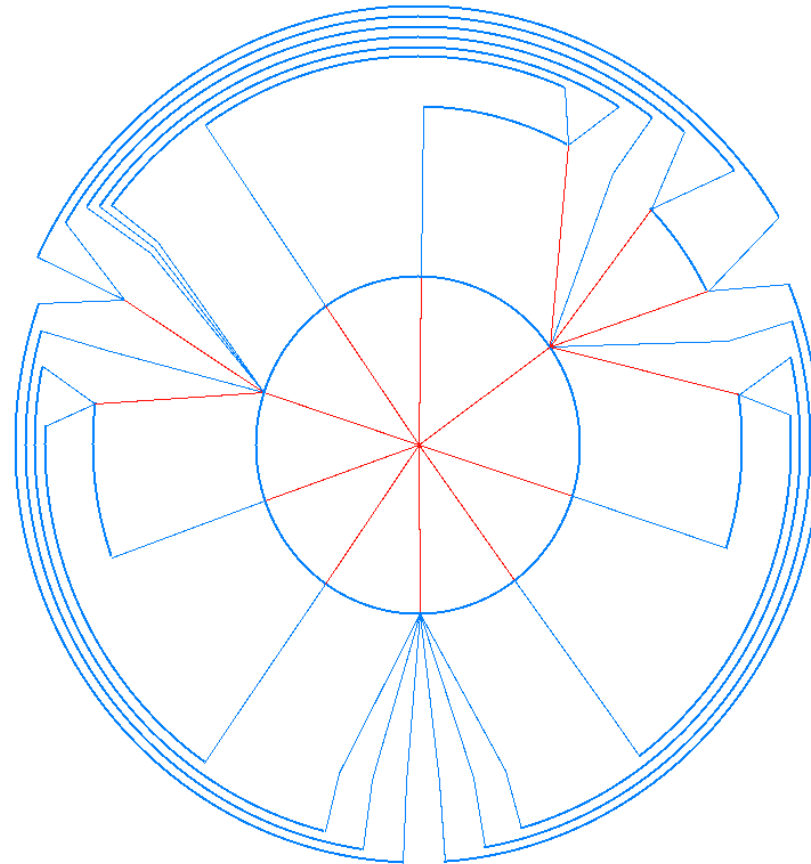
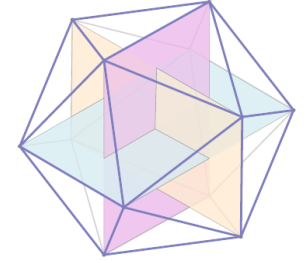


Parcours



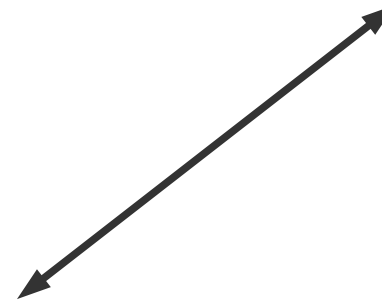
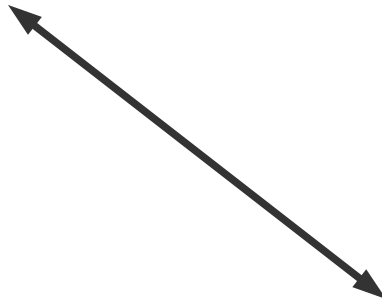
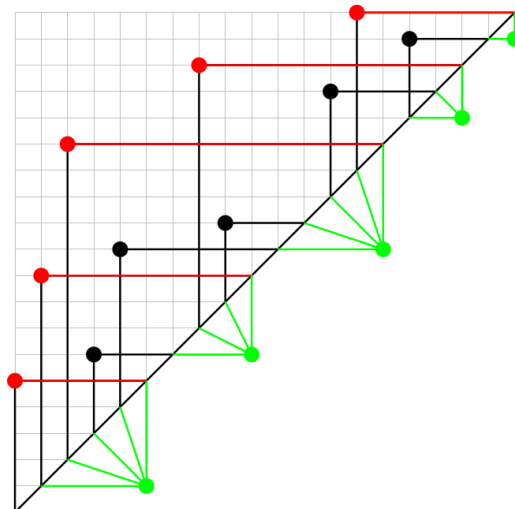
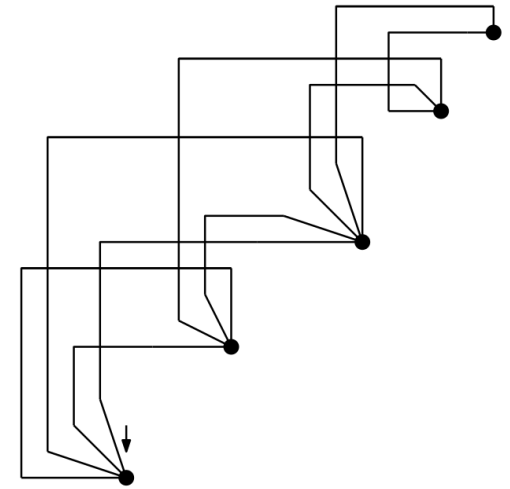
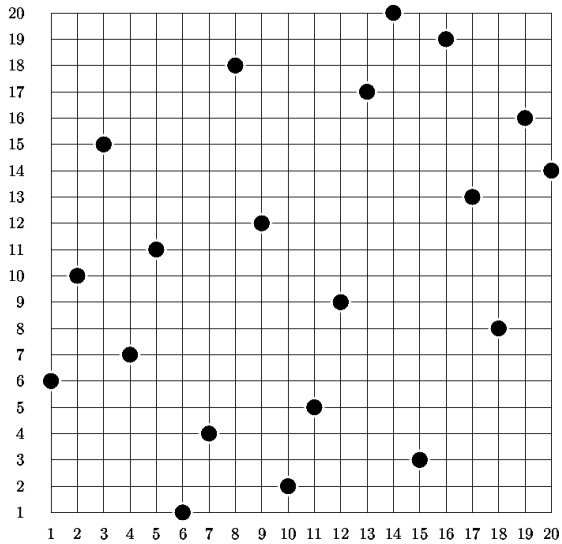
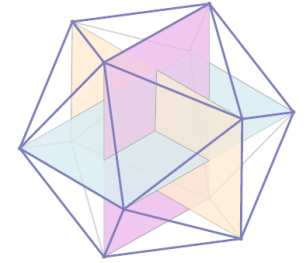


Breadth-First Search



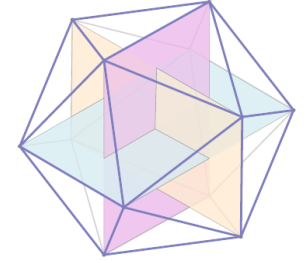


Codage de cartes

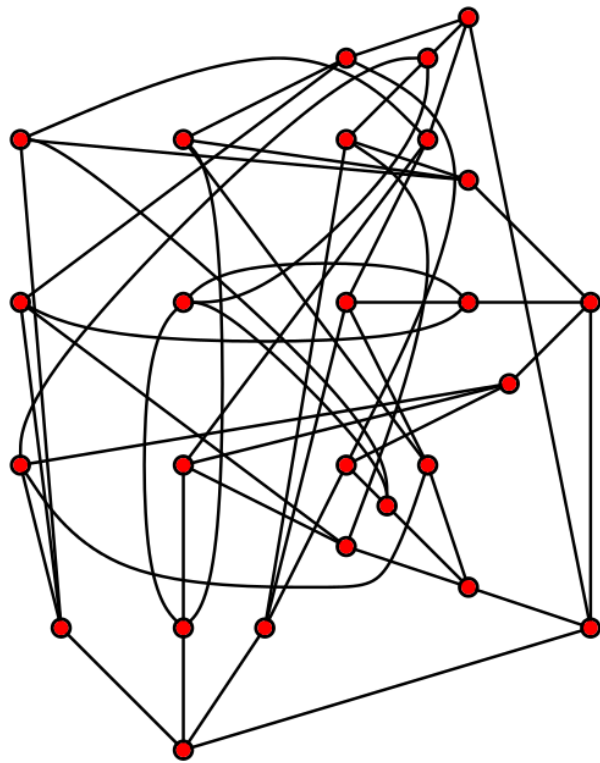




Recherche de sous-graphes



Le graphe G contient-il une copie de F ?



$n^{\gamma|F|}$ en général (Nešetřil, Poljak)

$d^{|F|-\alpha(F)} n^{\alpha(F)}$ pour les graphes d -degenérés

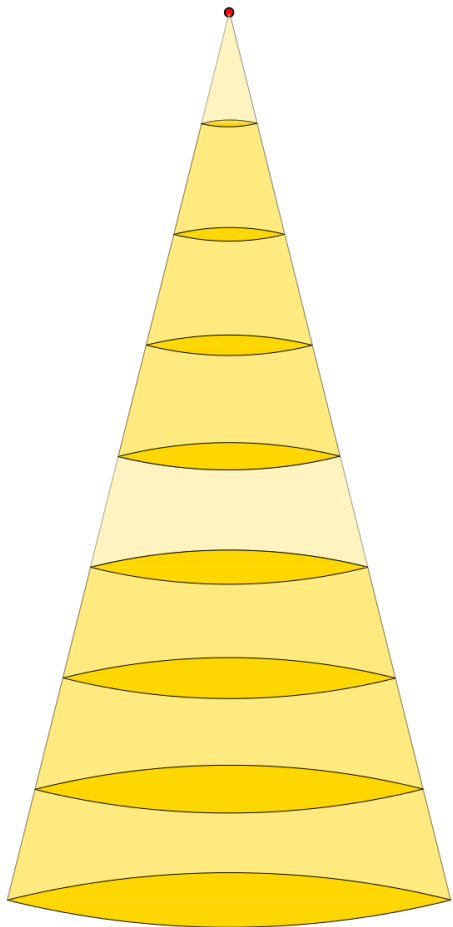
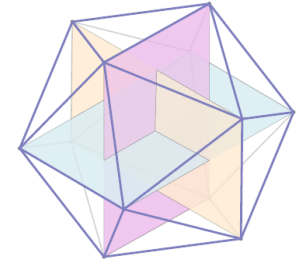
$O(n)$ pour ...

- Les graphes de tree-width borné (Courcelle '90)
- Les graphes planaires (Eppstein '95)
- Les graphes de genre borné (Epstein '00)
- Les classes de graphes d'expansion borné (Nešetřil, POM '06)

$n^{1+o(1)}$ pour les classes nulle part denses (Nešetřil, POM)



Recherche de sous-graphes

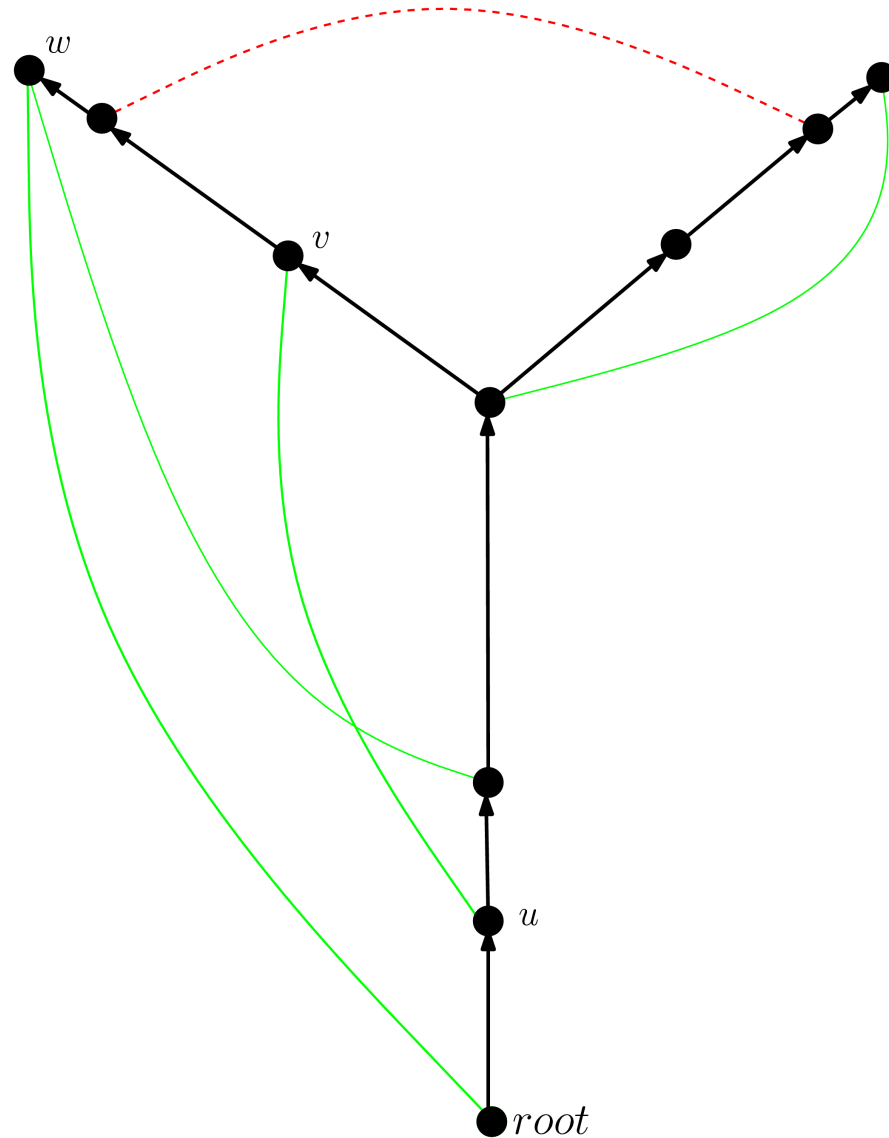
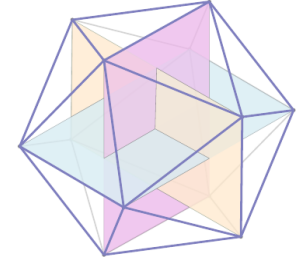


- Partitionner les sommets par distances à la racine mod $|F|+1$
- L'union des parts $\not\equiv i \pmod{|F|+1}$ induit un sous-graphe G_i de tw borné
- Résoudre le problème pour chaque G_i

Graphes planaire
Largeur d'arbre localement bornée

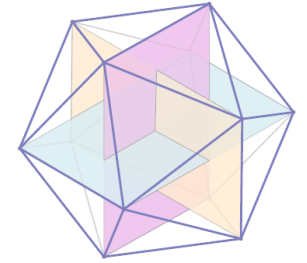


Depth-First Search

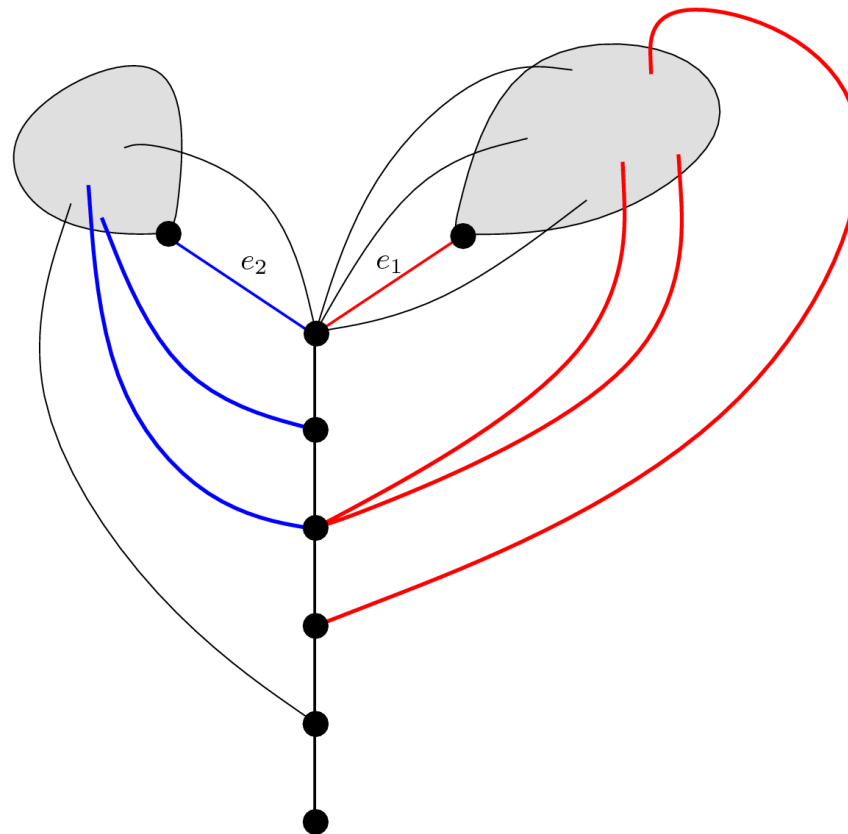




Planarité

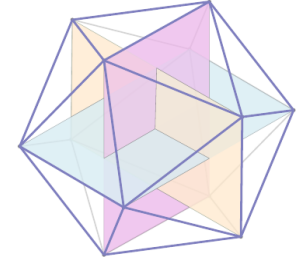


Caractérisation & Algorithme (de Fraysseix, POM, Rosenstiehl)





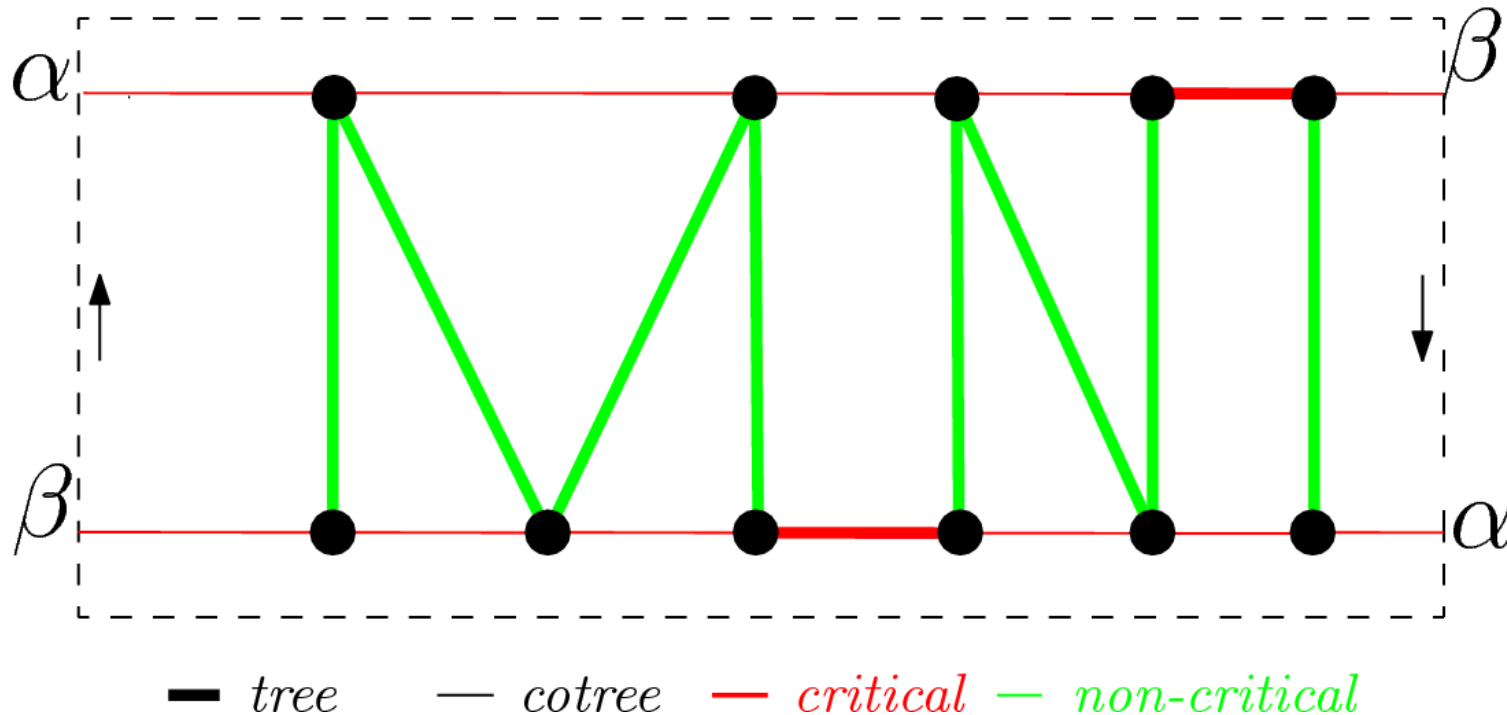
Kuratowski



Caractérisation des coarbres critiques

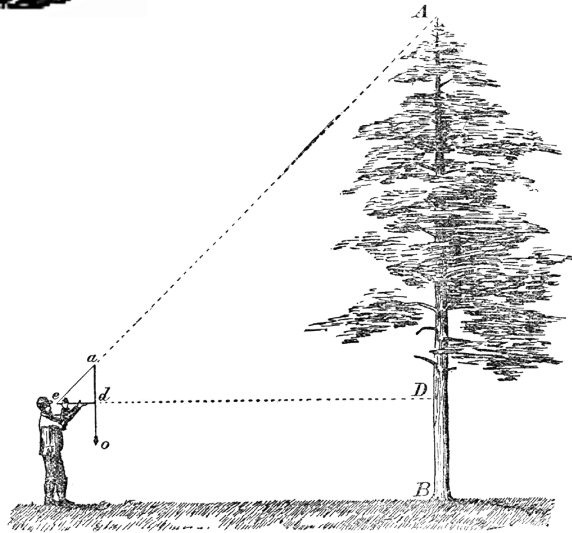
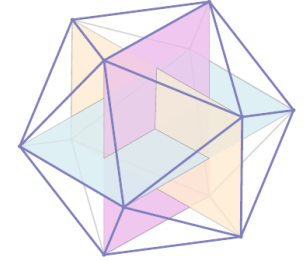
(de Fraysseix, POM)

Si >9 sommets et connexe, alors arbre DFS = chaîne et G = union d'un cycle d'arêtes critiques et de cordes 2 à 2 non-entrelacées.



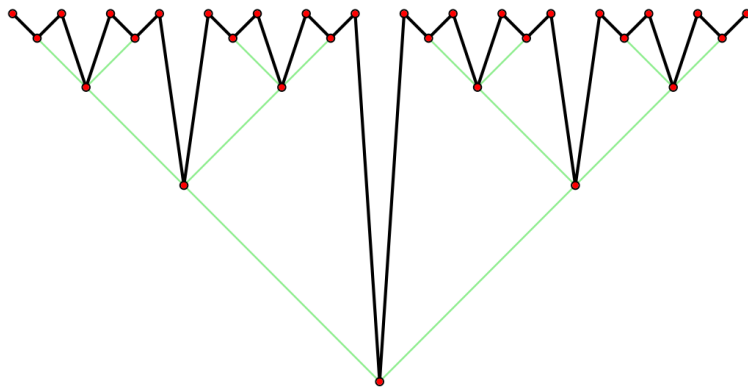


Profondeur d'arbre



Profondeur d'arbre $td(G)$

Hauteur minimum d'une forêt enracinée dont la fermeture contient G .

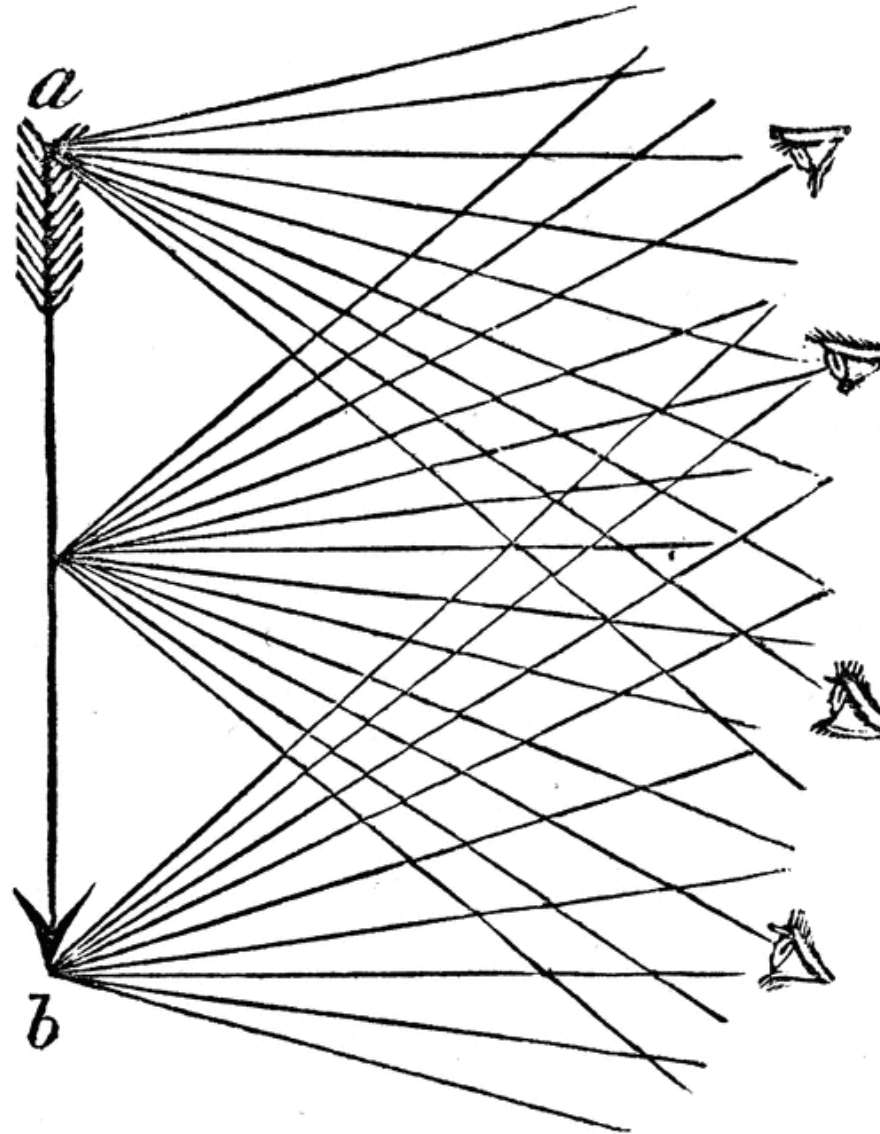
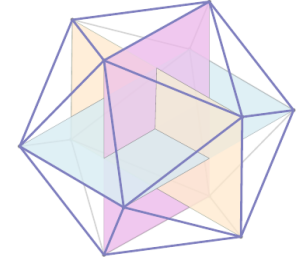


= profondeur minimale d'un DFS d'un graphe H qui contient G

$$td(P_n) = \log_2(n+1)$$

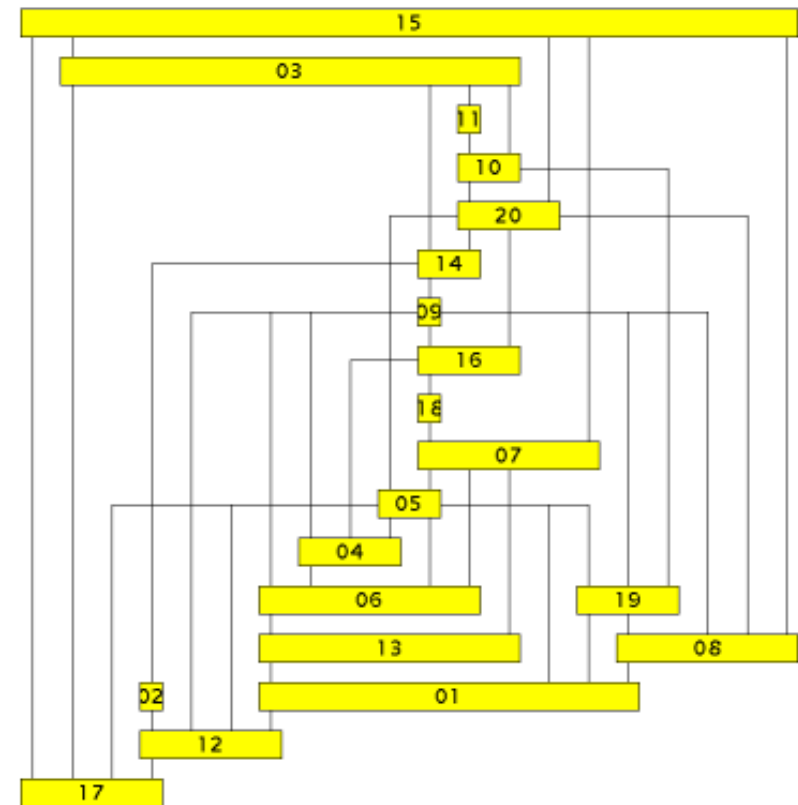
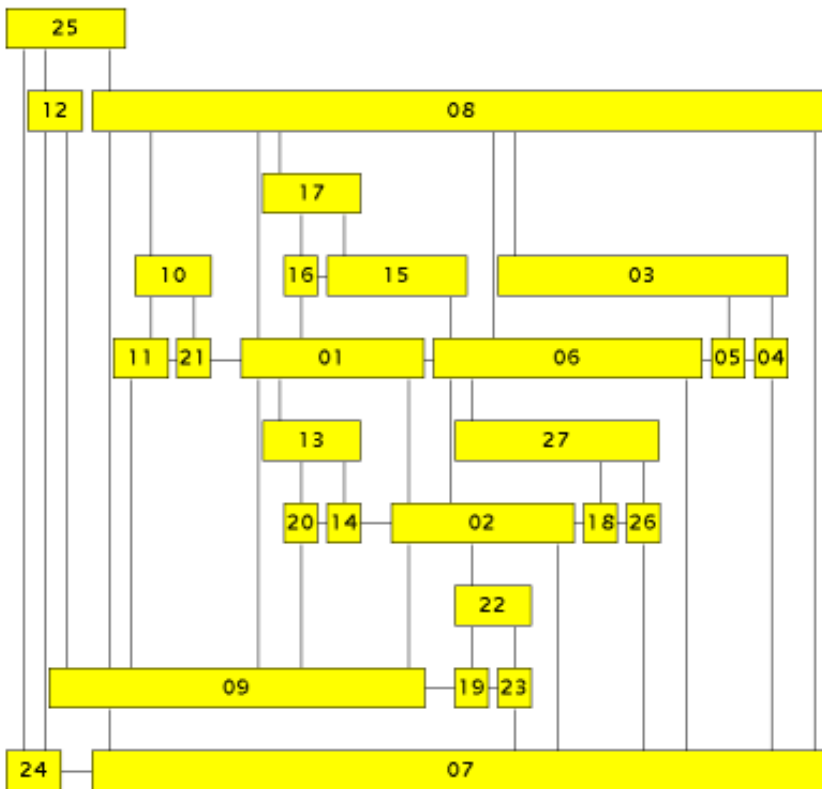
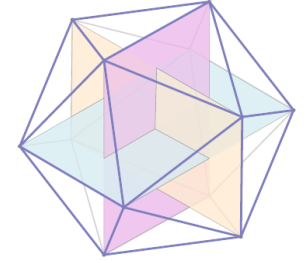


Orientations





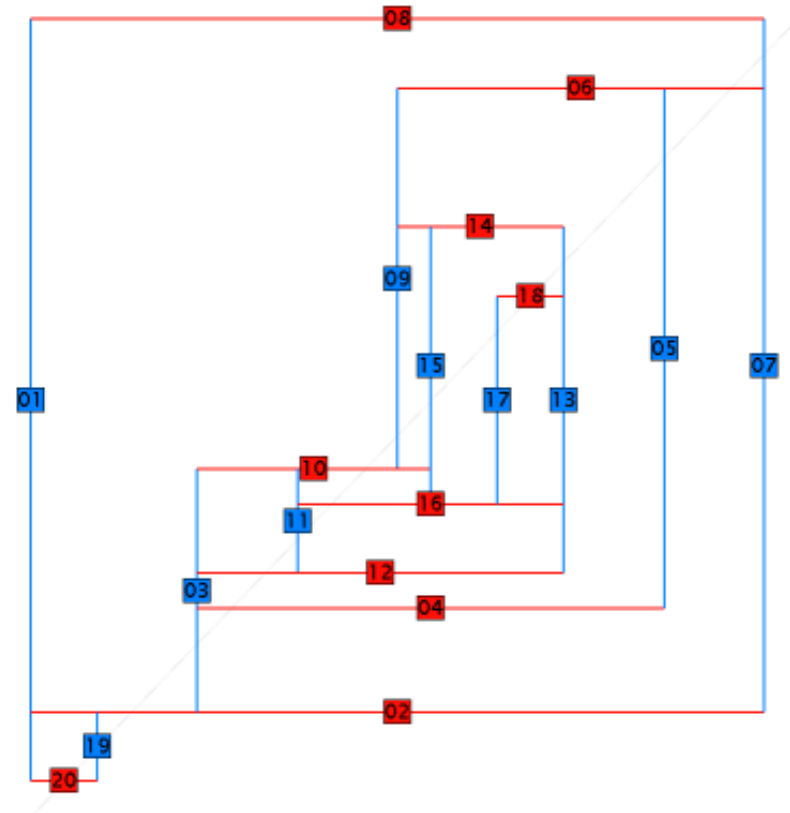
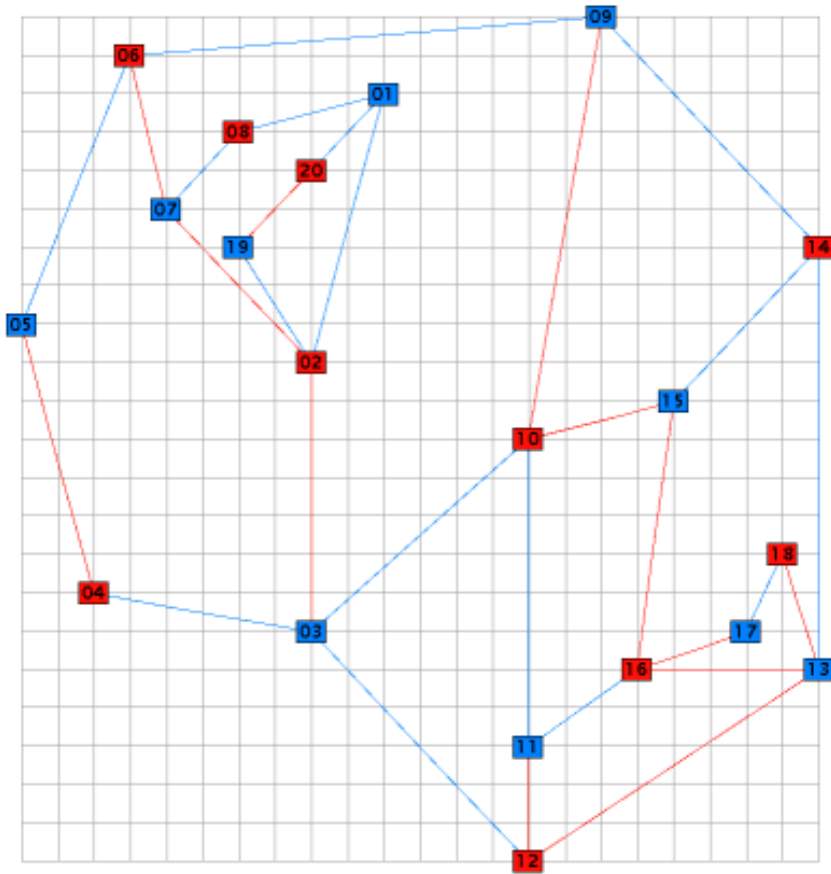
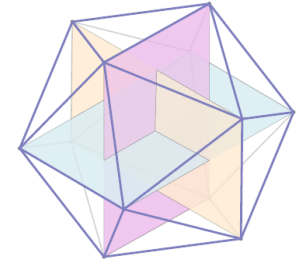
Tracé de visibilité



(de Fraysseix, Rosenstiehl)



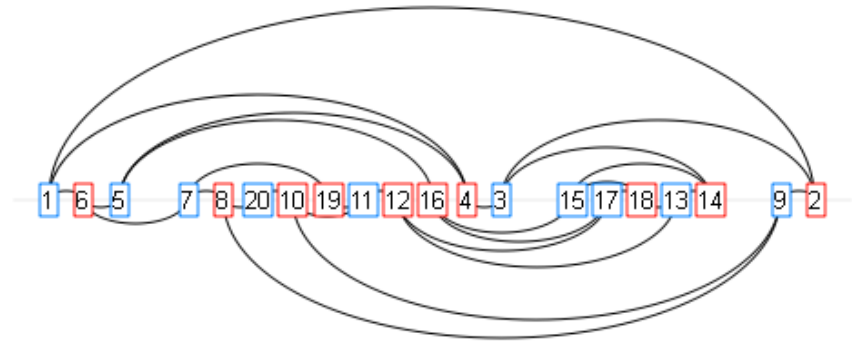
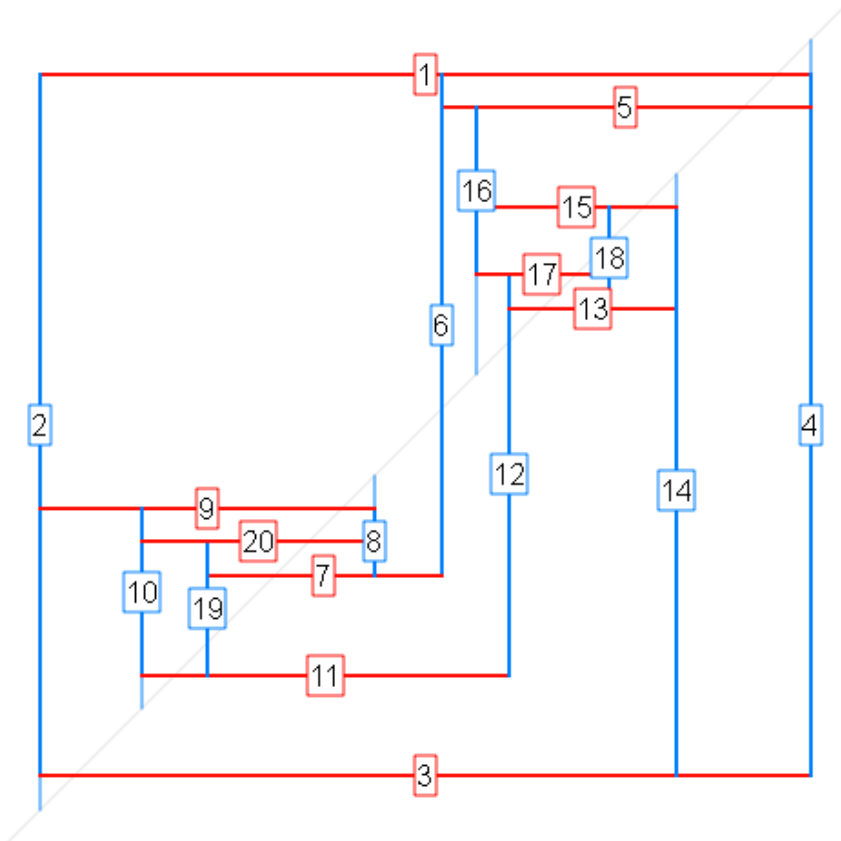
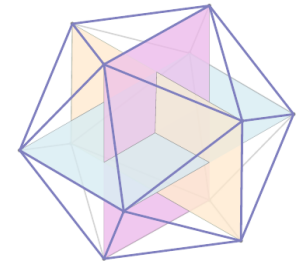
Bipartis planaires et contacts de segments



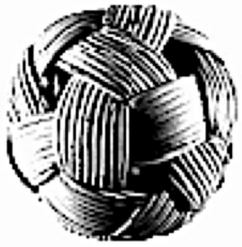
(de Fraysseix, POM, Pach)



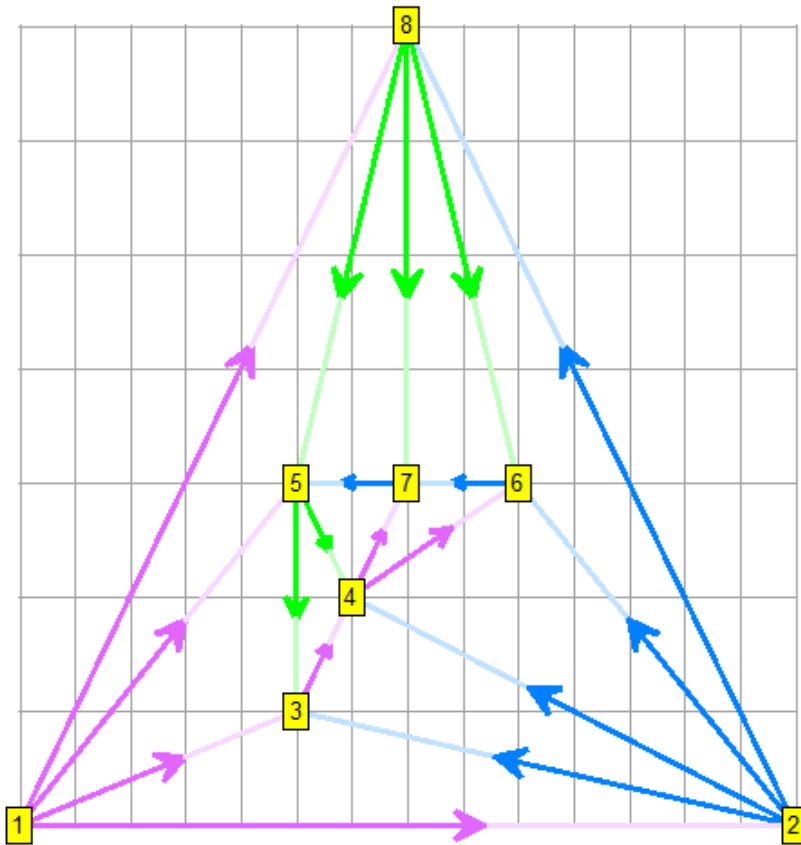
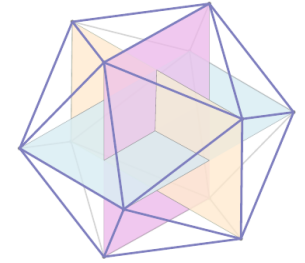
Deux arbres sur deux pages



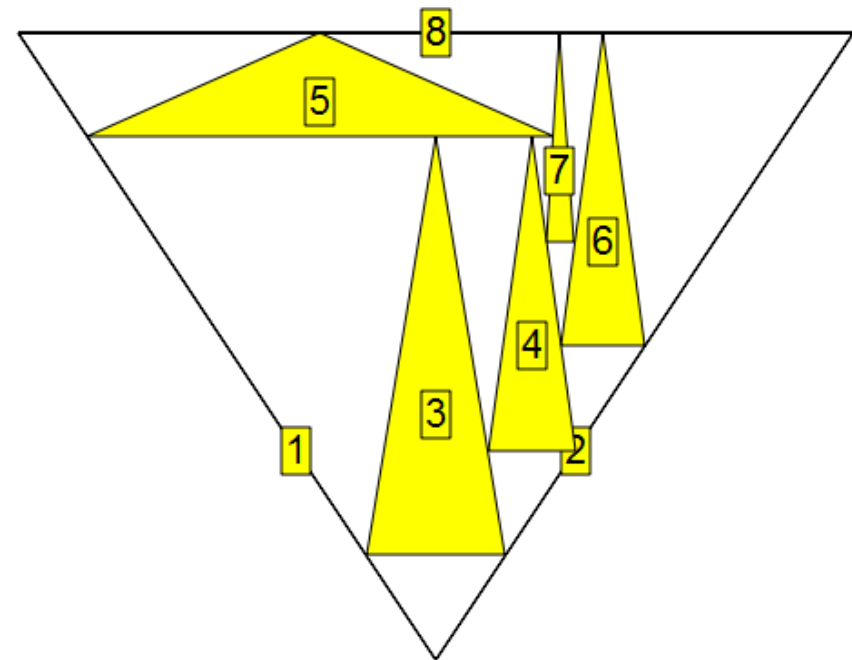
(de Fraysseix, POM, Pach)



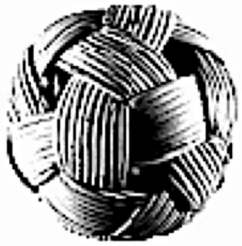
Tracé de Fary et contacts de triangles



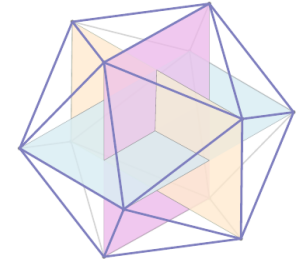
de Fraysseix, Pach, Pollack
Schnyder



de Fraysseix, POM, Rosenstiehl

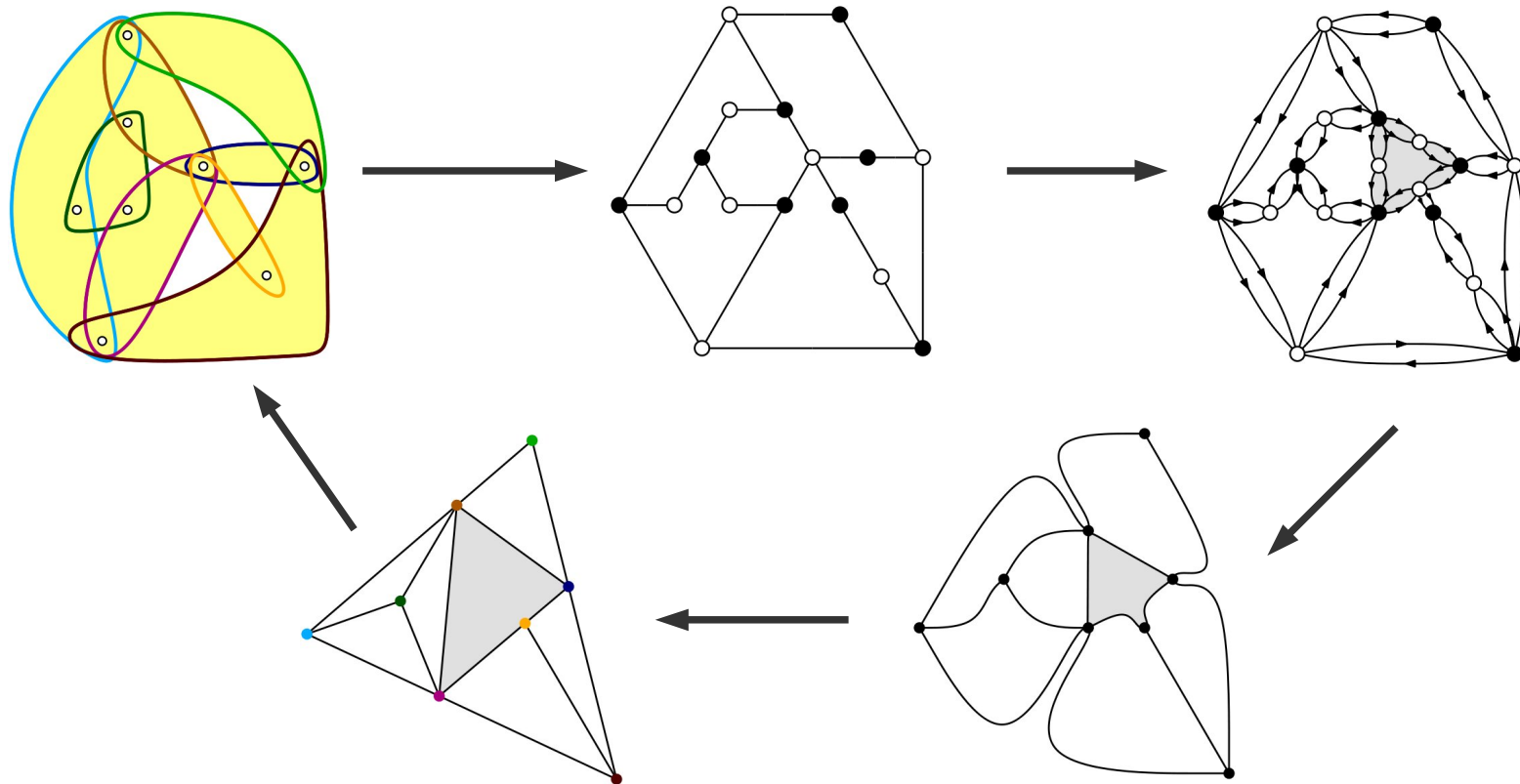


Orientations contraintes et contacts



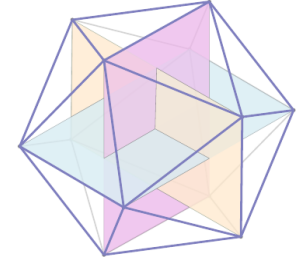
Caractérisation des systèmes de contacts de segments et/ou de triangles

(de Fraysseix, POM)

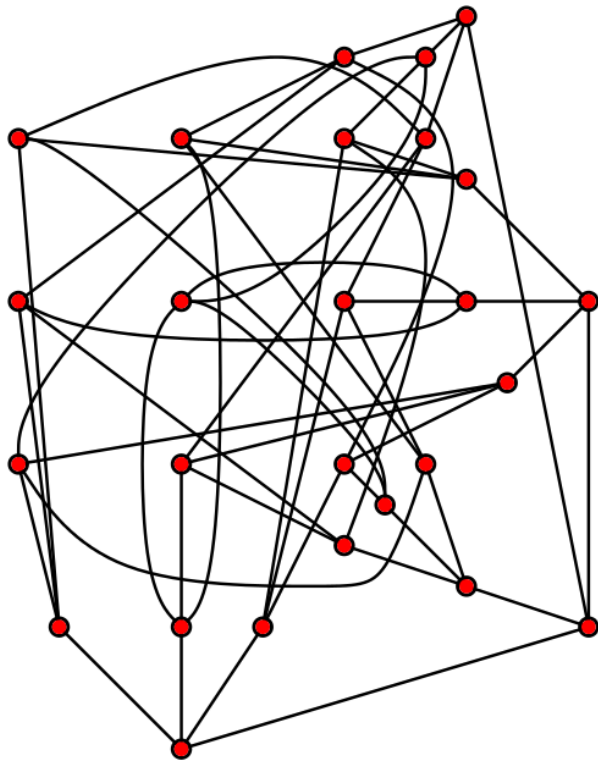




Recherche de sous-graphes



Le graphe G contient-il une copie de F ?



$n^{\gamma|F|}$ en général (Nešetřil, Poljak)

$d^{|F|-\alpha(F)} n^{\alpha(F)}$ pour les graphes d -degenérés

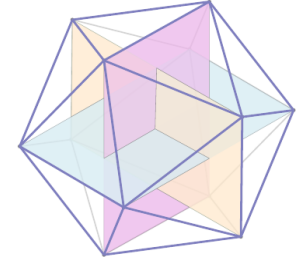
$O(n)$ pour ...

- Les graphes de tree-width borné (Courcelle '90)
- Les graphes planaires (Eppstein '95)
- Les graphes de genre borné (Epstein '00)
- Les classes de graphes d'expansion borné (Nešetřil, POM '06)

$n^{1+o(1)}$ pour les classes nulle part denses (Nešetřil, POM)



Graphes d -dégénérés



Un graphe G est d -dégénéré si tout sous-graphe de G contient un sommet de degré au plus d .

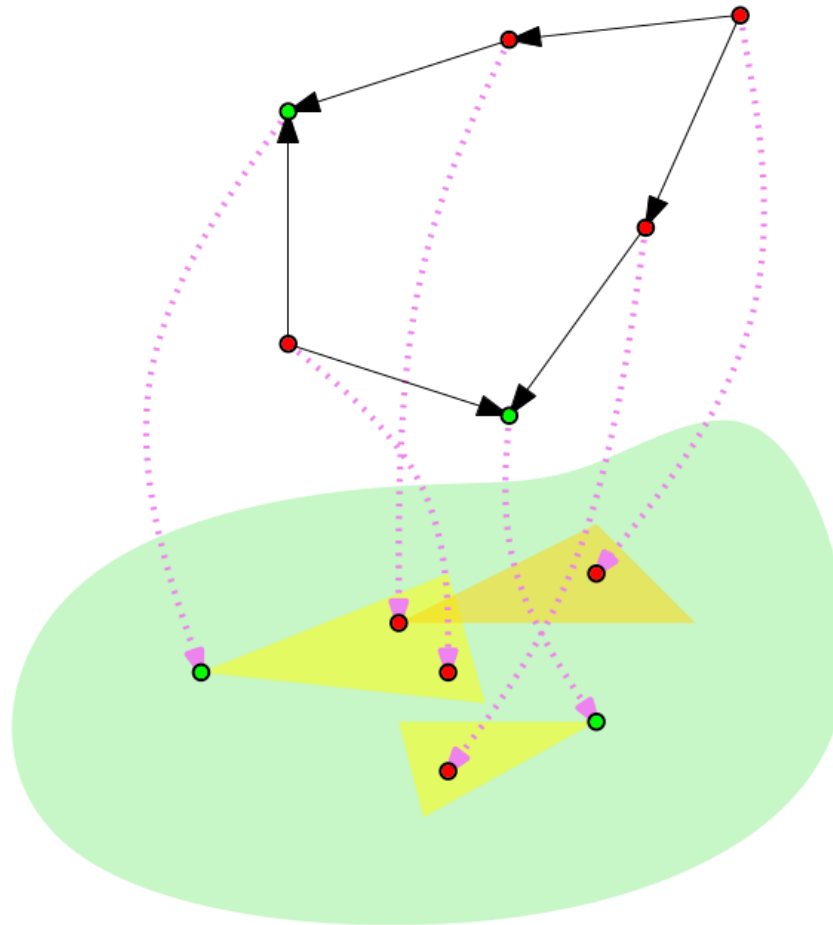
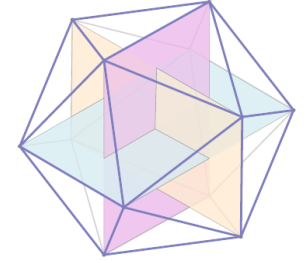
G est d -dégénéré



\exists orientation acyclique de degré entrant $\leq d$.

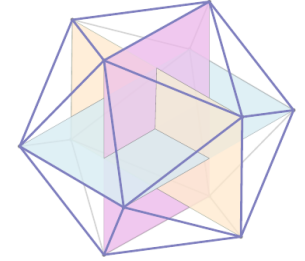


Graphes d -dégénérés

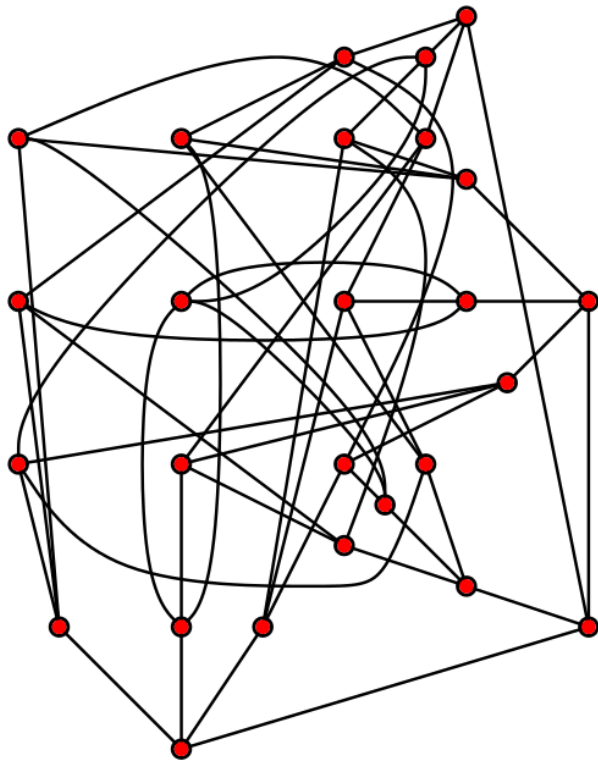




Recherche de sous-graphes



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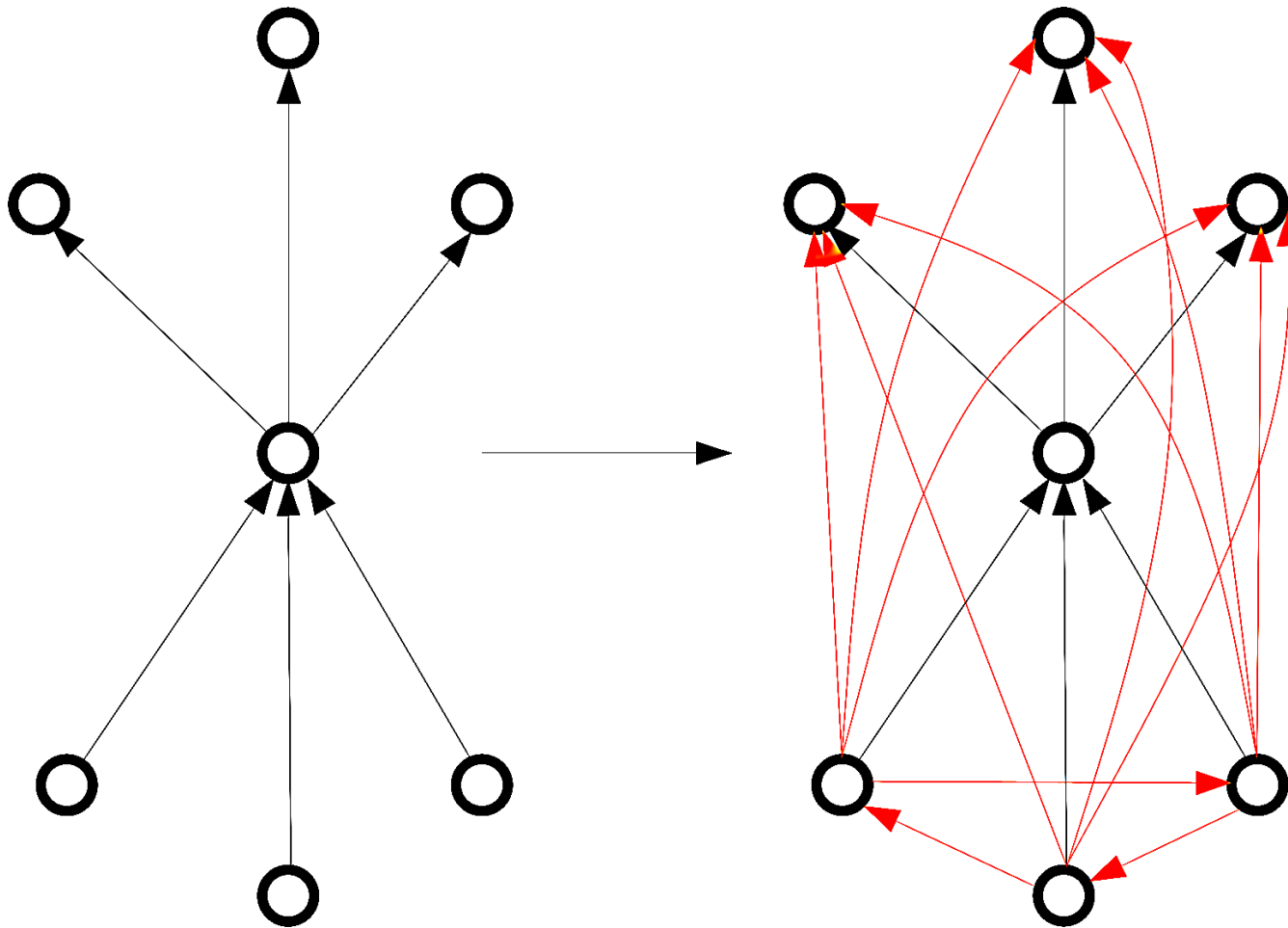
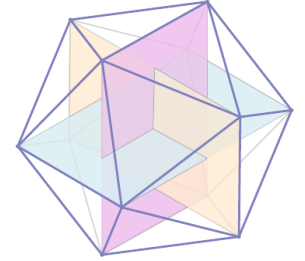
$O(n)$ pour ...

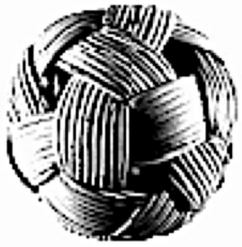
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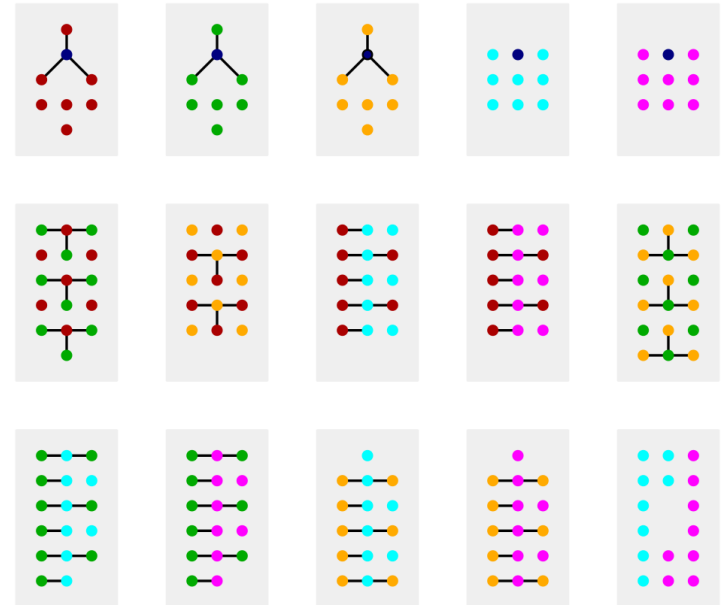
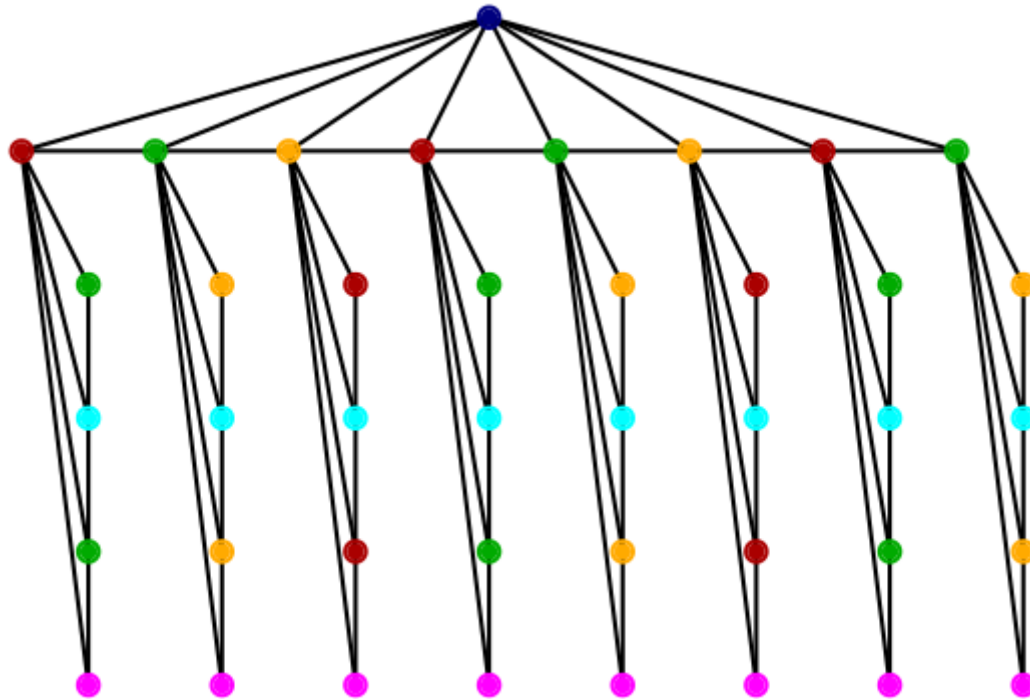
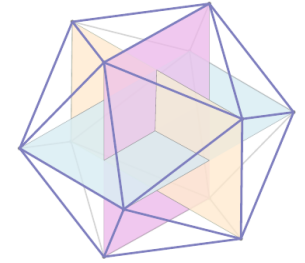


Méthode





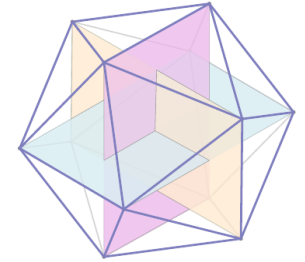
Décomposition de faible profondeur d'arbre



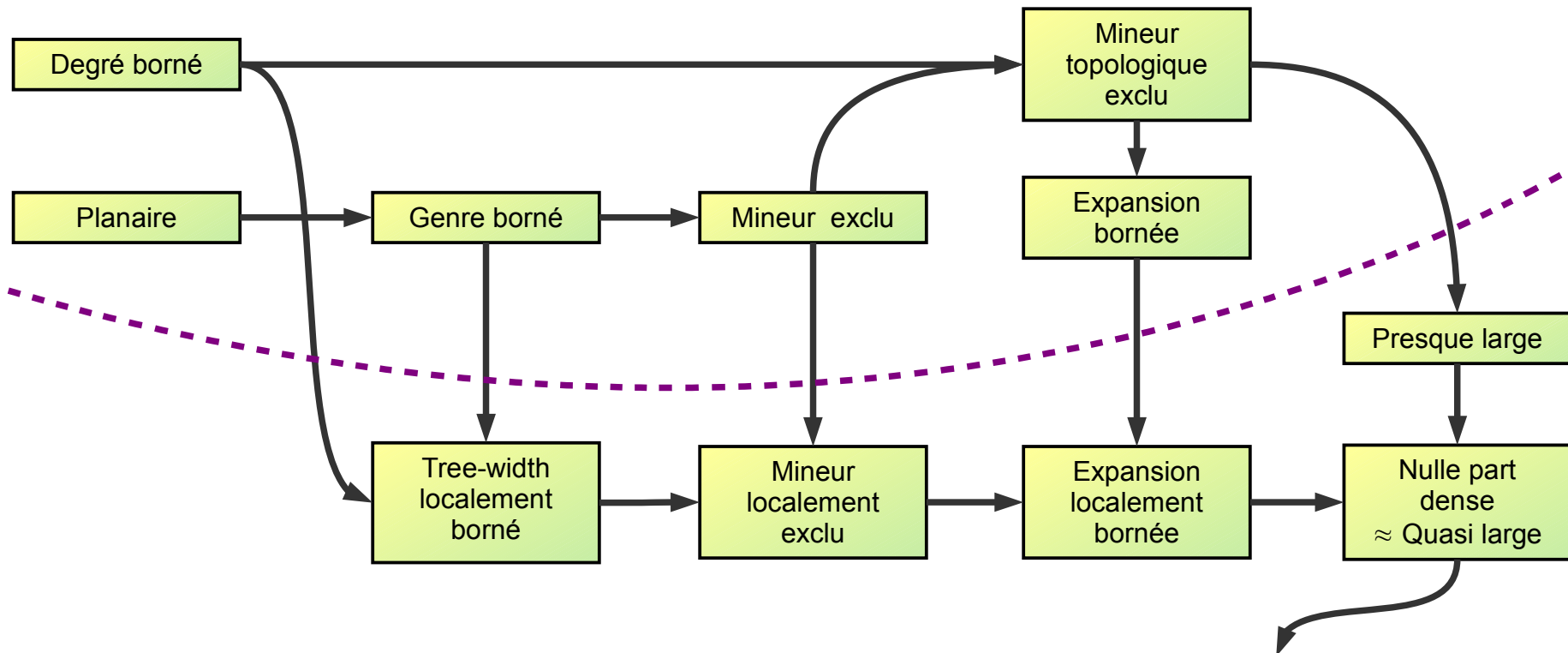
$$\chi(G) = \chi_1(G) \leq \chi_2(G) \leq \dots \leq \chi_p(G) \leq \dots \leq \chi_\infty(G) = \text{td}(G)$$



Classes de graphes



Expansion bornée $\Leftrightarrow (\forall p: \sup_G \chi_p(G) < \infty)$



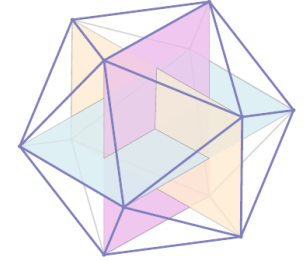
Nulle-part dense

$$\Leftrightarrow (\forall p: \chi_p(G) = |G|^{o(1)})$$

Quelque-part dense



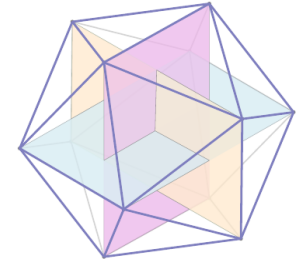
Applications



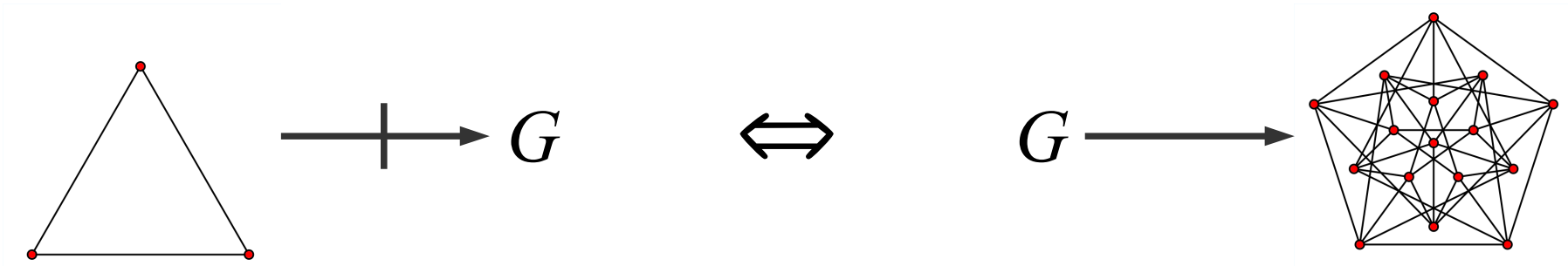
- Dualités restreintes d'homomorphismes
- Distance-coloration
- Comptage de motifs
- Test de propriétés du premier ordre
(Dvořák, Král', Thomas)



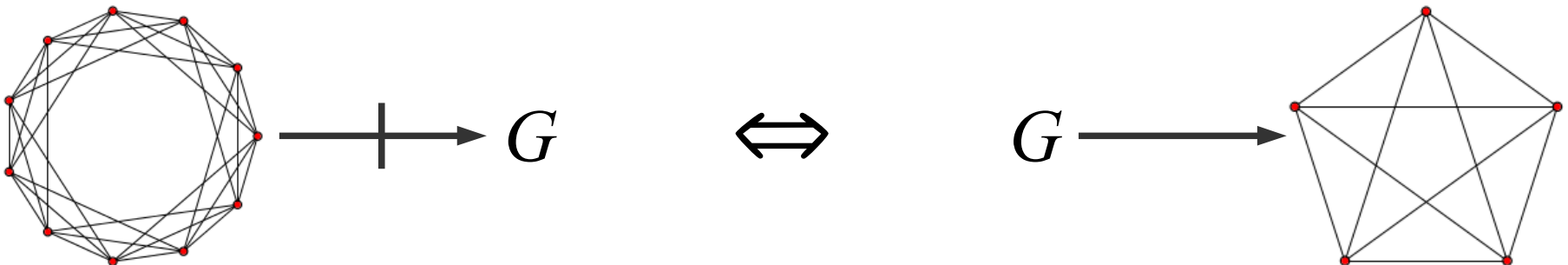
Dualités d'homomorphismes



- Pour tout G planaire :

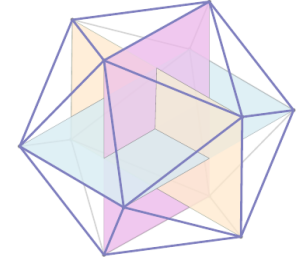


- Pour tout G toroïdal :



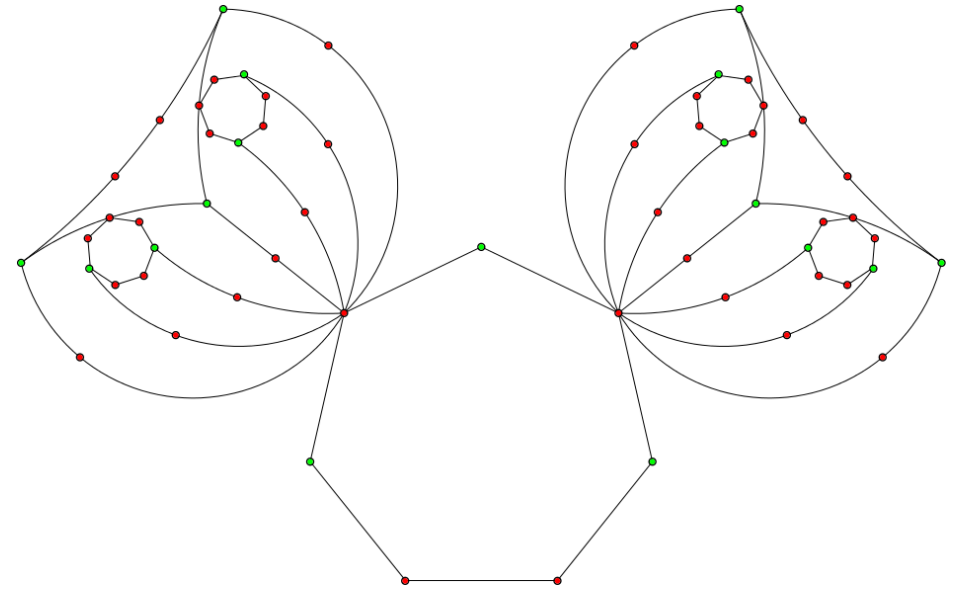


Distance-coloration



Problème (Van den Heuvel et Naserasr) :

Existe-t-il une constante C telle que pour tout p impair, tout graphe planaire G peut être distance- p -colorié avec C couleurs ?

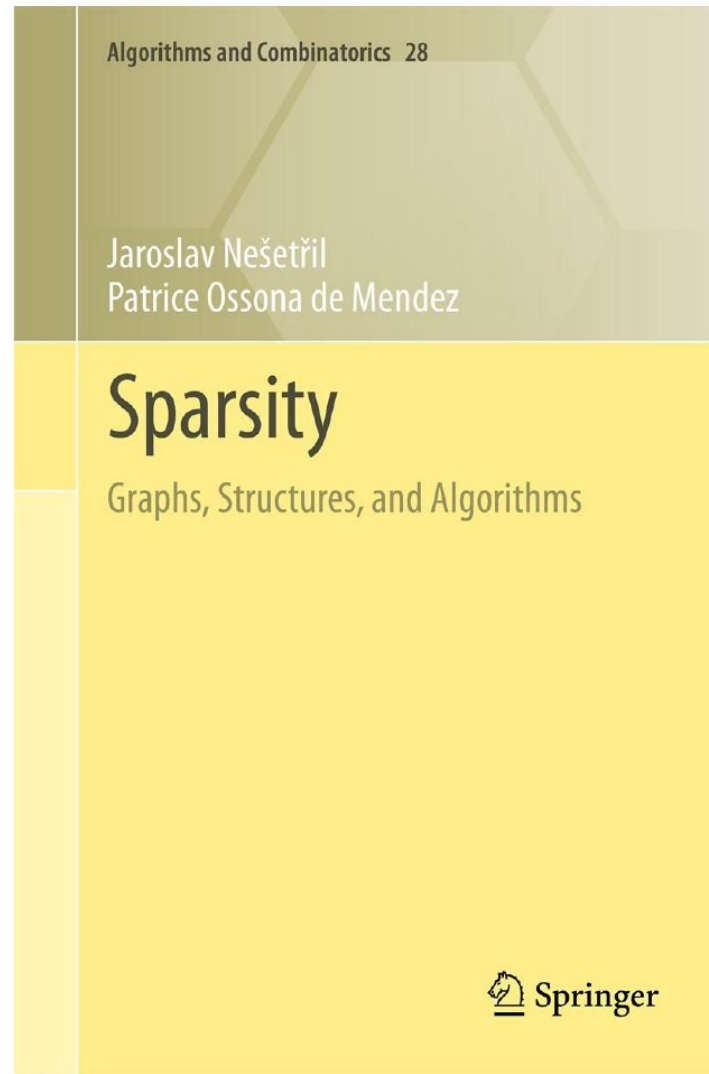
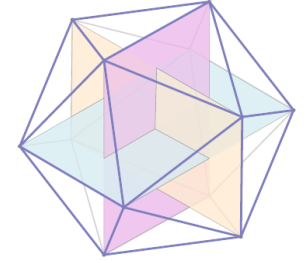


Théorème (Nešetřil, POM) :

Pour toute classe d'expansion bornée \mathcal{C} et pour tout p impair, il existe un entier $N(p)$ tel que tout graphe G de \mathcal{C} peut être distance- p -colorié avec $N(p)$ couleurs.

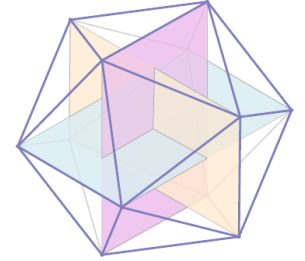


Pub





Merci !



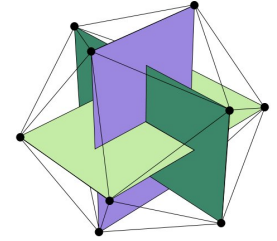


P.I.G.A.L.E.

Public Implementation of a Graph Algorithm
Library and Editor

SOURCEFORGE.NET®

[H. de Fraysseix](#) [P. Ossona de Mendez](#)



- Programmation (> 40000 lignes de code):
 - C++, multiplateforme (Qt4 © Trolltech), licence GPL, accessible sur Sourceforge
 - Structures originales optimisées,
 - Librairie, application graphique, interface réseau.
- Audience (> 11000 téléchargements, > 500 000 pages visitées):
 - Chercheurs (représentation de graphes, expérimentation grâce aux générateurs de graphes aléatoires de Schaeffer & Bonichon, et aux macros).
 - Industriels (vitrine).
- Contenu dérivé de nos travaux théoriques (> 20 articles):
 - algorithmes de planarité,
 - recherche de configuration de Kuratowski,
 - représentations de graphes (grille de taille linéaire, visibilité, contact de triangles, contact de T, etc.),
 - heuristique de détection des symétries d'un graphe,
 - test de 3-connexité, algorithmes d'augmentation, etc.