Solving network design and routing problems for urban freight distribution

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1 Problem statement

The last mile problem in freight transportation and distribution in cities is a complex issue. Each day, great amounts of merchandise are transported to and from the outside of the city via long-haul vehicles, capable of carrying great quantities of freight but inappropriate for retail pickup and delivery. Thus, transshipment is necessary to reach final customers. In this paper, we deal with the problem of designing a 2-level transportation network, based on publicly held *Urban Distribution Centers* (*UDC*) to collect inbound and outbound freight. Such a network allows bulk-breaking operations on the long-haul transport side, and permits retail trips on the city side. Let U be a set of potential sites where to install such centers, such that each site $u \in U$ has an associated installation cost and capacity. Let I be a set of *clients*, where each $i \in I$ is characterized by a combination of delivery and pickup *demands*. Moreover, let P be a set of *gates* in the outskirts of the city, which represent the terminal points of the main roads freight can come from, or go to. Gates are the sources of delivery demands and the destinations of pickup demands. A complete graph A_U (called the *external network*) is associated with set U, and all sites $u \in U$ can be reached by all gates $p \in P$ via a further network A_P. A_P and A_U form what we call the *1st level network*. Let us suppose that pickup and delivery duties must be serviced separately. In order to minimize empty trips and thus achieve environmental purposes, retail trips are allowed to be open. Furthermore, a self-service van hiring system is available and its stations, a set K of *self-service parking lots* (*SPL*), can act as start or end points of open service paths. UDCs and SPLs consequently share the same fleet of vans with low-level environmental impact. The network that connects UDCs to clients, clients between them, and clients to SPLs is called the *2nd level network*. The aim is to determine a subset of potential UDCs to open, a Hamiltonian circuit (ring) to connect them, the flows between gates and UDCs, the flows on the constructed ring, and the assignment of final customers to UDCs, in such a way as to minimize the sum of installation costs, 1^{st} level transportation costs and 2^{nd} level routing costs. Goods are characterized only in terms of quantity, disregarding the commodity type. Furthermore, we consider a time-independent scenario. Despite the strategic level of this planning problem, we account for some more operational aspects. We impose a classical capacity constraint on 2^{nd} level vans, but also a maximal trip length, as it is often the case when dealing with low-level environmental impact vehicles. We also impose fleet rebalancing constraints on both SPLs and UDCs.

2 Related problems of the literature

This problem is close to the Two-Echelon Capacitated Vehicle Routing Problem (2E-CVRP), which performs deliveries from a central depot to customers by means of a two-level distribution network, i.e. via intermediate depots called *satellites*, and a set of both 1^{st} and 2^{nd} level tours. As to 2E-CVRP, [1] presents a flow model and some valid inequalities, while [2] provides an exact method. The proposed problem is also close to the Capacitated Location-Routing Problem (CLRP): [3], [4] and more recently [5] delve into this wide class of location problems. In the canonical two-level version, potential facilities are opened, and clients are assigned to facilities, depending on routing aspects. The problem presented in this paper is very close to some known variants: in [6], a Set Partitioning formulation is proposed, which associates a variable to each possible delivery trip, including open trips; in [7], the selected depots are connected via a TSP; in [8], a third level of warehouses is introduced in the role of depots' suppliers. Anyway, to the best of our knowledge, there are no previous studies in the literature that encompass all the characteristics we have briefly described.

3 Model elements

We model our problem as a MILP with a Set Partitioning-like formulation of 2^{nd} level trips, and arc-flow elements to describe 1^{st} level goods transport. Due to space reasons, we only give the model guidelines instead of the model itself. We use super/subscripts $\lambda = 1$ or 2 to refer to delivery or pickup duties. The MILP model is based on mixed graph $G = (V, A, E)$, where the arcs in $A = A_P \cup A_U$ represent the 1^{st} level, while the edges in E model the 2^{nd} level. A_P and A_U are defined as $A_P = (P \times U) \cup (U \times P)$, $A_U = \{(u, u') \in U \times U : u \neq u'\}$. To describe the 2^{nd} level, we define a demand as a triplet $d = (i_d, p_d, q_d)$ (client, gate, quantity), then the sets of demands D^λ, $\lambda = 1, 2$. We define a node v_d for each demand d, so that each $i \in I$ results in as many nodes as the duties it is involved in to allow different demands of the same client i and type λ to be serviced by different paths. Node sets $V_{D^\lambda} = \{v_d : d \in D^\lambda\}$, along with P, U and K, complete the definition of V. We create two complete disjoint subgraphs $G^\lambda = (V_{D^\lambda}, E_{D^\lambda})$, where the cost c_e of edge $e \equiv (v_d, v_{d'}) \in E_{D^\lambda}$ is the euclidean distance between clients i_d and $i_{d'}$, so $c_e = 0$ if $i_d \equiv i_{d'}$. We outline source and terminal sets

$V_s^1 = V_t^2 = U$ and $V_s^2 = V_t^1 = U \cup K$, so as to define a feasible path $r \in R^\lambda$ as a path $(v_0^r..v_{n_r}^r)$, with $v_0^r \in V_s^\lambda, v_1^r..v_{n_r-1}^r \in V_{D^\lambda}$, $v_{n_r}^r \in V_t^\lambda$, whose load $q(r)$ and length $c(r)$ do not exceed 2^{nd} level vehicles given bounds Q and C. E is then defined as $E = \bigcup_\lambda((V_s^\lambda \times V_{D^\lambda}) \cup E_{D^\lambda} \cup (V_{D^\lambda} \times V_t^\lambda))$. We have three types of binary decision variables: location variables y_u, $u \in U$ to model UDCs opening decisions; $z_{u,u'}, u, u' \in U$, which is equal to 1 if we decide to bidirectionally connect u and u' in the ring; and routing variables x_r, $r \in R^\lambda$, $x_r = 1$ if r is in the solution. We also have real nonnegative 1^{st} level flow variables: flows f_{pu} and f_{up} from gate p to site u and viceversa; *p-outflows* f_{pa}^1 and *p-inflows* f_{pa}^2 which represent the quantity of goods on $a \in A_U$ that come from, or go to gate p. Indexing flows by the source/sink gate is needed to impose that each demand d is satisfied by flow coming from/going to the proper gate p_d, i.e. to correctly model flows on A_U. We impose the following constraints:

(1) flow balance constraints on UDCs; (2) flow balance constraints for gates;
(3) demands assignment constraints; (4) capacity constraints on A_U arcs;
(5) capacity constraints on UDCs; (6) upper bound on the number of UDCs to open;
(7) logic constraints linking the selection of UDC, A_U arcs and paths;
(8) connection constraints to design a Hamiltonian circuit among UDCs;
(9) vehicles rebalancing constraints on both UDCs and SPLs.

Constraints (1) are defined for each $u \in U$ and $p \in P$ to balance f_{pu}, p-outflows on A_U arcs and 2^{nd} level shipments (same for pickup); in (4), the capacity of each $a \in A_U$ is shared among outflows and inflows of all $p \in P$; (5) are linearizations of quadratic constraints. (1) and (2) outline the arc flow modeling of 1^{st} level goods moves; (3) and (9), along with definition of paths $r \in R^1 \cup R^2$, outline the SP-like description of 2^{nd} level; (4)-(8) are the location constraints. The objective function accounts for routing costs $c(r)$ of chosen 2^{nd} level paths; costs b_u of selected UDCs, and costs $g_{u,u'}$ to connect them in the ring; per-flow-unit costs h_a on A_U arcs; per-flow-unit costs m_{pu} and m_{up} on A_P arcs.

4 A heuristic algorithm

The problem is NP-hard since it generalizes the classical CVRP, which we obtain in the special case where $|U| = 1$, $|K| = 0$ and $|D^2| = 0$. To find solutions to real world instances, we have defined a heuristic algorithm that seeks for a solution. We use the following symbols:

- $\mathbb{P}_S$ keeps track of how solution values and 2^{nd} level arcs usage are related in solution set S;
- $M_\rho^\gamma(\mathbb{P}_S)$ is a set of paths $m \in R^1 \cup R^2$ s.t. $c(m) \leq \beta^\rho C$, $\beta < 1$; paths in $M_\rho^\gamma(\mathbb{P}_S)$ are randomly seeded and expanded via nearest neighbor and 2-opt techniques; generation of $M_\rho^\gamma(\mathbb{P}_S)$ considers $\mathbb{P}_S$ and assures that each $d \in D^1 \cup D^2$ is visited by at least γ paths;
- A_ρ is the MILP subproblem of opening a subset of UDCs, assign each piece of flow of a gate p, and exactly one visiting $m \in \overline{M}_\rho^\gamma(\mathbb{P}_S) = \bigcup_{\upsilon=0}^{\rho} M_\upsilon^\gamma(\mathbb{P}_S)$ for each $d \in \bigcup_\lambda D^\lambda$, to a UDC, so as to minimize opening and routing costs, while respecting capacity of chosen UDCs;

- $F(\overline{U}, R_{\overline{U}}, \Phi_{\overline{U}})$ is a multiflow on a ring with demands MILP, with $R_{\overline{U}} \subset A_U$ a directed ring on node set $\overline{U}$, and $\Phi_{\overline{U}}(u, w)$ the flow to be sent from u to w, $u, w \in \overline{U}$.

$\boxed{\textsc{SearchSolutions}(I,\ \beta,\ \tau,\ \gamma,\ \delta)\text{:}\quad \beta < 1,\ \tau < \beta,\ \gamma \geq 1,\ \delta > 1}$

0: $S \leftarrow \emptyset$; $i \leftarrow 0$; $reset(\mathbb{P}_S)$
1: **while**$_1$$(i < I)$
2: $\quad\rho \leftarrow 0$; $a \leftarrow 0$; **while**$_2$$(\beta^\rho \geq \tau)$ $\{generate(M_\rho^\gamma(\mathbb{P}_S))$; ρ += 1;$\}$
3: $\quad$**if**$(solve(A_\rho))$ collect $a \leq \delta$ diversified solutions **else** $\{i$ += 1; **continue while**$_1$;$\}$
4: $\quad$**for each** α_j collected solution of A_ρ, $j = 1 \dots a$
5: $\qquad\overline{U_j} \leftarrow$ selected UDC in α_j; $\Phi_{\overline{U_j}}(u, w) \leftarrow$ flow from u to w according to α_j;
6: $\qquad H_{\overline{U_j}} \leftarrow$ solution of TSP on $\overline{U_j}$ and costs g_{uw}; $R_{\overline{U_j}} \leftarrow$ directed ring derived by $H_{\overline{U_j}}$;
7: $\qquad F_j = F(\overline{U_j}, R_{\overline{U_j}}, \Phi_{\overline{U_j}})$; **if**$(solve(F_j))$ $\{S = S \cup \{(\alpha_j,\ H_{\overline{U_j}},\ opt(F_j))\}$; $update(\mathbb{P}_S)$;$\}$
8: $\quad i$ += 1;
9: **return** $\sigma \in S$: $z(\sigma) = \min_{s \in S} z(s)$

At row 3 we expressly collect diversified solutions of A_ρ to avoid cases in which A_ρ optimality leads to multiflow infeasibility; at row 7 we update both solutions set S and $\mathbb{P}_S$. We tested this algorithm on CLRP instances taken from the literature with up to 5 potential depots and 50 clients which we completed by adding gates and SPLs, 1^{st} level network features and pickup/delivery clients partition. Promising results have been obtained.

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