Université Paris 13
Institut Galilée
Master 2 PLS

SITH – Partie 2.1
Systèmes temporisés

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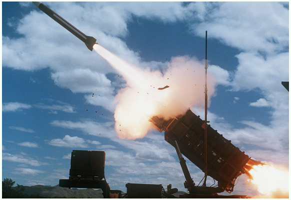
Partie 2.1: SITH – Plan

Context: Verifying complex timed systems

- Need for early bug detection
  - Bugs discovered when final testing: expensive
  - ⇝ Need for a thorough specification and verification phase



The Therac-25 radiation therapy machine (1/2)

- Radiation therapy machine used in the 1980s
- Involved in accidents between 1985 and 1987, in which patients were given massive overdoses of radiation
  - Approximately 100 times the intended dose!
  - Numerous causes, including race condition

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"The failure only occurred when a particular nonstandard sequence of keystrokes was entered on the VT-100 terminal which controlled the PDP-11 computer: an `X` *to (erroneously) select 25MV photon mode followed by ↑,* `E` *to (correctly) select 25 MeV Electron mode, then* `Enter`*, all within eight seconds."*

The Therac-25 radiation therapy machine (2/2)

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Limits of testing

This case illustrates the difficulty of bug detection without formal methods.

Bugs can be difficult to find

. . . and can have dramatic consequences for critical systems:

- health-related devices
- aeronautics and aerospace transportation
- smart homes and smart cities
- military devices
- etc.

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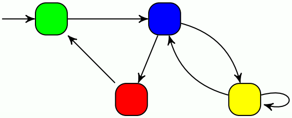
- etc.

Hence, high need for formal verification

Plan: Finite-State Automata

Model checking concurrent systems

- Use formal methods [Baier and Katoen, 2008]



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 is unreachable

A property to be satisfied

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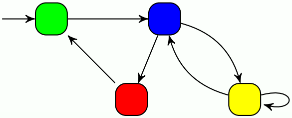

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- Question: does the model of the system satisfy the property?

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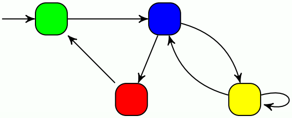

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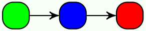

Counterexample

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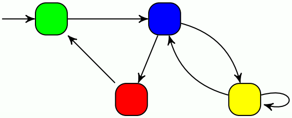

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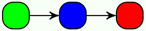

Counterexample

Turing award (2007) to Edmund M. Clarke, Allen Emerson and Joseph Sifakis

Transition systems

Définition (Transition system)

A *transition system (TS)* is a tuple $TS = (S, \Sigma, S_I, S_F, \Rightarrow)$, where

- S is a set of states;
- Σ is an alphabet of events;
- $S_I \subseteq S$ is a set of initial states;
- $S_F \subseteq S$ is a set of final (or accepting) states; and,
- $\Rightarrow : S \times \Sigma \to 2^S$ is a transition relation.

Usually, we write $s_1 \stackrel{a}{\Longrightarrow} s_2$ when $(s_1, a, s_2) \in \Rightarrow$.

Finite-state automata

Définition (Finite automaton)

A *Finite automaton (FA)* $FA = (L, \Sigma, l_I, L_F, \rightarrow)$ is a tuple where

- L is a finite set of locations;
- Σ is a finite set of actions;
- $l_I \in L$ is the initial location;
- $L_F \subseteq L$ is a set of final (or accepting) locations;
- $\rightarrow : L \times \Sigma \rightarrow 2^L$ is a transition relation.

Usually, we write $l_1 \xrightarrow{a} l_2$ when $(l_1, a, l_2) \in \rightarrow$.

Example 1

$FA = (L, \Sigma, l_I, L_F, \rightarrow)$, with

- $L = \{l_1, l_2, l_3\}$
- $\Sigma = \{a, b, c, d\}$
- $l_I = l_1$
- $L_F = \{l_2\}$
- $\rightarrow = \{(l_1, a, l_1), (l_1, b, l_2), (l_2, c, l_1), (l_2, d, l_2), (l_3, b, l_2)\}$

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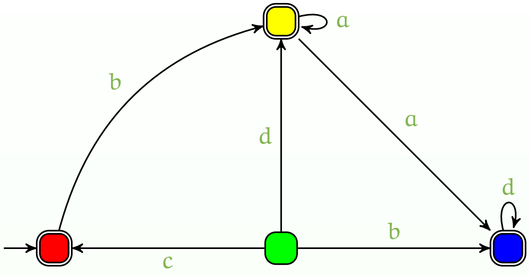

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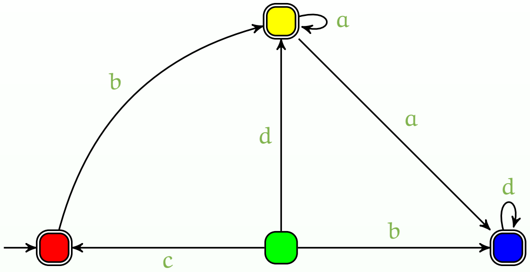


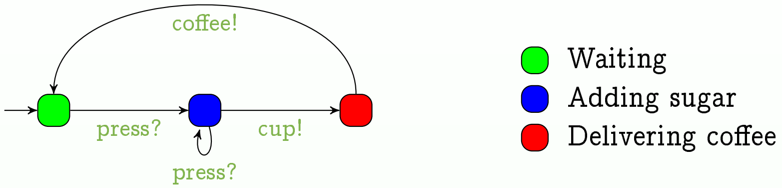
Semantics of finite automata

Définition (Semantics of finite automata)

Let $FA = (L, \Sigma, l_I, L_F, \Rightarrow)$ be a Finite Automaton.
The semantics of FA is the transition system $TS = (S, \Sigma, S_I, S_F, \Rightarrow)$, with

- $S = L$;
- Σ the same;
- $S_I = \{l_I\}$;
- $S_F = L_F$; and,
- $\Rightarrow = \rightarrow$.

A coffee machine $\mathcal{A}_C$



- Example of runs
  - Coffee with no sugar

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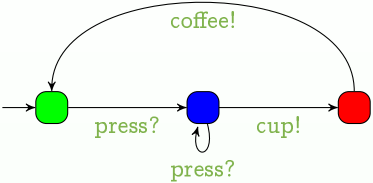

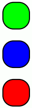

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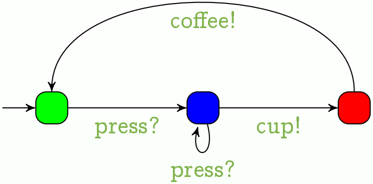

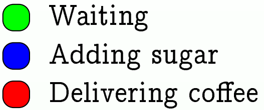

- Example of runs
  - Coffee with no sugar
  - Coffee with 2 doses of sugar
  - And so on

A coffee drinker (1/2)

- Specify a coffee drinker automaton $\mathcal{A}_{\mathrm{D1}}$ that performs forever the following actions:
  1. press the button once
  2. place the cup
  3. wait for the coffee
  4. drink the coffee
  5. put the cup to the washing machine

A coffee drinker (2/2)

- Specify a coffee drinker automaton $\mathcal{A}_{D2}$ that works just as $\mathcal{A}_{D1}$ except that (s)he can nondeterministically ask for 0, 1 or 2 doses of sugar.

A washing machine

- Specify a washing machine automaton $\mathcal{A}_W$ that accepts up to 5 cups, and washes all cups when the machine is full.

Systems as components

Often, a complex system is made of components or modules

Components can interact with each other:

- using strong synchronization
- using shared variables
- using one-to-one synchronization
- in an interleaving manner

Here, we show that FAs can be composed easily using strong synchronization on actions.

Composition of finite automata

$FA_1 = (L_1, \Sigma_1, (l_I)_1, (L_F)_1, \rightarrow_1)$
$FA_2 = (L_2, \Sigma_2, (l_I)_2, (L_F)_2, \rightarrow_2)$

Then we define $FA_1 \parallel FA_2$ as

Composition of finite automata: Example 1

Draw the automaton composed of the automata $\mathcal{A}_{\mathrm{C}} \parallel \mathcal{A}_{\mathrm{D1}}$

Composition of finite automata: Example 2

Draw the automaton composed of the automata $\mathcal{A}_{\text{C}} \parallel \mathcal{A}_{\text{D2}}$

Composition of finite automata: Example 3

Start to draw the automaton composed of the automata $\mathcal{A}_{\mathrm{C}} \parallel \mathcal{A}_{\mathrm{D2}} \parallel \mathcal{A}_{W}$. What do you notice?

Temporal logics

Modal logics expressing timing information over a set of atomic propositions, and can be used to formally verify a model.

Some temporal logics:

- LTL (Linear Temporal Logic) [Pnueli, 1977]
- CTL (Computation Tree Logic) [Clarke and Emerson, 1982]
- MITL
- CTL*
- μ-calculus

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Warning

Temporal logics express the ordering between events over time, but do not (in general) contain timed information.

LTL (Linear Temporal Logic) [Pnueli, 1977]

LTL expresses formulas about the future of one path, using a set of atomic propositions AP

Minimal syntax:

$$\varphi ::= p \in AP \mid \neg\varphi \mid \varphi \vee \varphi \mid \mathsf{X}\varphi \mid \varphi \mathsf{U} \varphi$$

Explanation and additional operators:

$p \in AP$	atomic proposition	
X	Next	"at the next step"
U	Until	
F	Finally (eventually)	"now or sometime later"
G	Globally	"now and anytime later"
R	Release	
W	Weak until	

LTL: Examples

Express in LTL the following properties:

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- "If I ask for food infinitely often, then I will get food infinitely often" (strong fairness property)

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CTL (Computation Tree Logic) [Clarke and Emerson, 1982]

CTL expresses formulas on the order between the future events for some or for all paths, using a set of atomic propositions AP

Quantifiers over paths:

$$\varphi ::= p \in AP \mid \neg\varphi \mid \varphi \vee \varphi \mid \mathsf{E}\psi \mid \mathsf{A}\psi$$

Quantifiers over states:

$$\psi ::= \mathsf{X}\varphi \mid \varphi\mathsf{U}\varphi$$

Explanation:

E	Exists	"along some of the future paths"
A	ForAll	"along all the future paths"

CTL: More on quantifiers

A path quantifier must always be followed by a state quantifier.

Some useful combinations:

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The reachability problem

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Given FA, given a given location $\mathfrak{l}$, does there exist a path from an initial location of FA leading to $\mathfrak{l}$?

Applications:

- Is there an execution of the therapy machine leading to the delivery of high radiations?
- Can the coffee machine deliver a coffee with five doses of sugar?

Forward reachability

Let S be the set of all reachable states.
Given a subset $S' \subseteq S$ of states, which states of S are reachable from S' in just one step?

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By extension, we write $Post^*(S')$ for the set of all states reachable from states of S'.

Forward reachability: Algorithm

Algorithm *isReachable*(TS, S_I, S_F)

input : Set S_I of initial states, set S_F of final states

output : `true` if S_F is reachable from S_I, `false` otherwise

$S \leftarrow S_I$; $i \leftarrow 0$;

Forward reachability: Algorithm

Algorithm $isReachable(TS, S_I, S_F)$

input : Set S_I of initial states, set S_F of final states

output : `true` if S_F is reachable from S_I, `false` otherwise

```
S ← S_I ; i ← 0 ;
repeat
    if S ∩ S_F ≠ ∅ then
        ;
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repeat
  if $S \cap S_F \neq \emptyset$ **then**
    ;
  $S \leftarrow$;
  $i \leftarrow i + 1$;
until ;
return ;

Forward reachability: Applications

Verifying properties using observers

An observer is an automaton that observes the system behavior

- It synchronizes with other automata's actions
- It must be non-blocking (see example on the white board)
- Its location(s) give an indication on the system property

Then verifying the property reduces to a reachability condition on the observer (in parallel with the system)

Observers for the coffee machine (1/2)

Design an observer for the coffee machine and the drinker verifying that it is possible to order a coffee with at least one dose of sugar.
(. . . and check the validity of the property)

Observers for the coffee machine (2/2)

Design an observer for the coffee machine and the drinker verifying that whenever the coffee comes, the cup was not put to the washing machine before.

(. . . and check the validity of the property)

Plan: Timed Automata

Beyond finite state automata

Finite State Automata give a powerful syntax and semantics to model qualitative aspects of systems

- Executions, sequence of actions
- Modular definitions (parallelism)
- Powerful checking (reachability, safety, liveness. . .)

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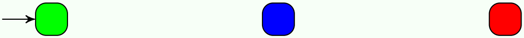
But what about quantitative aspects:

- Time ("the airbag always eventually inflates, but maybe 10 seconds after the crash")
- Temperature ("the alarm always eventually ring, but maybe when the temperature is above 75 degrees")

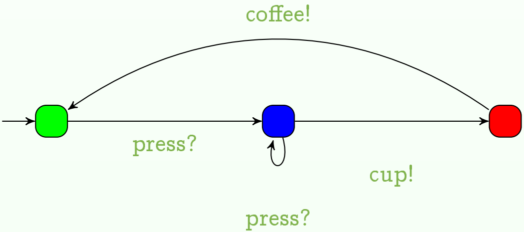
Timed automaton (TA)

- Finite state automaton (sets of locations)



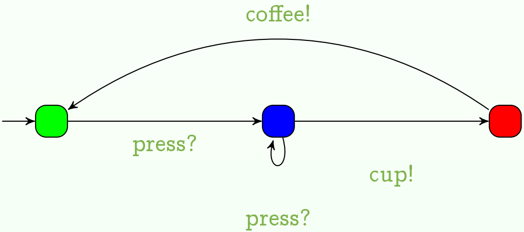
Timed automaton (TA)

- Finite state automaton (sets of locations and actions)



Timed automaton (TA)

- Finite state automaton (sets of locations and actions) augmented with a set X of clocks [Alur and Dill, 1994]
  - Real-valued variables evolving linearly at the same rate



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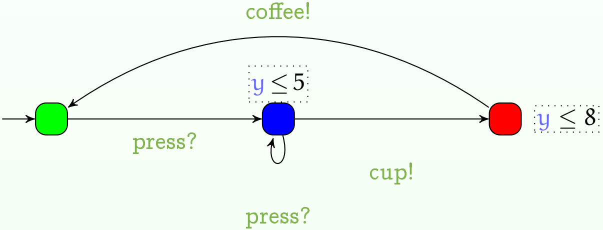

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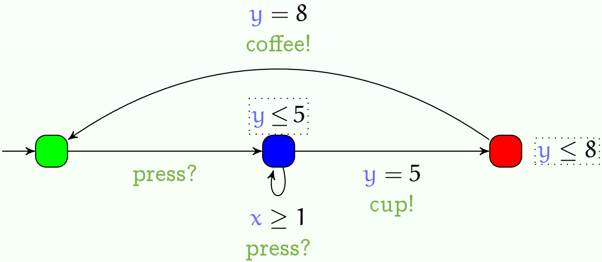

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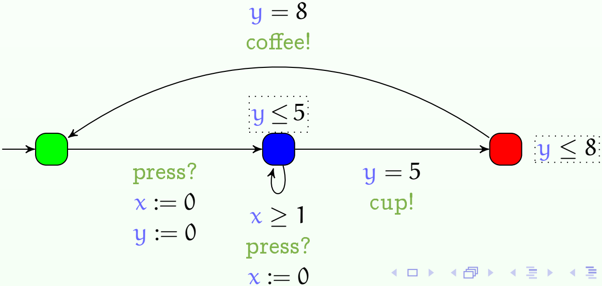
  - Clock reset: some of the clocks can be set to 0 at each transition



Formal definition of timed automata

Définition (Timed automaton)

A *timed automaton (TA)* $\mathcal{A}$ is a 6-tuple of the form $\mathcal{A} = (L, \Sigma, l_I, X, Inv, \rightarrow)$, where

- L is a finite set of locations, $l_I \in L$ is the initial location,
- Σ is a finite set of actions,
- X is a set of clocks,
- Inv is the invariant, assigning to every $l \in L$ a constraint $Inv(l)$ on the clocks, and
- $\rightarrow$ is a step (or "transition") relation consisting of elements of the form $e = (l, g, a, R, l')$, also denoted by $l \xrightarrow{g,a,R} l'$, where $l, l' \in L$, $a \in \Sigma$, $R \subseteq X$ is a set of clock variables to be reset by the step, and g (the step guard) is a constraint on the clocks.

Example 1

Draw the TA $\mathcal{A} = (L, \Sigma, l_1, X, Inv, \rightarrow)$ such that

- $L = \{l_1, l_2, l_3, l_4\}$,
- $\Sigma = \{a_1, a_2, a_3\}$,
- $X = \{x_1, x_2\}$,
- $Inv(l_1) = x_1 \leq 3$, and $Inv(l_3) = x_2 \geq 2$,
- $\rightarrow = \{(l_1, x_1 \geq 2, a_1, \{x_1\}, l_2)$,
  $(l_1, x_2 \leq 1, a_2, \emptyset, l_3)$,
  $(l_2, x_2 = 1, a_3, \{x_2\}, l_2)$,
  $(l_2, \texttt{true}, a_1, \emptyset, l_3)$,
  $(l_3, \texttt{true}, a_2, \{x_1, x_2\}, l_4)$,
  $(l_4, x_2 > 2, a_3, \emptyset, l_3)\}$

Example 2

Give the formal TA corresponding to the timed coffee machine.

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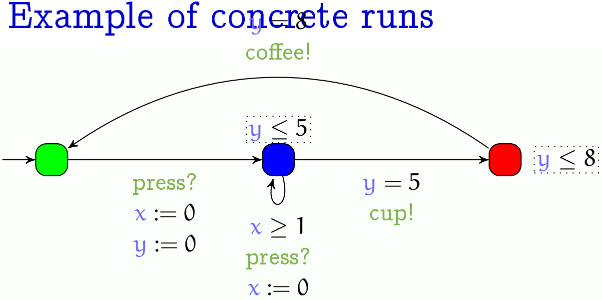
Concrete semantics of timed automata

- Concrete state of a TA: pair (l, w), where
  - l is a location,
  - w is a valuation of each clock

  Example: (, $\binom{x=1.2}{y=3.7}$)

- Concrete run: alternating sequence of concrete states and actions or time elapse

Example of concrete runs



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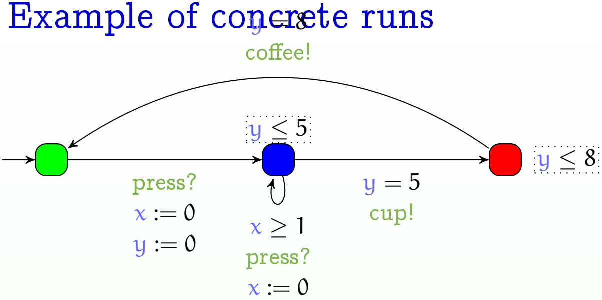

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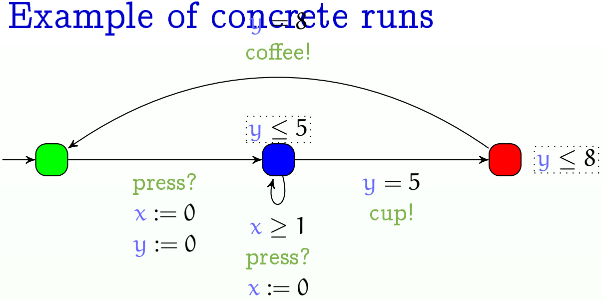

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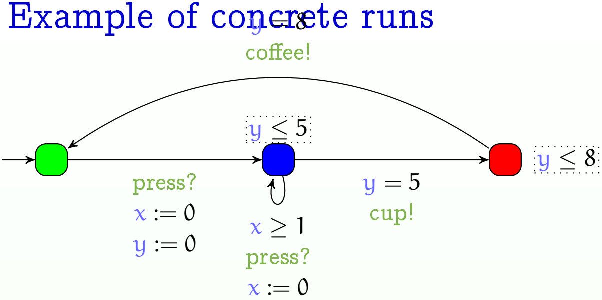

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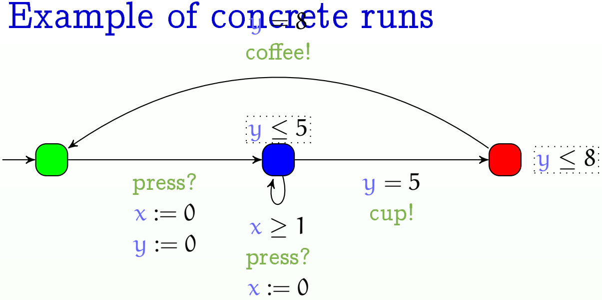

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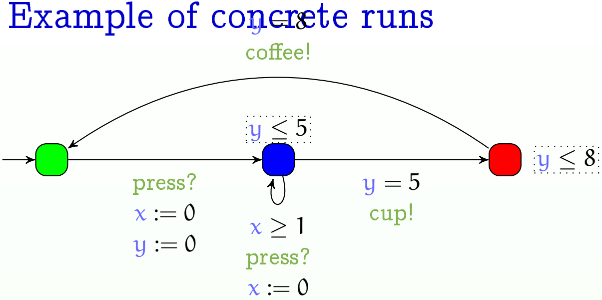

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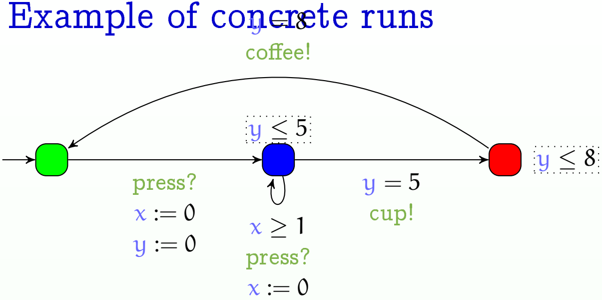

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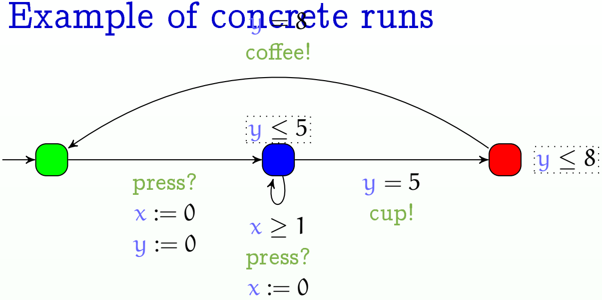

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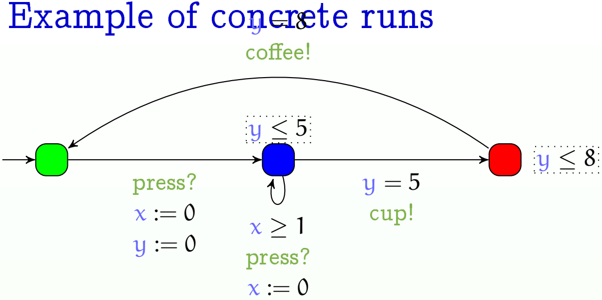

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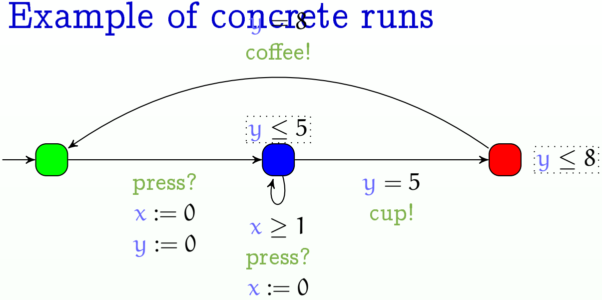

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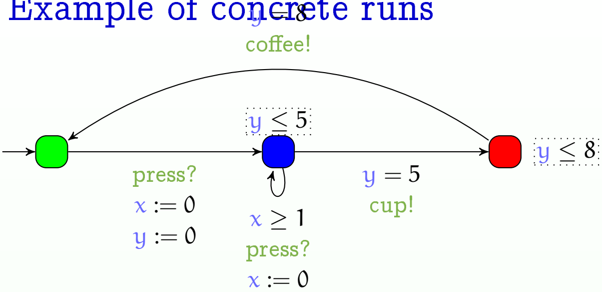

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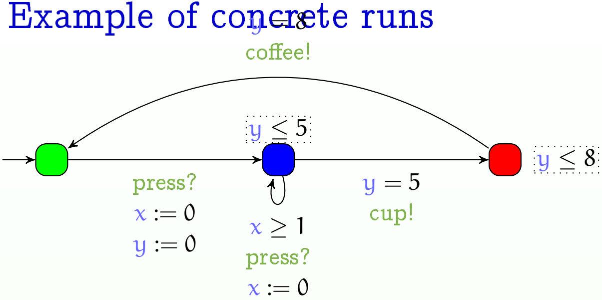

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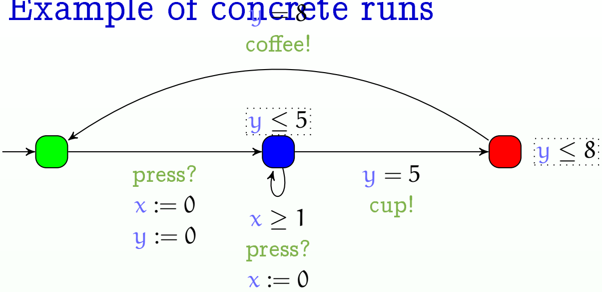

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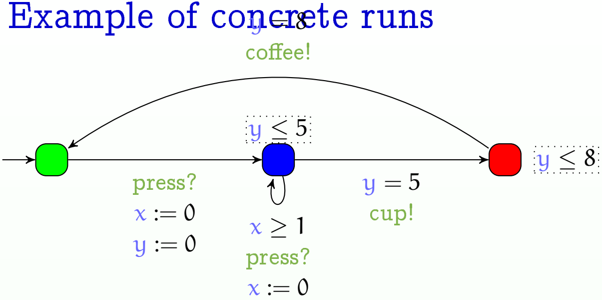

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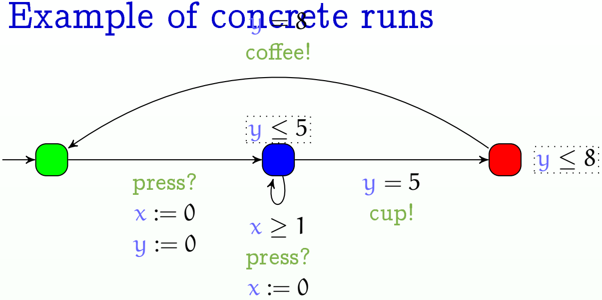

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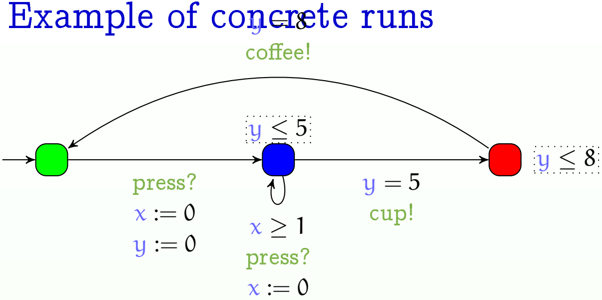

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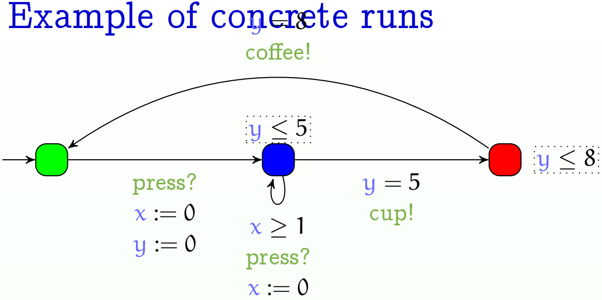

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Dense time

- Time is dense: transitions can be taken anytime
  - Infinite number of timed runs
  - Model checking needs a finite structure!
- Some runs are equivalent
  - Taking the press? action at $t = 1.5$ or $t = 1.57$ is equivalent w.r.t. the possible actions
- Idea: reason with abstractions
  - Region automaton [Alur and Dill, 1994], and zone automaton
  - Example: in location , all clock values in the following zone are equivalent

    $$y \leq 5 \land y - x \geq 4$$

  - This abstraction is finite

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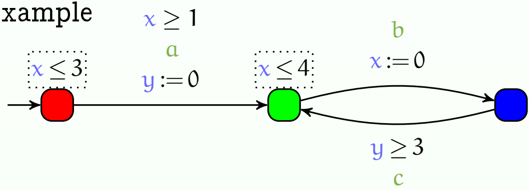

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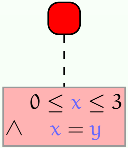

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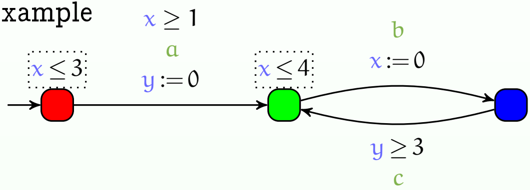

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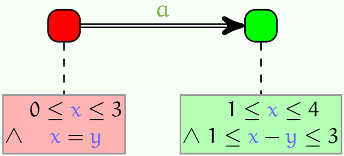

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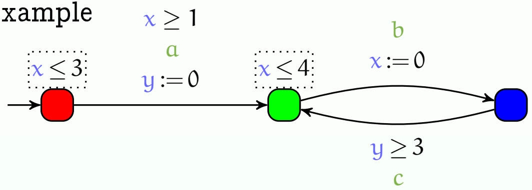

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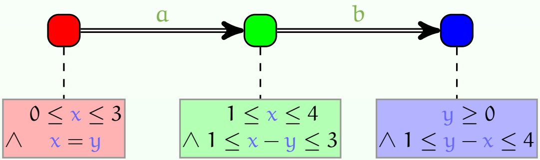

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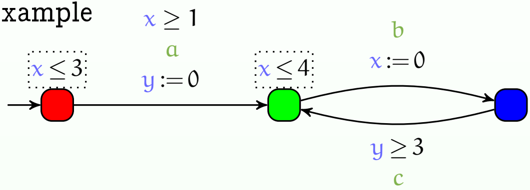

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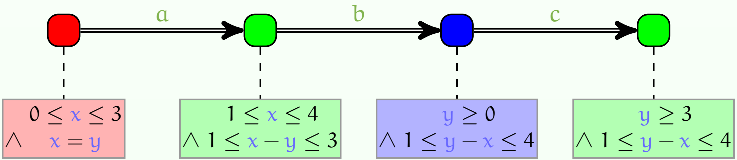

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However, one can:

- design semi-algorithms: if the algorithm halts, then its result is correct
- design algorithms yielding over- or under-approximations

Decision problems for timed automata

The finiteness of the region automaton allows us to check properties

- ☺ Reachability of a location (PSPACE-complete)
- ☺ Liveness (Büchi conditions)

Some problems impossible to check using the region automaton (but still decidable)

- ☺ non-Zenoness emptiness check

Some undecidable problems (and hence impossible to check in general)

- ☹ universality of the timed language
- ☹ timed language inclusion

Software supporting timed automata

Timed automata have been successfully used since the 1990s

Tools for modeling and verifying models specified using TA

- HyTech (also hybrid, parametric timed automata) [Henzinger et al., 1997]
- Kronos [Yovine, 1997]
- TReX (also parametric timed automata) [Annichini et al., 2001]
- Uppaal [Larsen et al., 1997]
- Roméo (parametric time Petri nets) [Lime et al., 2009]
- PAT (also other formalisms) [Sun et al., 2009]
- IMITATOR (also parametric timed automata) [André et al., 2012]

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TCTL (Timed CTL) [Alur et al., 1993]

TCTL expresses formulas on the order and the time between the future events for some or for all paths, using a set of atomic propositions AP

Quantifiers over paths:

$$\varphi ::= p \in AP \mid \neg p \mid \varphi \vee \varphi \mid \varphi \wedge \varphi \mid E\psi \mid A\psi$$

Quantifiers over states:

$$\psi ::= \varphi U_I \varphi$$

I is an interval of the form $[a, b]$, $[a, b)$, $(a, b]$, (a, b), $[a, \infty)$, or (a, ∞), where $a, b \in \mathbb{N}$

TCTL: Examples

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Remarks on timed automata

- Timed automata can be composed just as finite-state automata
- Symbolic states can be efficiently computed using Difference Bound Matrices (DBMs)
- *isReachable* can be applied to the abstract semantics of timed automata (the underlying finite transition system)
- Observers (both untimed and timed) can be used for timed automata

  The expressive power of observers for timed automata has been studied in [Aceto et al., 1998b, Aceto et al., 1998a]

Exercise: An observer for timed automata

Design an observer for the coffee machine verifying that it must never happen that the button can be pressed twice within a time strictly less than 1 unit of time.

Towards a parametrization. . .

- Challenge 1: systems incompletely specified
  - Some delays may not be known yet, or may change
- Challenge 2: Robustness [Markey, 2011]
  - What happens if 8 is implemented with 7.99?
  - Can I really get a coffee with 5 doses of sugar?
- Challenge 3: Optimization of timing constants
  - Up to which value of the delay between two actions press? can I still order a coffee with 3 doses of sugar?
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- A solution: Parametric analysis
  - Consider that timing constants are unknown (parameters)
  - Find good values for the parameters s.t. the system behaves well

Source et références

General References

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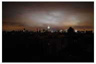

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