4th Workshop on Reachability Problems

Brno

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Behavioral Cartography of Timed Automata

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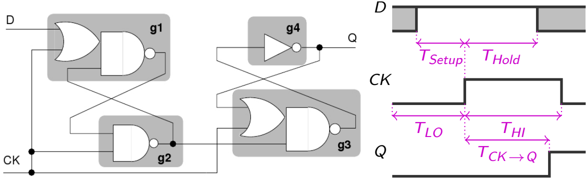
The Good Parameters Problem

- Context: Verification of Timed Systems
- Good parameters problem
  - Synthesize a set of values of the timing parameters guaranteeing that the system behaves well (e.g., avoids any bad state)
- Classical approaches
  - Computation of all the reachable states, and intersection with the set of bad states [Alur et al., 1995]
  - Approach based on CEGAR (Counter-Example Guided Abstraction Refinement [Clarke et al., 2000, Frehse et al., 2008])
- New approach: method of behavioral cartography

An Example: Flip-Flop Circuit (1/2)

- Schematics [Clarisó and Cortadella, 2007]



  - 4 elements: G_1, G_2, G_3, G_4 with internal signals g_1 to g_4
  - 2 input signals (D and CK), 1 output signal (Q)
- Timing parameters
  - Traversal delays of the gates by the electric current
    - ★ Parametric interval; example for G_1: $[\delta_1^-, \delta_1^+]$
  - Durations of low (T_{LO}) and high (T_{HI}) levels of CK
  - Stabilization time of D: T_{Setup}, T_{Hold}

An Example: Flip-Flop Circuit (2/2)

- We suppose given a valuation π_0 of the parameters (called point)

$$
\begin{array}{llll}
T_{HI} = 24 & T_{LO} = 15 & T_{Setup} = 10 & T_{Hold} = 17 \\
\delta_1^- = 7 & \delta_1^+ = 7 & \delta_2^- = 5 & \delta_2^+ = 6 \\
\delta_3^- = 8 & \delta_3^+ = 10 & \delta_4^- = 3 & \delta_4^+ = 7
\end{array}
$$

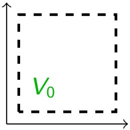
  - This point guarantees a good behavior:
    - $Q^\uparrow$ occurs before $CK^\downarrow$

- We are looking for a set of points (containing π_0) for which the system behaves well

General Problems

We consider a system modeled by a parametric timed automaton.

- The good parameters problem:
  - "Given a bounded parameter domain V_0, find a set of points of good behavior in V_0 (ideally the largest one)"



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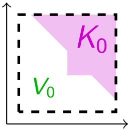

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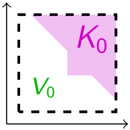
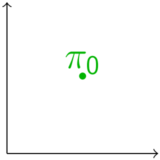


- This problem reduces to the inverse problem:
  - "Given a reference point π_0, find other points around π_0 of same behavior"



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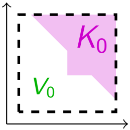

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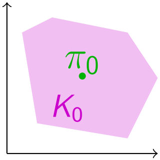

Outline

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Timed Automaton

- Finite state automaton (sets of locations)

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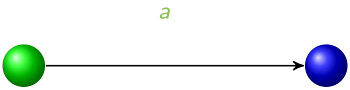
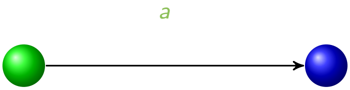
- Finite state automaton (sets of locations and actions)



Timed Automaton

- Finite state automaton (sets of locations and actions) augmented with
  - A set X of clocks (i.e., real-valued variables evolving linearly at the same rate)



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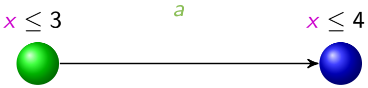

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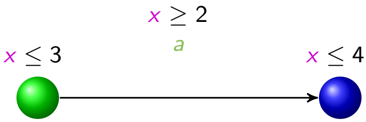

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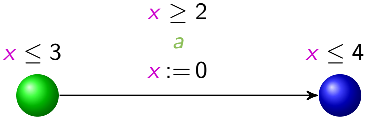
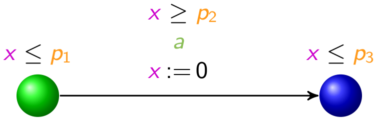


Parametric Timed Automaton (PTA)

- Finite state automaton (sets of locations and actions) augmented with
  - A set X of clocks (i.e., real-valued variables evolving linearly at the same rate)
  - A set P of M parameters (i.e., unknown constants), used in guards and invariants
- Features
  - Location invariant: property to be verified by the clocks and the parameters to stay at a location
  - Transition guard: property to be verified by the clocks and the parameters to enable a transition
  - Clock reset: clocks can be set to 0 at each transition



States and Traces

- Given a PTA $\mathcal{A}$ and a point π, we denote by $\mathcal{A}[\pi]$ the (non-parametric) timed automaton where all parameters are instantiated by π

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  - q is a location,
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- Given a PTA $\mathcal{A}$ and a point π, we denote by $\mathcal{A}[\pi]$ the (non-parametric) timed automaton where all parameters are instantiated by π
- (Parametric) state of a PTA: couple (q, C), where
  - q is a location,
  - C is a constraint (conjunction of inequalities) over the parameters
- Trace over a PTA: finite alternating sequence of locations and actions

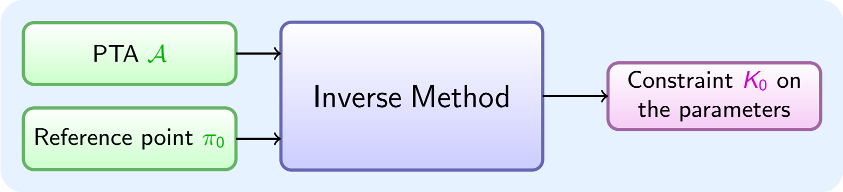
Good and Bad Traces w.r.t. a Given Property

- A trace is said to be a good trace if it verifies a given property
  - Example of good trace for the flip-flop ($Q^{\uparrow}$ occurs before $CK^{\downarrow}$)

  - Example of bad trace for the flip-flop

Outline

The Inverse Problem (1/2)

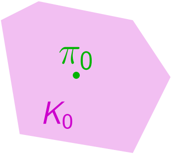


The Inverse Problem (2/2)

- Input
  - A PTA $\mathcal{A}$
  - A reference valuation π_0 of all the parameters of $\mathcal{A}$
    - ★ Exemplifying a good behavior
      (all traces of $\mathcal{A}[\pi_0]$ correspond to good behaviors)

The Inverse Problem (2/2)

- Input
  - A PTA $\mathcal{A}$
  - A reference valuation π_0 of all the parameters of $\mathcal{A}$
    - Exemplifying a good behavior
      (all traces of $\mathcal{A}[\pi_0]$ correspond to good behaviors)
- Output: tile K_0
  - Convex constraint on the parameters such that
    - $\pi_0 \models K_0$
    - For all point $\pi \models K_0$, $\mathcal{A}[\pi]$ and $\mathcal{A}[\pi_0]$ have the same trace sets



The Inverse Method: General Idea [André et al., 2009a]

Start with $K_0 = True$

REPEAT

1. Compute the set S of reachable parametric states under K_0
2. Refine K_0 by removing a π_0-incompatible state from S
   - Select a π_0-incompatible state (q, C) within S (i.e., $\pi_0 \not\models C$)
   - Select a π_0-incompatible inequality J within C (i.e., $\pi_0 \not\models J$)
   - Add $\neg J$ to K_0

UNTIL no more π_0-incompatible state in S

Application to the Flip-Flop Circuit (1/2)

- Input: π_0

$$
\begin{array}{llll}
T_{HI} = 24 & T_{LO} = 15 & T_{Setup} = 10 & T_{Hold} = 17 \\
\delta_1^- = 7 & \delta_1^+ = 7 & \delta_2^- = 5 & \delta_2^+ = 6 \\
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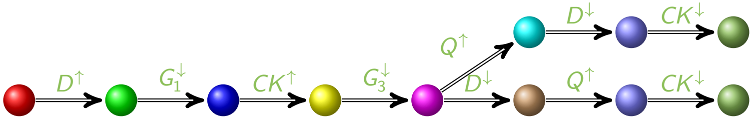
\end{array}
$$

- Output: K_0

$$
\begin{array}{ll}
T_{Setup} > \delta_1^+ & \wedge\ \delta_3^+ + \delta_4^+ \geq T_{Hold} \\
\wedge\ T_{Hold} > \delta_3^+ & \wedge\ \delta_3^+ + \delta_4^+ < T_{HI} \\
\wedge\ T_{Setup} \leq T_{LO} & \wedge\ \delta_3^- + \delta_4^- \leq T_{Hold} \\
\wedge\ \delta_1^- > 0 &
\end{array}
$$

  - Corresponding trace set



Application to the Flip-Flop Circuit (2/2)

π_0 :

$\delta_1^- = 7$	$\delta_1^+ = 7$	$T_{HI} = 24$
$\delta_2^- = 5$	$\delta_2^+ = 6$	$T_{LO} = 15$
$\delta_3^- = 8$	$\delta_3^+ = 10$	$T_{Setup} = 10$
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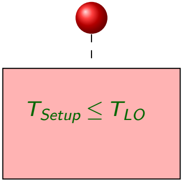
$K_0 = True$



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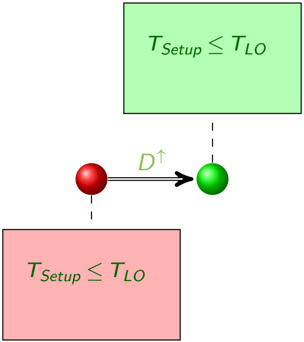

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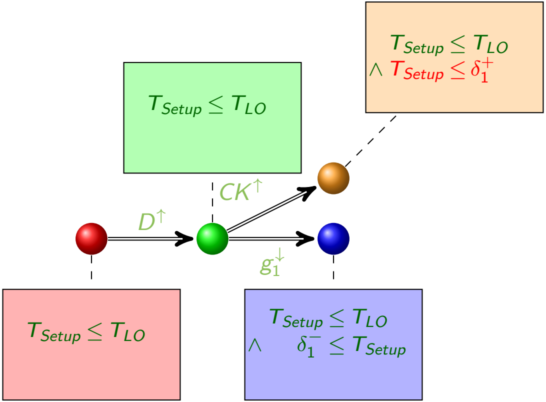

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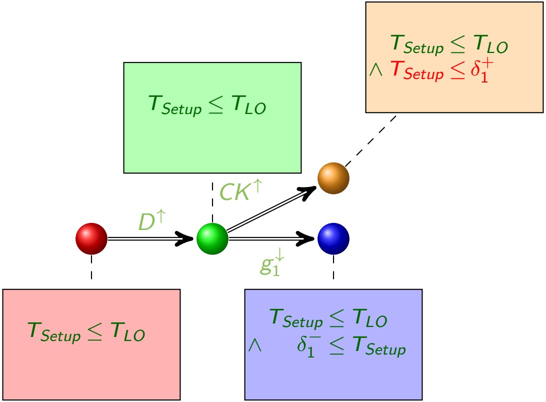
$K_0 =$

$T_{Setup} > \delta_1^+$



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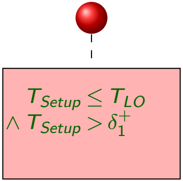

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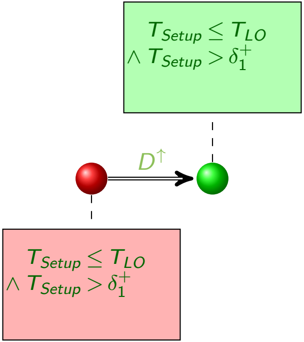

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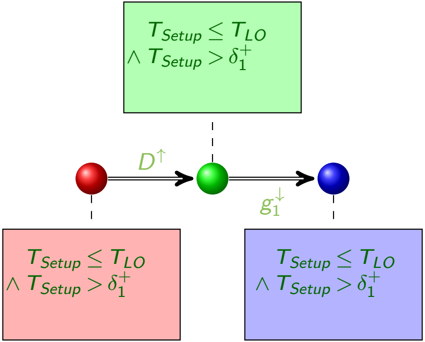

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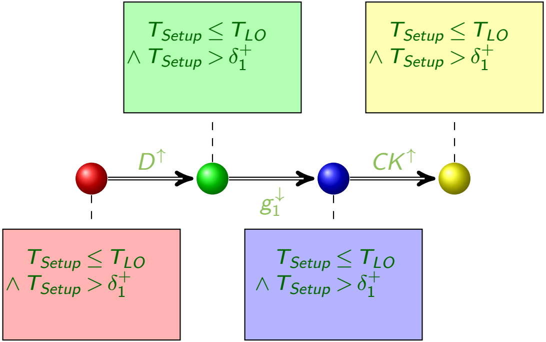

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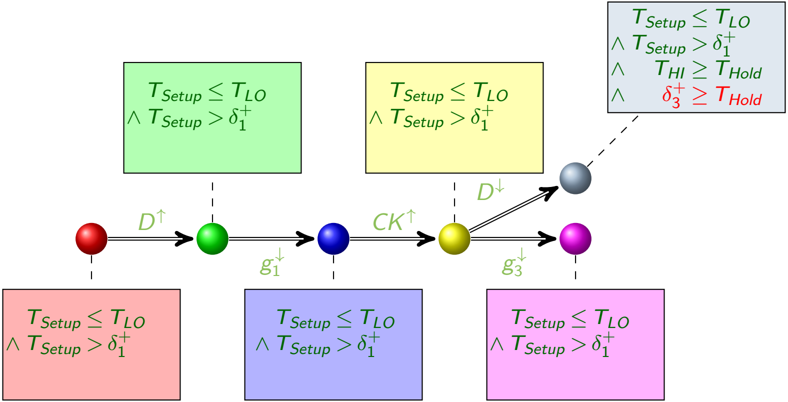

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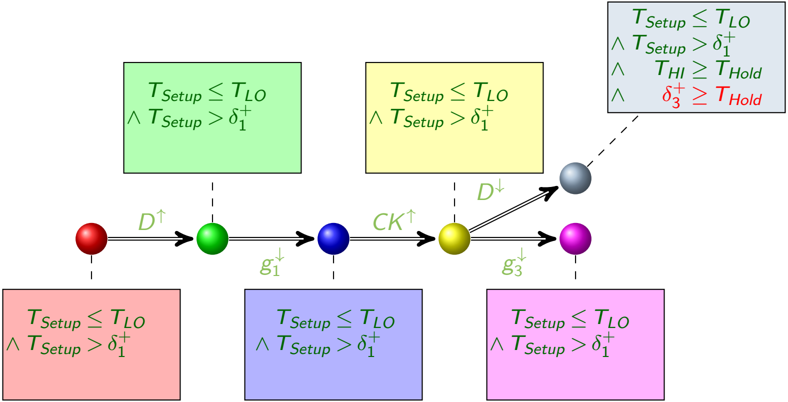

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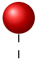

$$T_{Setup} \leq T_{LO} \wedge T_{Setup} > \delta_1^+ \wedge T_{Hold} > \delta_3^+$$

Application to the Flip-Flop Circuit (2/2)

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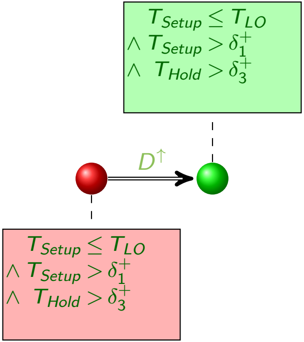

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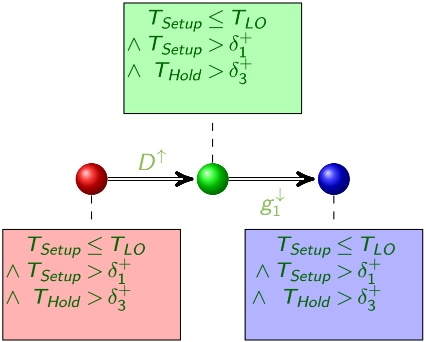

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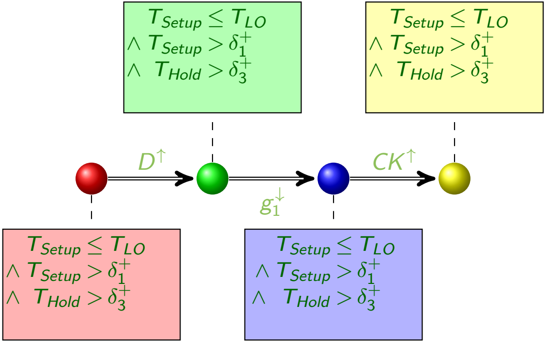

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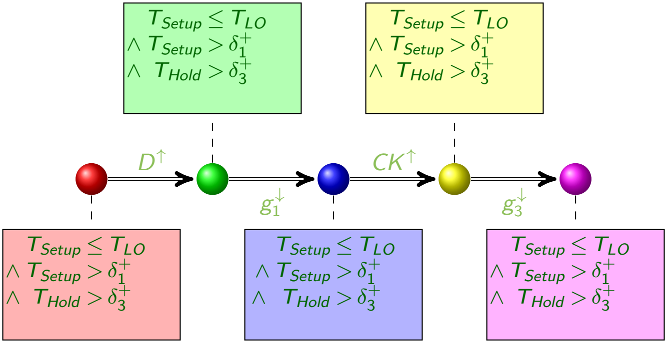

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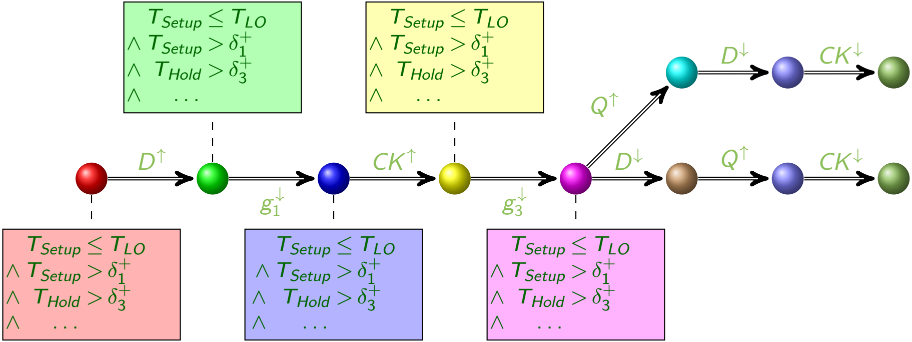
$\delta_2^- = 5$	$\delta_2^+ = 6$	$T_{LO} = 15$
$\delta_3^- = 8$	$\delta_3^+ = 10$	$T_{Setup} = 10$
$\delta_4^- = 3$	$\delta_4^+ = 7$	$T_{Hold} = 17$

$$K_0 = \begin{array}{lll} T_{Setup} > \delta_1^+ & \wedge & \delta_3^+ + \delta_4^+ \geq T_{Hold} \\ \wedge\ T_{Hold} > \delta_3^+ & \wedge & \delta_3^+ + \delta_4^+ < T_{HI} \\ \wedge\ T_{Setup} \leq T_{LO} & \wedge & \delta_3^- + \delta_4^- \leq T_{Hold} \\ \wedge\ \delta_1^- > 0 & & \end{array}$$



Advantages and Drawbacks of the Inverse Method

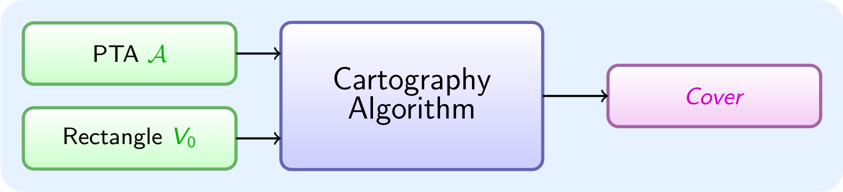
- Advantages
  - Useful to optimize timing bounds of systems
  - Terminates often in practice (unlike a brute reachability analysis, e.g., using HyTech)
  - Allows to handle dozens of parameters
- Drawbacks
  - The generated constraint K_0 is not maximal: there are points $\pi \notin K_0$ which give the same trace sets as π_0
  - The criterion of equality of trace sets may be too restrictive: for a given property ϕ, there may be different trace sets satisfying ϕ

Outline

Beyond the Inverse Method

- Goal: Find the maximal set of points corresponding to a good behavior
- Method: Iterate the inverse method for all the integer points of a given rectangle V_0
- Output: set of tiles for all the integer points of V_0
  - $\rightsquigarrow$ behavioral cartography of the parameter space

The Behavioral Cartography Algorithm



repeat
    select an integer point $\pi \in V_0$;
    if $\pi \notin Cover$ **then**
        $Cover \leftarrow Cover \cup IM(\mathcal{A}, \pi)$;
until *Cover contains all the integer points of the rectangle*;

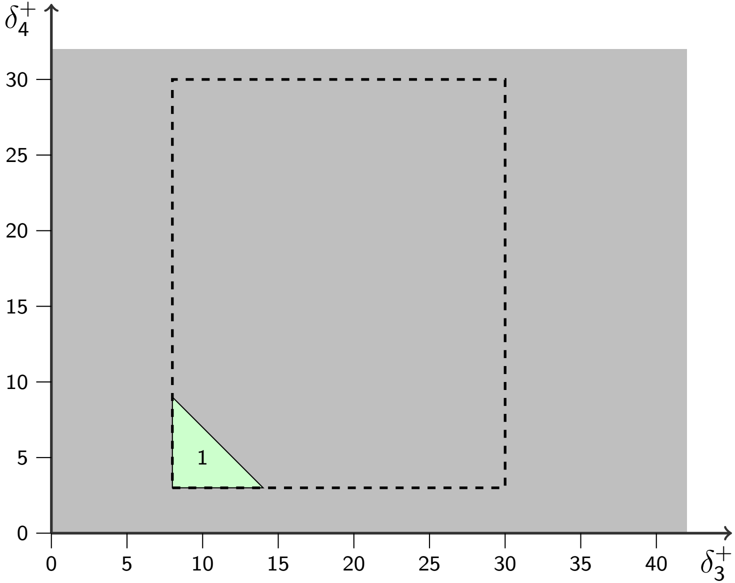
Partition into Good and Bad Tiles

- A tile is said to be a good tile if all its corresponding traces are good traces
- According to the nature of the trace sets, we can partition the tiles into good and bad ones

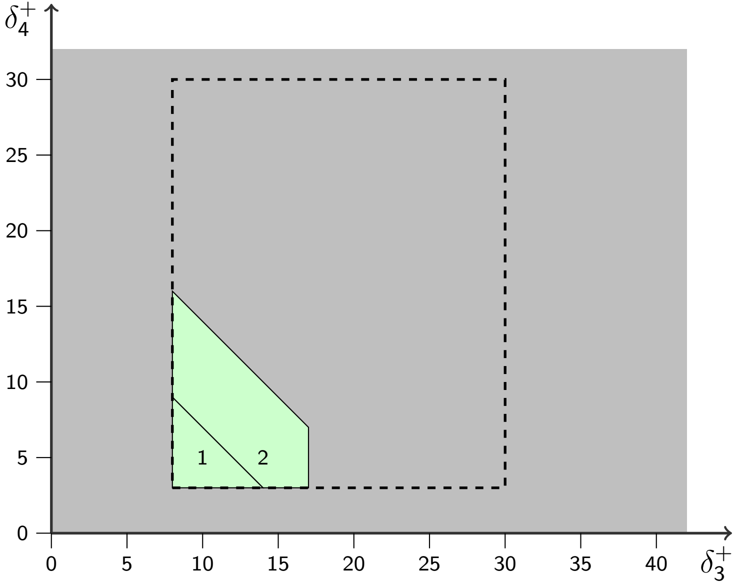
Application to our Flip-Flop Example

- We consider only parameters δ_3^+ and δ_4^+
  - The other parameters are instantiated
- Goal: Perform the behavioral cartography of the flip-flop circuit according to δ_3^+ and δ_4^+
  - Find the values for δ_3^+ and δ_4^+ such that the flip-flop has a good behavior

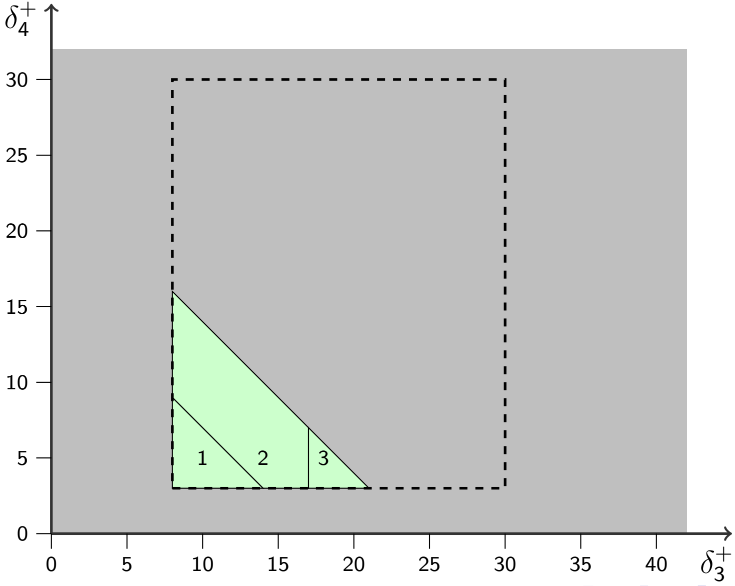
Behavioral Cartography of the Flip-Flop (δ_3^+ and δ_4^+)



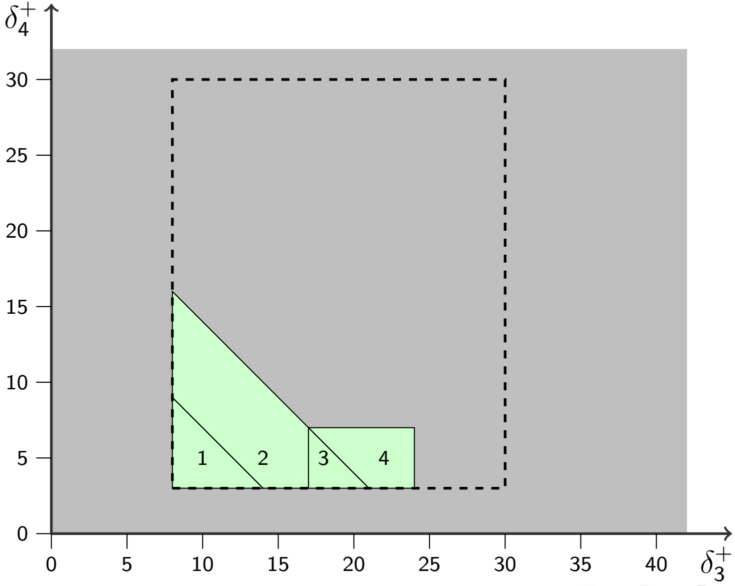
Behavioral Cartography of the Flip-Flop (δ_3^+ and δ_4^+)



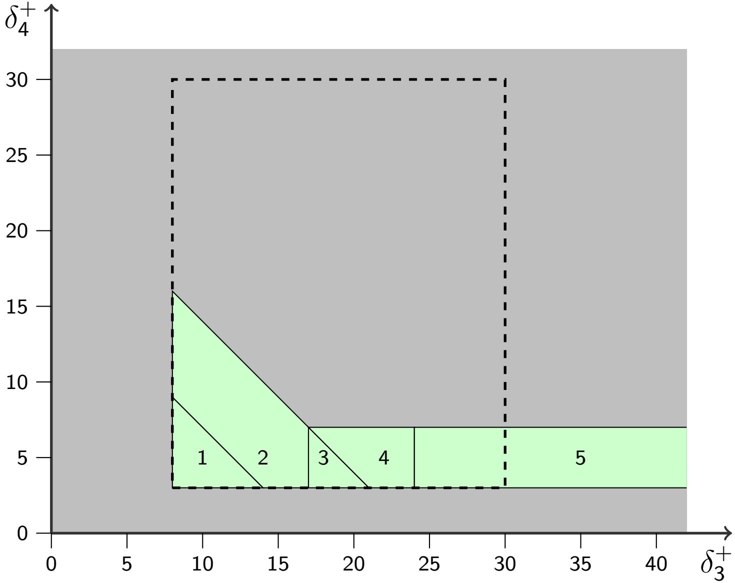
Behavioral Cartography of the Flip-Flop (δ_3^+ and δ_4^+)



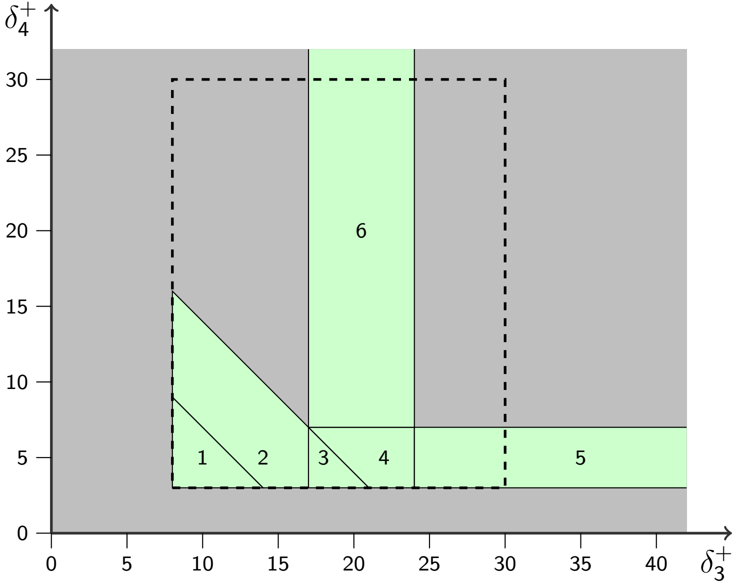
Behavioral Cartography of the Flip-Flop (δ_3^+ and δ_4^+)



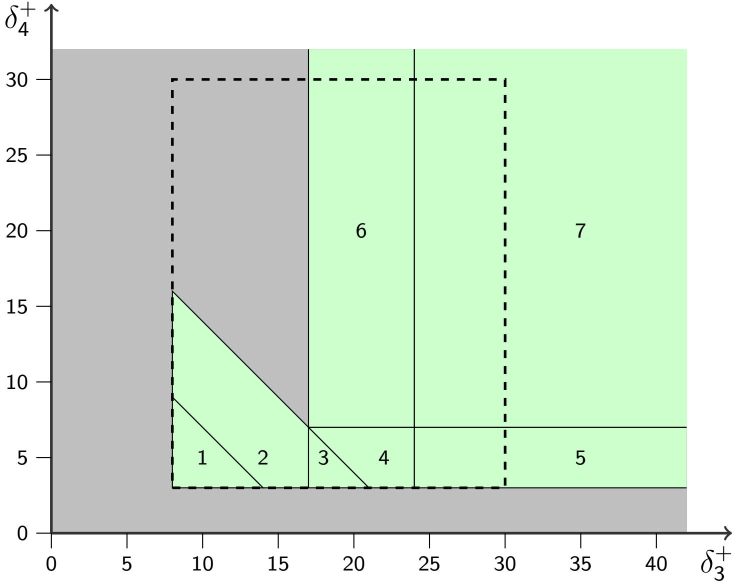
Behavioral Cartography of the Flip-Flop (δ_3^+ and δ_4^+)



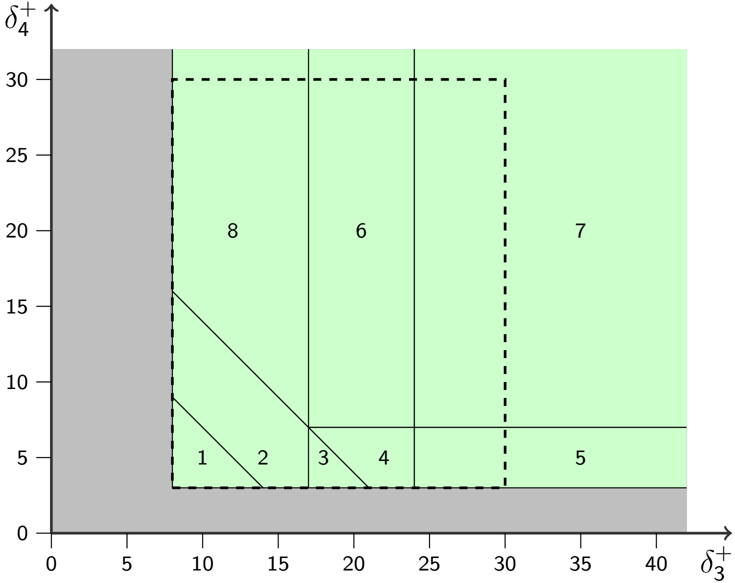
Behavioral Cartography of the Flip-Flop (δ_3^+ and δ_4^+)



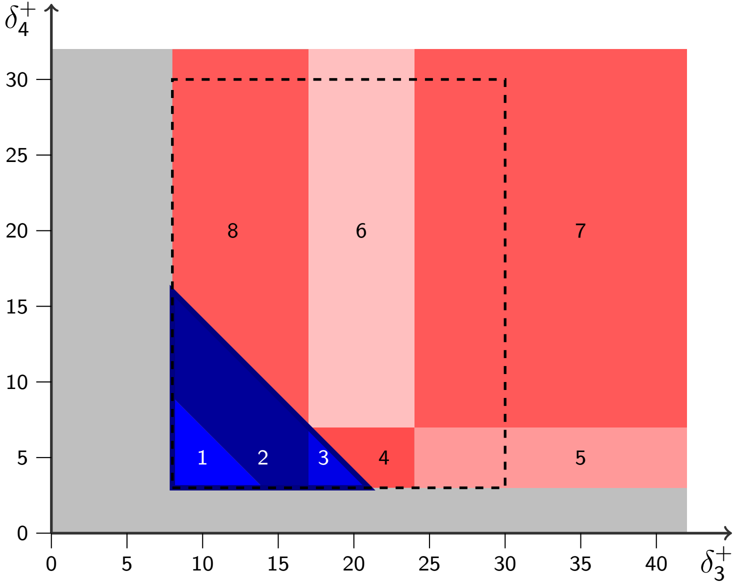
Behavioral Cartography of the Flip-Flop (δ_3^+ and δ_4^+)



Behavioral Cartography of the Flip-Flop (δ_3^+ and δ_4^+)

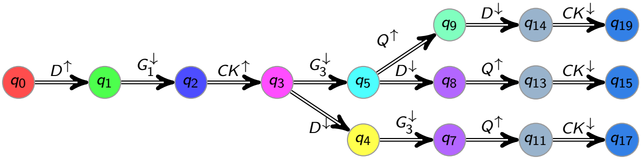


Behavioral Cartography of the Flip-Flop (δ_3^+ and δ_4^+)

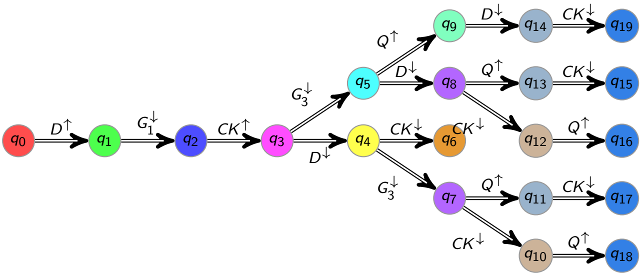


Example of good and bad tiles

- Good tile 3



- Bad tile 7



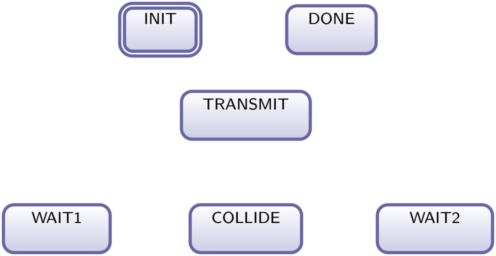
Behavioral Cartography of the Flip-flop: Remarks

- Remarks on the cartography
  - For this example, all the real-valued part of the parametric space within and outside V_0 is covered
- The set of good tiles (in blue) corresponds to the maximal set of good values for δ_3^+ and δ_4^+
  - $\delta_3^+ + \delta_4^+ \geq 24 \quad \wedge \quad \delta_3^+ \geq 8 \quad \wedge \quad \delta_4^+ \geq 3$

Outline

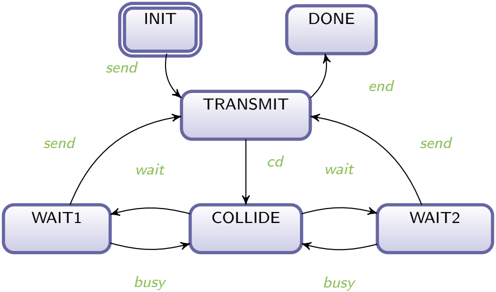
Probabilistic Timed Automaton

- Parametric Timed Automaton
  - Set of locations



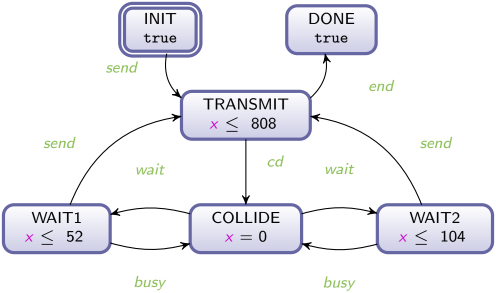
Probabilistic Timed Automaton

- Parametric Timed Automaton
  - Set of locations, set of actions



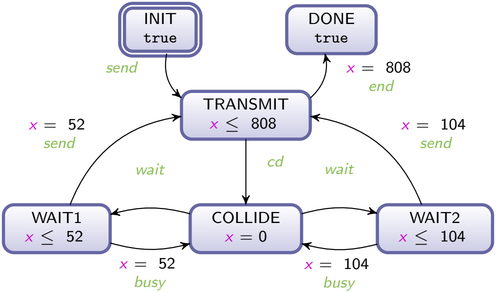
Probabilistic Timed Automaton

- Parametric Timed Automaton
  - Set of locations, set of actions
  - Set of clocks (real-valued variables increasing at the same linear rate)
    - Features: Location invariant



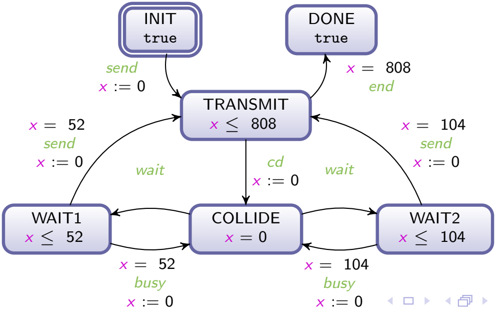
Probabilistic Timed Automaton

- Parametric Timed Automaton
  - Set of locations, set of actions
  - Set of clocks (real-valued variables increasing at the same linear rate)
    - Features: Location invariant, transition guard



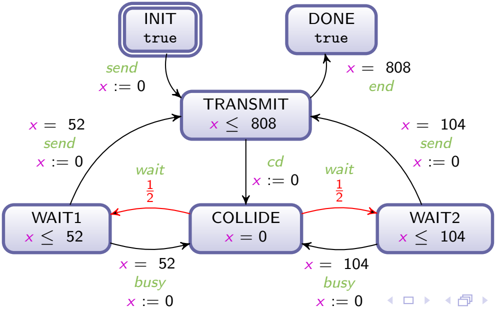
Probabilistic Timed Automaton

- Parametric Timed Automaton
  - Set of locations, set of actions
  - Set of clocks (real-valued variables increasing at the same linear rate)
    - ★ Features: Location invariant, transition guard, clock reset



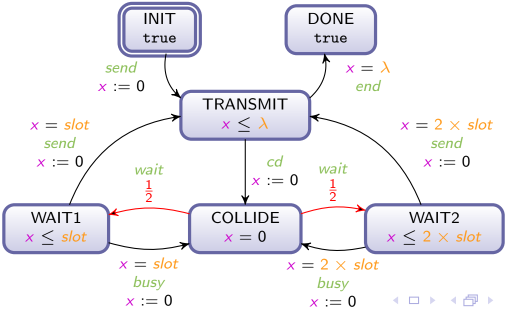
Probabilistic Timed Automaton

- Parametric Timed Automaton
  - Set of locations, set of actions
  - Set of clocks (real-valued variables increasing at the same linear rate)
    - Features: Location invariant, transition guard, clock reset
- Augmented with probabilities [Kwiatkowska et al., 2002]
  - The sum of the probabilities leaving a given location through a given action is equal to 1



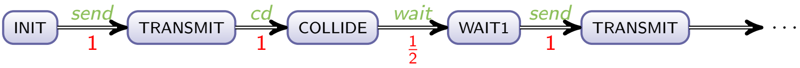
Parametric Probabilistic Timed Automaton (PPTA)

- Parametric Timed Automaton
  - Set of locations, set of actions, set of parameters (unknown constants) [André et al., 2009b]
  - Set of clocks (real-valued variables increasing at the same linear rate)
    - ★ Features: Location invariant, transition guard, clock reset
- Augmented with probabilities [Kwiatkowska et al., 2002]
  - The sum of the probabilities leaving a given location through a given action is equal to 1



Semantics

- Semantics for timed automata
  - Time elapsing in a location, and
  - Discrete actions: instantaneous transition from a location to another one
- Semantics for probabilistic timed automata
  - Time elapsing in a location, and
  - Discrete actions: instantaneous transition from a location to a distribution of locations
- Probabilistic traces
  - Finite alternating sequence of locations and actions with probabilities



Minimum and Maximum Probabilities of Reaching a State

- A scheduler s associates to every state one output distribution
  - Denoted by $\mathcal{A}^s$

- Given a scheduler, one can define the probability of reaching a location

- Minimum and maximum probabilities of reaching a given location
  - Minimum and maximum for all possible schedulers

- Derandomized form $\mathcal{A}^*$ of a PPTA $\mathcal{A}$: replace distributions by non-determinism: $\mathcal{A}^*$ becomes a PTA
  - Given some π, we have:

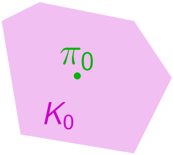
$$Traces(\mathcal{A}^*[\pi]) = \bigcup_{s \in Sched} Traces(\mathcal{A}^s[\pi])$$

The Inverse Problem for PPTAs

- Inputs
  - A PPTA $\mathcal{A}$
  - A reference valuation π_0 of $\mathcal{A}$

The Inverse Problem for PPTAs

- Inputs
  - A PPTA $\mathcal{A}$
  - A reference valuation π_0 of $\mathcal{A}$
- Output: tile K_0
  - Convex constraint on the parameters such that
    - $\pi_0 \models K_0$
    - For all $\pi \models K_0$, the sets of probabilistic traces of $\mathcal{A}[\pi]$ and $\mathcal{A}[\pi_0]$ are equal



The Inverse Problem for PPTAs

- Inputs
  - A PPTA $\mathcal{A}$
  - A reference valuation π_0 of $\mathcal{A}$
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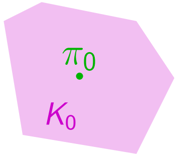


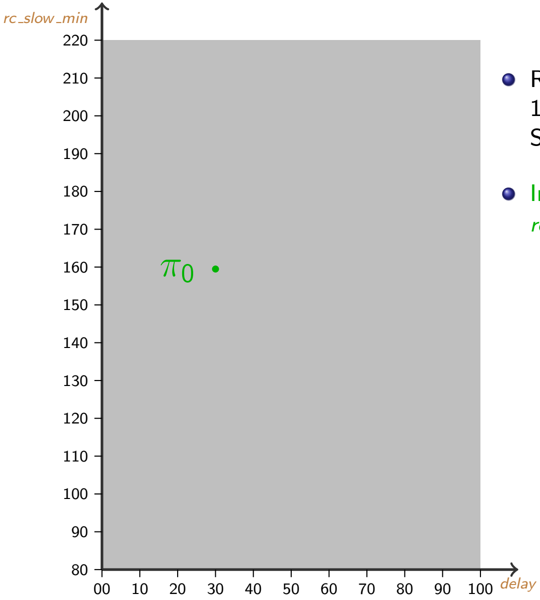
As a consequence, the minimum and maximum probabilities for reachability properties are the same in $\mathcal{A}[\pi]$ and $\mathcal{A}[\pi_0]$

Extension of the Inverse Method to Probabilistic Systems

1. Construct a derandomized (non-probabilistic) version $\mathcal{A}^*$ of $\mathcal{A}$
2. Compute a constraint K_0 by applying the inverse method to $\mathcal{A}^*$ and π_0

Then the minimum (resp. maximum) probability of reaching a given location of $\mathcal{A}$ is the same for all $\pi \in K_0$.

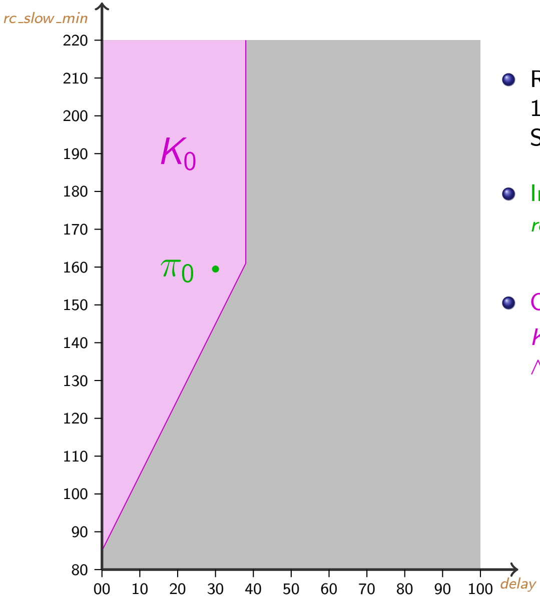
Application to The Root Contention Protocol



- Root contention protocol of the IEEE 1394 ("FireWire") High Performance Serial Bus [Hune et al., 2002]
- Input: IEEE reference valuation

$$rc\_slow\_min = 159ns$$
$$delay = 30ns$$

Application to The Root Contention Protocol



- Root contention protocol of the IEEE 1394 ("FireWire") High Performance Serial Bus [Hune et al., 2002]
- Input: IEEE reference valuation

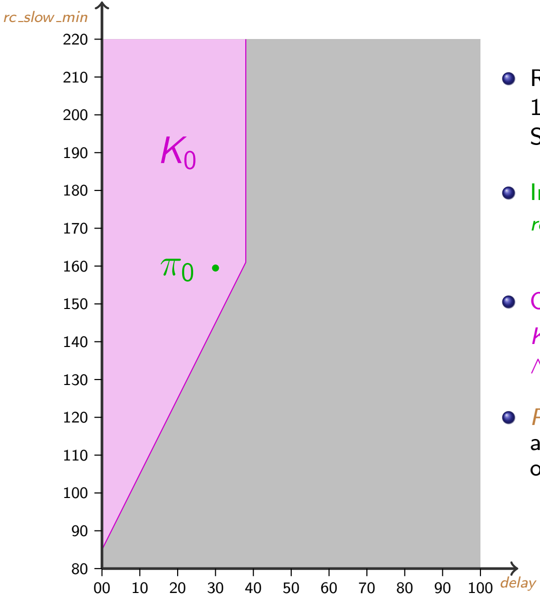
  $rc\_slow\_min = 159ns$

  $delay = 30ns$
- Output:

  $K_0 : 2delay < 76$

  $\wedge \ 2delay + 85 < rc\_slow\_min$

Application to The Root Contention Protocol



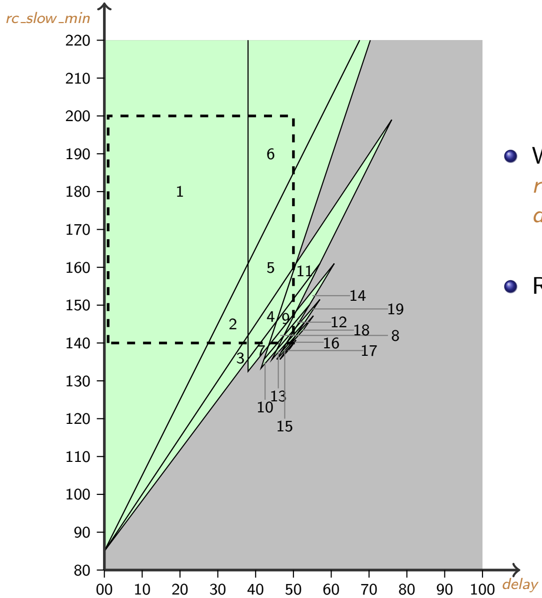
- Root contention protocol of the IEEE 1394 ("FireWire") High Performance Serial Bus [Hune et al., 2002]
- Input: IEEE reference valuation
  $rc\_slow\_min = 159ns$
  $delay = 30ns$
- Output:
  $K_0 : 2delay < 76$
  $\wedge\ 2delay + 85 < rc\_slow\_min$
- $Prop_3$: The minimum probability that a leader is elected after three rounds or less is equal to 0.75
  - For all $\pi \models K_0$, $Prop_3$ is satisfied

Extension of the Cartography to Probabilistic Systems

1. Construct a derandomized (non-probabilistic) version $\mathcal{A}^*$ of $\mathcal{A}$
2. Apply the cartography algorithm to $\mathcal{A}^*$ and V_0

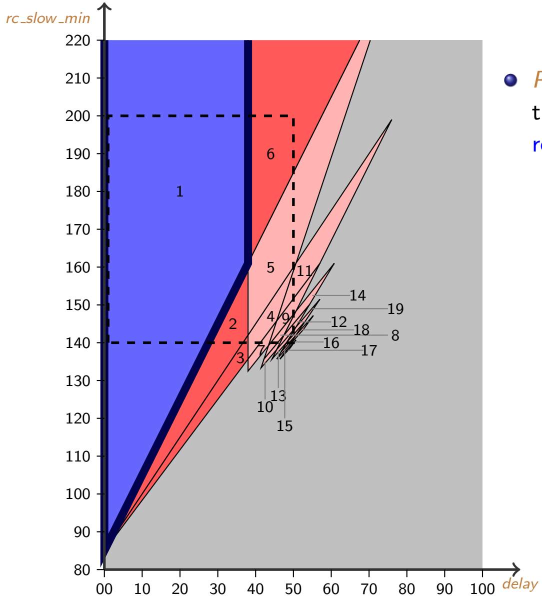
Then the minimum (resp. maximum) probability of reaching a given location of $\mathcal{A}$ is uniform within each tile of the cartography.

The Root Contention Protocol: Cartography (1/2)



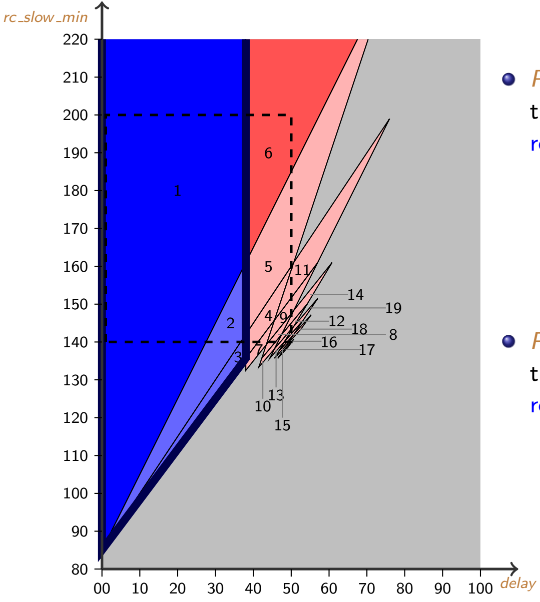
- We consider the following V_0 : $rc\_slow\_min \in [140; 200]$, and $delay \in [1; 50]$
- Remarks
  - Tiles 1 and 6 are infinite towards one dimension
  - The cartography does not cover the whole real-valued space within V_0 (holes in the lower right corner of V_0)

The Root Contention Protocol: Cartography (2/2)



- $Prop_3$: "The minimum probability that a leader is elected after three rounds or less is equal to p"
  - Tile 1: $p = 0.75$
  - Tiles 2, 3, 6: $p = 0.625$
  - Other tiles: : $p = 0.5$
  - Good tile if $p \geq 0.75$

The Root Contention Protocol: Cartography (2/2)



- *Prop*$_3$: "The minimum probability that a leader is elected after three rounds or less is equal to p"
  - Tile 1: $p = 0.75$
  - Tiles 2, 3, 6: $p = 0.625$
  - Other tiles: : $p = 0.5$
  - Good tile if $p \geq 0.75$
- *Prop*$_5$: "The minimum probability that a leader is elected after five rounds or less is equal to p"
  - Tile 1: $p = 0.94$
  - Tiles 2 and 3: $p = 0.79$
  - Tile 6: $p = 0.66$
  - Other tiles: : $p = 0.5$
  - Good tile if $p \geq 0.75$

Advantages of the Probabilistic Cartography

- Quantitative refinement of the good parameters problem
  - Instead of a partition with a binary criterion (good / bad), we have a partition according to various probabilities
- The cartography is independent from the probabilistic property
  - Only the probability associated to each tile depends on the property
  - No need to compute a cartography for each property

Implementation

- Tool IMITATOR II [André, 2010]
  - IMITATOR: "Inverse Method for Inferring Time AbstracT BehaviOR"
  - 8000 lines of code
  - 6 man-months of work
  - Program written in OCaml
  - Makes use of the PPL library

- IMITATOR II is available on its Web page
  - http://www.lsv.ens-cachan.fr/~andre/IMITATOR2

Case Studies

- Implementation in IMITATOR II
  - outputs a list of tiles with their corresponding trace set under a graphical form
  - outputs the cartography under a graphical form (for 2 parameter dimensions)
- Computation times of various case studies
  - Experiments conducted on an Intel Core2 Duo 2.4 GHz with 2 Gb

Example	PTAs	loc./PTA	$\|X\|$	$\|P\|$	$\|V_0\|$	tiles	states	trans.	Time
SR-latch	3	[3, 8]	3	3	1331	6	5	4	0.3
Flip-flop	5	[4, 16]	5	2	644	8	15	14	3
Latch circuit	7	[2, 5]	8	4	73062	5	21	20	96.3
And–Or	3	[4, 8]	4	6	75600	4	64	72	118
CSMA/CD	3	[3, 8]	3	3	2000	140	349	545	269
SPSMALL	10	[3, 8]	10	2	3149	259	60	61	1194
RCP	5	[6, 11]	6	3	186050	19	5688	9312	7018

Outline

Summary (1/2)

- Inverse Method: Algorithm *IM*
  - Modeling of a system with parametric timed automata
  - Starting with a valuation π_0 of the system, we generate a constraint K_0 with the same trace set as π_0
- Behavioral cartography: Algorithm *BC*
  - Solves the good parameters problem: synthesizes the largest set of points within a rectangle V_0 corresponding to a given good behavior
  - Under certain conditions, covers the whole real-valued parametric space

Summary (2/2)

- Extension to probabilistic systems
  - Synthesizes a set of tiles, with uniform min/max reachability probabilities within each tile
  - Useful to compute probabilities (e.g., using Prism) for systems with large constants (notion of rescaling)
  - Avoid the repeated computation of probabilities for many different values of the parameters

Future Work

- Extend the behavioral cartography to hybrid automata
  - Allow to consider different clock rates
- Consider a weaker property than equality of trace sets
  - Reference trace with partial orders

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The Inverse Method: Algorithm

Algorithm 1: $IM(\mathcal{A}, \pi_0)$

input : A PTA $\mathcal{A}$ of initial state s_0
input : Reference point π_0 of the parameters
output: Constraint K_0 on the parameters

$i \leftarrow 0$; $K_0 \leftarrow True$; $S \leftarrow \{s_0\}$
while $True$ **do**
  while *there are* π_0*-incompatible states in* S **do**
    Select a π_0-incompatible state (q, C) of S (i.e., s.t. $\pi_0 \not\models C$) ;
    Select a π_0-incompatible J in C (i.e., s.t. $\pi \not\models J$) ;
    $K_0 \leftarrow K_0 \wedge \neg J$;
    $S \leftarrow \bigcup_{j=0}^{i} Post^{j}_{\mathcal{A}(K_0)}(\{s_0\})$;
  if $Post_{\mathcal{A}(K_0)}(S) = \emptyset$ **then return** $K_0 \leftarrow \bigcap_{(q,C) \in S} (\exists X : C)$
  $i \leftarrow i + 1$;
  $S \leftarrow S \cup Post_{\mathcal{A}(K_0)}(S)$; // $S = \bigcup_{j=0}^{i} Post^{j}_{\mathcal{A}(K_0)}(\{s_0\})$