Philippe Flajolet
and
Analytic Combinatorics

Conference in the memory of Philippe Flajolet

Program

Paris - Jussieu, amphi 25
14-15-16 December 2011

Philippe Flajolet and Analytic Combinatorics

This conference pays homage to the man as well as the multi-faceted mathematician and computer-scientist. It also helps more people understand his rich and varied work, through talks aimed at a large audience.

The first afternoon is devoted to testimonies and official talks. The next two days are dedicated to scientific talks. These talks are intended to people who want to learn about Philippe Flajolet's work, and are given mostly by co-authors of Philippe. In 30 minutes, they give a pedagogical introduction to his work, identify his main ideas and contributions and possibly show their evolution.

Most of the talks form a basis for an introduction to the corresponding chapter in Philippe Flajolet's collected works, to be edited soon.

Frédérique Bassino, Mireille Bousquet-Mélou, Brigitte Chauvin, Julien Clément, Antoine Genitrini, Cyril Nicaud, Bruno Salvy, Robert Sedgewick, Michèle Soria, Wojciech Szpankowski and Brigitte Vallée.

Support: Virginie Collette and Chantal Girodon.

Wednesday, December 14 - Testimonies

13:30 - 14:00 Welcome.

14:00 - 15:30

- Inria: Welcome by Michel Cosnard.
- UPMC: Welcome by Serge Fdida.

- *Jean-Marc Steyaert*. Being 20 with Philippe.
- *Maurice Nivat*. Un savant modeste, Philippe Flajolet.
- *Jean Vuillemin*. Beginnings of Algo.

- *Bruno Salvy*. Life at Algo from 1988 on.
- *Laure Reinhart*. Philippe at the Helm of Rocquencourt Research Activities.
- *Gérard Huet*. Philippe and Linguistics.

- *Marie Albenque, Lucas Gérin, Éric Fusy, Carine Pivoteau, Vlady Ravelomanana*. Short Testimonies.

16:00-17:30

- *Michèle Soria*. Teaching with Philippe.
- *Brigitte Vallée*. Philippe and the French "MathInfo" Interface.
- *Brigitte Chauvin*. the Alea Group.

- *Robert Sedgewick*. Writing with Philippe.
- *Hsien-Kuei Hwang*. Overview of the Scientific Works.

- *Wojciech Szpankowski*. History of AofA.
- *Conrado Martínez*. Philippe Flajolet's Disciples around the World: the Barcelona Case.

- *Mark Ward*. Collected Papers.
- *Cyril Banderier*. Scientific Family Tree.
- *Brigitte Vallée*. Philippe's Library.

18:00-19:00 Official Event

- *Michel Cosnard* (Chairman and CEO of Inria).
- *Philippe Taquet* (Vice-president of the French Academy of Sciences).
- *Philippe Baptiste* (Scientific Director of the INS2I Institute of the CNRS).
- *Jacques Stern* (Adviser to the Minister of Higher Education and Research).

19:00 Cocktail, Caves Esclangon, near Tower 66, underground

Thursday, December 15 - Scientific Part 1

9:00-10:30

- *Robert Sedgewick* (Princeton University, USA). From Analysis of Algorithms to Analytic Combinatorics.
- *Hsien-Kuei Hwang* (Academia Sinica, Taiwan). The Ubiquitous Gaussian Limit Law in Analytic Combinatorics.
- *Xavier Viennot* (LaBRI, Bordeaux). Combinatorial Aspects of Continued Fractions and Applications.

11:00-12:30

- *Wojciech Szpankowski* (Purdue University, USA). Analytic Information Theory.
- *Brigitte Vallée* (GREYC, Caen). Philippe Flajolet and Dynamical Combinatorics.
- *Michael Drmota* (TU Wien, Austria). Relations to Number Theory in Philippe Flajolet's Work.

12:30-14:00 Buffet, Caves Esclangon, near Tower 66, underground.

14:00-16:00

- *Luc Devroye* (McGill University, Canada). Random trees in the Work of Philippe Flajolet.
- *Nicolas Broutin* (INRIA Rocquencourt). Heights of Trees.
- *Philippe Jacquet* (INRIA Rocquencourt). Flajolet's Works on Networking and Telecommunication Protocols.
- *Basile Morcrette* (INRIA Rocquencourt & LIP6, Paris) and *Nicolas Pouyanne* (LMV, Versailles). Balanced Pólya Urn Processes: the Analytic Approach.

16:30-18:30

- *Jean-Marc Steyaert* (LIX, Palaiseau). Term Rewriting Systems.
- *Bruno Salvy* (INRIA Rocquencourt). Automatic Analysis and Computer Algebra.
- *Paul Zimmermann* (LORIA, Nancy). Random Generation with Philippe Flajolet.
- *Michèle Soria* (LIP6, Paris). Boltzmann Sampling and Simulation.

20:00 Social dinner, Moulin Vert, 34 bis rue des Plantes, Paris 14 (Métro Alésia, line 4).

Friday, December 16 - Scientific Part 2

9:00-10:30

- *Marc Noy* (Universitat Politècnica de Catalunya, Spain). Planar Maps and Planar Configurations.
- *Mireille Bousquet-Mélou* (LaBRI, Bordeaux). Lattice Paths.
- *Conrado Martínez* (Universitat Politècnica de Catalunya, Spain). Search Trees.

11:00-12:30

- *Frédérique Bassino* (LIPN, Villetaneuse) and *Cyril Nicaud* (LIGM, Marne-la-Vallée). Inherent Ambiguity of Context-free Languages.
- *Jérémie Lumbroso* (LIP6, Paris). How Philippe Flipped Coins to Count Data.
- *Helmut Prodinger* (University of Stellenbosch, South Africa). The Register Function.

12:30-14:00 Buffet, Caves Esclangon, near Tower 66, underground.

14:00-15:30

- *Andrew Odlyzko* (University of Minnesota, USA). A Singular Mathematician and the Singularity Analysis of Generating Functions.
- *Cyril Banderier* (LIPN, Villetaneuse) and *Guy Louchard* (Université Libre de Bruxelles, Belgium). Philippe Flajolet's Contributions on the Airy Distributions.
- *Alfredo Viola* (Universitad de la República, Montevideo, Uruguay). What do we Learn from the Analysis of Hashing Algorithms?

16:00-18:00

- *Philippe Dumas* (INRIA Rocquencourt). Dr Flajolet's Elixir.
- *Mordecai Golin* (Hong Kong University of Science & Technology, Hong Kong). Divide & Conquer Recurrences and The Mellin-Perron Formula.
- *Pierre Nicodème* (LIPN, Villetaneuse). Motif Statistics in the Work of Philippe Flajolet.
- *Julien Clément* (GREYC, Caen) and *Mark Ward* (Purdue University, USA). The Digital Tree Process.

Abstracts

The citations in the following abstracts refer to the complete bibliography of Philippe Flajolet, which is listed pages 15-23.

Thursday, 9:00-10:30

Robert Sedgewick (Princeton University, USA). From Analysis of Algorithms to Analytic Combinatorics.

Analytic Combinatorics aims to enable precise quantitative predictions of the properties of large combinatorial structures. Primarily due to the efforts of Philippe Flajolet and his many research collaborators, the theory has emerged over recent decades as essential both for the scientific analysis of algorithms in computer science and for the study of scientific models in many other disciplines, including probability theory, statistical physics, computational biology and information theory. This talk surveys thirty years of joint work with Philippe that was inspired by learning the analysis of algorithms from Knuth and that culminated in the publication of two books: "An Introduction to the Analysis of Algorithms" [130, 131] and "Analytic Combinatorics" [201].

Hsien-Kuei Hwang (Academia Sinica, Taiwan). The Ubiquitous Gaussian Limit Law in Analytic Combinatorics.

Philippe gave a talk with exactly the same title in Poznan in August 1997. More than fourteen years later, his slides are still very inspiring and most ideas, frameworks, tools, and guidelines presented in his talk become clearer but still lead to many challenging research themes. In this talk, I will give a brief historical account of Gaussian limit law in Analytic Combinatorics, centering on Philippe's contribution, which will be further highlighted [45, 47, 88, 96, 107, 112, 117, 135, 142, 144, 149, 155, 159, 167, 168, 183, 186, 193, 198].

Xavier Viennot (LaBRI, Bordeaux). Combinatorial aspects of continued fractions and applications

In his 1980 seminal paper [22, 23], Philippe Flajolet stated a fundamental theorem interpreting any analytic continued fractions in terms of certain weighted lattice paths (the so-called Motzkin paths). This theory is equivalent to give an interpretation of the moments of any family of orthogonal polynomials in term of these weighted paths. Combining this general statement with some specific bijections between classical combinatorial objects and the so-called "histories" related to weighted Motzkin paths, Flajolet deduces several combinatorial proofs for many continued fraction expansions of well known power series. In particular the classical "Françon-Viennot bijection" between permutations and "Laguerre histories" play a key role and give rise to a combinatorial theory of the Sheffer class of orthogonal polynomials (*i.e.*, Hermite, Laguerre, Charlier, Meixner I and II).

Using Françon's concept of "data histories", a spectacular application of this combinatorial theory of continued fraction has been made by P. Flajolet (with J. Françon and J. Vuillemin) for the evaluation of the integrated cost of data structures subject to arbitrary sequences of insert, delete and search operations [18, 19, 25]. Each classical data structures is related to a classical family of Sheffer type polynomials [21, 30].

Extensions of the theory has been made by E. Roblet for Padé approximants and T-fractions. Further combinatorial developments have been made more recently by P. Flajolet with E. van Fossen Conrad [186] and R. Bacher [203] for continued fractions expansions of some elliptic functions. Some recent applications are also related to physics with some discrete integrable systems and positivity in cluster algebras involving continued fraction rearrangements (P. Di Francesco and R. Kedem) or the work of J. Bouttier and E. Guitter relating random planar maps and continued fractions. One of the last paper of Philippe (with P. Blasiak) connects continued fractions with the normal ordering of creation-annihilation operators in quantum physics [205].

Thursday, 11:00-12:30

Wojciech Szpankowski (Purdue University, USA). Analytic Information Theory.

Analytic information theory was born at INRIA in the 90's in a joint work with Philippe Flajolet and Philippe Jacquet. It deals with problems of information theory solved by complex-analytic methods known also as "flajolerie". In this talk, we survey recent results on the redundancy rate problem. In particular, we concentrate on the joint work with Philippe on the minimax redundancy for renewal processes [156, 173]. The redundancy rate problem for a class of sources consists in determining by how much the actual code length exceeds the optimal (ideal) code length. In a minimax scenario one finds the maximal redundancy over all sources within a certain class while in the average scenario one computes the average redundancy over all possible source sequences. The redundancy rate problem is typical of a situation where second-order asymptotics play a crucial role (since the leading term of the optimal code length is known to be the entropy of the source). This problem is an ideal candidate for analytic information theory. Here, the asymptotic expansion is derived in a typical "flajolerie" manner by applying a barrage of complex–analytic methods that includes generating function representations (of integer partition), Mellin transforms, singularity analysis, and saddle point estimates.

Brigitte Vallée (GREYC, Caen). Philippe Flajolet and Dynamical Combinatorics.

In the talk, I shall describe how dynamical systems arose into the domain of analytic combinatorics and analysis of algorithms, how they created a new point of view and gave rise to new results.

Period I: The beginning. [1990–1995] The story began in 1990, with our first attempt to analyse the Gauss algorithm that finds the shortest vector of a two dimensional Euclidean lattice. We proved in [90] that the execution of the algorithm creates a continued fraction, as the Euclid algorithm in the one dimensional–case. This led us to a precise expression of the mean number of steps of the algorithm. However, the distribution of the number of steps remained unknown. First, Daudé's PhD thesis [Da] showed that both analyses [Euclid and Gauss] involve the same fundamental objects, the so–called continuants. Second, Hensley's paper [He] and Mayer's chapter [Ma] in the "green book" [Gb] convinced us that the Ruelle-Mayer transfer operator (related to the dynamical system underlying the continued fraction algorithm) is a convenient tool for generating continuants. This led us to the precise analysis of the Gauss algorithm: we showed in [114, 132] that the number of steps asymptotically follows a geometric law, with a ratio which involves the dominant eigenvalue of the Ruelle-Mayer operator.

Period II: The developments. [1995-1998] During the previous work, we had discovered the power of the "dynamical approach", at least for continuous models of Euclidean type [in one or two dimensions]. Then, later, we understood two main facts:

(i) This approach happens to be fruitful to analyze discrete models (namely, here, the Euclid algorithm), via the use of generating functions (of Dirichlet type) which operate, as usual, a transfer between the discrete model and the continuous model. In this case, these generating functions are themselves "generated" by the Ruelle-Mayer operator. In [144], using these ideas, we obtained a very natural average-case analysis of the Euclid algorithm. Then, in [209], we (mostly Philippe...) performed precise computations of the spectrum of this operator (by approximating it with a family of finite matrices) and obtained a record on the number of digits for the Gauss-Kusmin constant.

(ii) The Ruelle-Mayer operator generates the fundamental intervals (a fundamental interval gathers all the reals whose continued fraction expansion begins with a prescribed finite sequence), and then the lengths of such intervals, i.e., the fundamental probabilities. This remark provided us a unified point of view on continued fraction expansions of real numbers, which led in [144] to various results in a very natural way.

Period III: The foundation of dynamical combinatorics. [1998–2006] We have further extended these two main ideas. We began together, and I often continued without Philippe, with other collaborators. Most of my works during the years [2000–2010] related to the framework of dynamical combinatorics have strongly benefited from regular discussions with Philippe, as I explained it in [Va3].

The direction (i) gave rise to a complete explanation of (almost) all facts related to any algorithm of Euclid type [Va1, BaVa]. Bivariate generating functions are needed, as usual, for distributional analyses, and they are generated by bivariate transfer operators, which "mark" some cost of interest.

The continued fraction expansion is just a particular case of a numeration system. This convinced me that the ideas of (ii) can be extended to any dynamical system. This led to the concept of dynamical sources [Va2]. Then, together with Julien Clément, in [140, 161], Philippe and I performed the analysis of *general* tries under the *general* model of dynamical sources. The Dirichlet series $\Lambda(s)$ of the source plays a central role in the analysis, and it is generated by a (generalized) transfer operator of the dynamical system. In [157], we returned to the particular case of the continued fraction source and related the behaviour of the mean trie-path-length to the Riemann hypothesis...

Period IV: Towards a more realistic analysis of algorithms? [2008–2010] The paper [202] –a joint work with Philippe and I, together with Jim Fill and Julien Clément– revisited classical sorting and searching algorithms when the keys are viewed as words, which are compared via their symbols. We understood that most of the analyses performed on a dynamical source could be extended to a general source (not even dynamical) via the study of $\Lambda(s)$. For a general source, even if there is no "explicit" expression of $\Lambda(s)$, the so–called "tameness" conditions on $\Lambda(s)$ may be precisely described. This made possible a fine probabilistic analysis of the main data structures built on the source words. In a joint paper with Mathieu Roux, we also revisited the tameness conditions in the case of a memoryless source, and obtained the optimal tameness region, that is characterized via diophantine properties of the probabilities [208].

Philippe and I were convinced that this double realistic point of view (a realistic comparison between words produced by a realistic general source) would be the beginning of a new joint story. We planed to revisit most of classical algorithms (sorting and searching) with this point of view, and we had began to study digital search trees, during December 2010...

Bibliography.

[BaVa] V. Baladi and B. Vallée, *Euclidean Algorithms are Gaussian*, Journal of Number Theory, Volume 110, Issue 2 (2005) pp 331–386.

[Da] H. Daudé, *Des fractions continues à la réduction des réseaux : analyse en moyenne*, PhD thesis of the University of Caen, 1993.

[Gb] T. Bedford, M. Keane, and C. Series, *Ergodic Theory, Symbolic Dynamics and Hyperbolic Spaces*, Oxford University Press (1991).

[He] D. Hensley, *The number of steps in the Euclidean algorithm*, Journal of Number Theory, 49 (2), (1994) pp 149–182.

[Ma] D.H. Mayer, *Continued fractions and related transformations*, pp 175–222, in [Gb].

[Va1] B. Vallée, *Euclidean Dynamics*, Discrete and Continuous Dynamical Systems, 15 (1) May 2006, pp 281–352.

[Va2] B. Vallée, *Dynamical sources in Information Theory: Fundamental Intervals and Word prefixes*, Algorithmica (2001), vol 29 (1/2) pp 262–306.

[Va3] B. Vallée, *Vingt-cinq ans de compagnonnage scientifique avec Philippe Flajolet*, Gazette des Mathematiciens, ISSN 0224-8999 (129) 2011, pages 118–120.

Michael Drmota (TU Wien, Austria). Relations to Number Theory in Philippe Flajolet's Work.

The work of Philippe Flajolet has astonishingly many connections to number theory, mostly to analytic number theory. First of all the Riemann zeta-function (its values and its analytic properties) appears in several of his papers [116, 120, 125, 143, 157, 197, 199, 207], also in relation to so-called digital sums.

Another major topic that is related to number theory are (random) polynomials over finite fields, where he and his co-authors studied (among other things) analogues to the celebrated Erdős-Kac theorem [88, 127, 145, 163].

Moreover he "borrowed" in his work several techniques from number theory such as continued fraction expansions or elliptic function that he applied in unexpected frameworks like in the enumeration of permutations or in the analysis of urn models [22, 32, 77, 87, 184, 186, 203].

Thursday, 14:00-16:00

Luc Devroye (McGill University, Canada). Random Trees in the Work of Philippe Flajolet.

Recursively defined trees are the perfect target for the lumberjacks of analytic combinatorics. So we will describe the trees that were axed by one of the main bush men of that field. We will also discuss what to do with the trees that were left standing upon his retirement (covers paper [206, 195, 208, 198, 187, 161, 147, 140, 135, 122, 117, 106, 96, 103, 101, 72, 66, 61, 53, 33]).

Nicolas Broutin (INRIA Rocquencourt). Heights of Trees.

Trees sampled uniformly at random are some of the most important random discrete structures: they appear for instance naturally in computer science (hashing algorithms, models for networks), biology (evolution), theoretical physics (percolation, quantum gravity, particle systems with coagulation/fragmentation). The understanding uniform trees thus underlies the understanding of a number of important models.

We will review the results of Philippe and his coauthors [45, 104, 195, 206] on the subject of heights of such random trees, which started with the pionneering Flajolet-Odlyzko paper [26, 33]. We will put the theorems and methods in their historical context, and emphasize the major influence this branch of Philippe's work had on the research that followed. We will in particular discuss the question of the universality of the asymptotic behaviour of large random trees, and sketch the main ideas of the proof.

Philippe Jacquet (INRIA Rocquencourt). Flajolet's Works on Networking and Telecommunication Protocols.

Philippe Flajolet has produced many results that are pertinent to telecom technologies. For example the approximate counting algorithms are a useful tool for the detection of a cyber-attack on servers. But in the eighties, Philippe has produced many results that were specific to telecommunications. In particular he has investigated the performance of collision resolution algorithms [54]. The tree (or stack) algorithms have analysis that are classically close to tries analysis, but with some interesting differencing details [49, 55, 65, 68]. This work has given the first to date complete performance analysis of the packet delays in a communication protocol under Poisson arrival in 1985.

Basile Morcrette (INRIA Rocquencourt & LIP6, Paris) and Nicolas Pouyanne (LMV, Versailles). Balanced Pólya Urn Processes: the Analytic Approach.

A Pólya urn random process is defined by one urn containing balls of finitely many colors and an invariable replacement rule. Originally introduced by Laplace and Jacob Bernoulli, these models had mostly been studied by probabilistic methods before the works by Philippe Flajolet and its co-authors [183, 188, 196]. The main tool of their analytic approach consists in considering the multivariate exponential generating function of histories of the urn's composition. These functions satisfy multiplicative properties so that they can be expressed by means of solutions of a linear PDE or of a monomial differential system. When it is possible, solving the associated differential equations provides an explicit parametrization of the generating functions. This is done for families of urns and for famous specific examples as well. Probabilistic consequences are drawn from the explicit parametrizations, like limit distributions, large deviations or local limits results.

Thursday, 16:30-18:30

Jean-Marc Steyaert (LIX, Palaiseau). Term Rewriting Systems.

Philippe devoted an intense activity to the combinatorics of trees so present in computer science under so many models: term algebra, search trees, tries, etc. The first family he considered was that of term trees, binary or general rooted plane trees, aiming at evaluating the (average) cost of finding given patterns or families of patterns ([15, 28, 43]). This constitutes the first step in analytic combinatorics. Since most algorithms on trees should be expressed recursively, the next step has been to develop a formal framework for the performance analysis of this natural and ubiquitous family of programs. This framework consists of a syntactic part to translate the code into equations on generating series, then of an analytic part to capture the asymptotics of the running costs: this complexity calculus ([31, 67]) is at the origin of the Luo computer algebra system for combinatorics and analysis of algorithms. In the same flavour is the study on the common subexpression problem, which gives precise information about tree compaction that can be achieved by sharing subtrees: Philippe was especially fond of this result ([89]).

Bruno Salvy (INRIA Rocquencourt). Automatic Analysis and Computer Algebra.
Philippe Flajolet was very much interested in computer algebra. His use of complex analysis was often targeted towards a symbolic computation of the quantities he was after. This is evident in singularity analysis, but also in many of his uses of the Mellin transform or other integral representations. Thus he made heavy use of Maple both in his research on algorithms and as a tool to solve nice problems [109, 136]. But his main relation to computer algebra was his desire to automate Analytic Combinatorics. This resulted in a number of works, first on a general plan of attack [31, 35, 67], then around the implementation of the Luo system [79, 80, 94] and later of more versatile libraries for combinatorial structures [123]. More recently, he was very much interested in the so-called D-finite functions where he used—not suprisingly—complex analysis to answer classification questions [184, 207].

Paul Zimmermann (LORIA, Nancy). Random Generation with Philippe Flajolet.
I had the great chance to do my PhD thesis under the supervision of Philippe. He was a great PhD advisor and colleague. He spent a lot of time with us (while smoking or searching for his Dunhill cigarettes in the pockets of his jacket), hearing about our last “findings”, being enthusiastic but not too much if he knew some idea would lead to a dead end, suggesting new directions, making sure not to spoil our work. In this talk I will focus on the work [113, 119] we did together with Bernard Van Cutsem on what is now called the “recursive method” for random generation of combinatorial structures. I will recall the contributions of Philippe to the “random generation calculus” and the “cost algebra”, his discovery of the “boustrophedon” method. I will explain how that work had a major influence on my research. I will also speak about the companion paper on the unlabelled case, which we wrote but never published, either because Philippe was not satisfied about the originality or soundness of that paper, or because he was too busy with other exciting work. I will never know.

Michèle Soria (LIP6, Paris). Boltzmann Sampling and Simulation.
Boltzmann model of random generation is deeply rooted in Analytic Combinatorics: given a suited parameter x, it provides a systematic translation of combinatorial specifications into simple and efficient sampling algorithms, which rely on the evaluation of the generating functions at x, and the simulation of a few discrete distributions. In the Boltzmann model, the complexity, in terms of real-arithmetic operations, is linear in the (fluctuating) size of the output random object. Moreover, the control parameter x can be tuned in order to maximize the probability of attaining objects near a target size n, and samplers with rejection (when the size of the produced object is not satisfactory) are considered: a precise average-case analysis based on the nature of the singularities, shows that for most specifiable combinatorial families, linear complexity still holds for Boltzmann samplers with rejection.

In the real-arithmetic Boltzmann model the required discrete distributions (Bernoulli, Geometric, Poisson, Logarithmic-series) are simulated from a generator which produces random numbers uniformly distributed over the real interval $(0, 1)$. Since the objects ultimately produced are discrete, it is natural to try and produce them by purely discrete means, and thus design discrete Boltzmann samplers which are solely based on binary coin flips.

The idea of simulating various distributions from a discrete source of unbiased coin flips, goes back to Knuth and Yao (1976). In this binary model, the complexity is measured in terms of the number of bits needed for the generation: for example, it is possible [60] to produce k bits of an exponentially distributed random variable by using and average of $k + 5.679$ coin flippings. The question of perfect simulation of discrete random variables was investigated in the study of Buffon machines: Bernouilli, geometric, Poisson and logarithmic-series distributions can be generated by algorithms that require a very few number of flips, on average.

This talk refers to Philippe *et* al. papers over a period of almost 30 years:

- the analysis of the perfect simulation of an exponentially distributed variate [42, 60],
- the founding articles on Boltzmann sampling [170, 179, 194],
- and the recent work on Buffon machines [200].

Friday, 9:00-10:30

Marc Noy (Universitat Politècnica de Catalunya, Spain). Planar Maps and Planar Configurations.

We discuss the contributions of Philippe Flajolet on planar maps and planar non-crossing configurations [85, 149, 160], specially those in the papers Random maps, coalescing saddles, singularity analysis and Airy phenomena [160] and Analytic combinatorics of non-crossing configurations [149]. In both cases we emphasize the attributes characteristic of Philippe's work: depth in the analysis and breadth in the applications.

Mireille Bousquet-Mélou (LaBRI, Bordeaux). Lattice Paths.

Lattice paths are ubiquitous in combinatorics, and hence in Philippe's work. We focus here on a few papers that explicitly deal with the enumeration and asymptotic properties of 1D or 2D lattice paths. This includes a uniform treatment of 1D excursions with arbitrary steps, via the kernel method and singularity analysis [168], as well as more sporadic results, often motivated by algorithmic applications (the maximum of a random walk is related to a packing problem, a random walk in a triangle is related to a storage allocation scheme [56]). We shall finish with a very recent work on a class of 2D self-avoiding polygons, called prudent polygons [204].

Conrado Martínez (Universitat Politècnica de Catalunya, Spain). Search Trees.

Search trees play a fundamental rôle in most, if not all, areas of Computer Science. Unsurprisingly, such prominent data structures couldn't escape the attention and intellectual curiosity of Philippe Flajolet. Broadly speaking, search trees can be classified as either data-driven (e.g., binary search trees) or space-driven (e.g., tries). In my talk I will only survey his work on the data-driven family.

In the first part of the talk, I will review Flajolet's contributions to the analysis of binary search trees (BSTs, for short) and its close relatives. In particular, I will review his early results on efficient storage of BSTs in bubble memories [51], the unified framework for the analysis of various quantities in increasing trees [96], and his results on patterns in random BSTs [135].

The second part of the talk will be devoted to the many contributions of Philippe Flajolet to the analysis of multidimensional search trees, in particular, k-d trees, quadtrees and multiattribute trees, starting with his seminal paper of 1986 in J. ACM with Claude Puech [58] (a preliminary version of which appeared as [41]), where they showed that the expected cost of partial matches in random k-d trees was $\Theta(n^{1-s/k+\theta(s/k)})$, with $\theta(x) > 0$, a surprising result contradicting the commonly conjectured expected cost $\Theta(n^{1-s/k})$. Besides this fundamental result, I'll also describe the results in [93, 103, 106, 117, 122] on quadtrees (with "Analytic Variations on Quadtrees" [106] as a notable milestone in this series), the results in [81, 82] on multiattribute trees, and some further results on k-d trees [75].

Friday, 11:00-12:30

Frédérique Bassino (LIPN, Villetaneuse) and Cyril Nicaud (LIGM, Marne-la-Vallée). Inherent Ambiguity of Context-free Languages.

A context-free language is inherently ambiguous when no unambiguous grammar can describe it. In this talk we will focus on three articles where Philippe Flajolet adressed the problem of determining whether a given context-free language is inherently ambiguous.

The general problem is undecidable, but by analyzing the properties of the counting generating function associated with a language, Philippe Flajolet gave [46, 62, 64] some sufficient conditions to ensure its inherent ambiguity. He started with the observation that an unambiguous context-free language has an algebraic generating function (this is a consequence of a classical theorem of Chomsky and Schutzenberger). A language that has a transcendental generating function can therefore not be unambiguous. By giving various criteria for establishing the transcendence of generating functions, he then proved the inherent ambiguity of various classical context-free languages.

Jérémie Lumbroso (LIP6, Paris). How Philippe Flipped Coins to Count Data.
With his overwhelming presence in the analytical and mathematical fields, it is easy to discount Philippe Flajolet's work in so-called "Probabilistic Counting" algorithms as a passing interest. In truth, Philippe is considered one of the grandfathers of streaming algorithms. His work in this domain is spread over more than two decades. And from shining light on Morris' powerful but then completely unknown "Approximate Counting", to introducing a controversial way of using hash function, it provides yet another demonstration of his sharp, visionary insight. His last paper on the topic, "HyperLogLog", reads as a beautiful and elegant testament to much of his legacy in Analysis of Algorithms. This lecture will do its best to give a glimpse of the key algorithmic ideas and relationships involved in the journey... (Covers papers [38, 48, 40, 50, 84, 134, 176, 193] and survey [180].)

Helmut Prodinger (University of Stellenbosch, South Africa). The Register Function.
Together with Raoult and Vuillemin, Flajolet analysed the average number of registers to evaluate a random binary tree optimally [13, 20]. That was his first research project in the area that made him famous, namely Analysis of Algorithms. Independently, Rainer Kemp solved this problem as well [190].

In the talk, the different approaches towards the analysis of the register function will be described and compared, also with respect to a later paper [57] (jointly with me) about the register function of unary-binary trees: The Flajolet-Raoult-Vuillemin approach uses an interesting result of H. Delange about the summatory function of the (binary) sum-of-digits function. Kemp used Mellin transforms (although using the nickname Gamma function method, which was used at the time by Knuth), and the later Flajolet-Prodinger paper uses a combination of Mellin transforms and singularity analysis [116].

Flajolet extended Delange's analysis from the binary number system to the GRAY code representation[27], in order to mimic his first analysis related to another problem, solved earlier by Sedgewick (odd-even merge). Later, he extended his analysis on digital sums vastly, by bringing in the Mellin-Perron summation formula. This is extremely interesting material, but would require another talk to cover it.

Friday, 14:00-15:30

Andrew Odlyzko (University of Minnesota, USA). A Singular Mathematician and the Singularity Analysis of Generating Functions.
Philippe Flajolet made numerous contributions to the evaluation of asymptotics of algorithms and combinatorial structures. Many were accomplished using singularity analysis of generating functions [26, 33, 45, 86, 117, 148, 150, 182]. His work included significant breakthroughs on some specific problems, as well as a pioneering venture to develop a general framework for automatic or at least semi-automatic translation from the singularities of analytic generating functions to the asymptotics of the coefficients of such functions.

Cyril Banderier (LIPN, Villetaneuse) and Guy Louchard (Université Libre de Bruxelles, Belgium). Philippe Flajolet's Contributions on the Airy Distributions.
This talk will first give a glimpse on 2 different kinds of limit laws involving the Airy function, that Flajolet showed to play a rôle in many combinatorial contexts: "The first cycles in an evolving graph" [78], "On the analysis of linear probing hashing" [142], "Random maps, coalescing saddles,

singularity analysis, and Airy phenomena" [152, 160], "Hasching, trees, paths, and graphs" [175], "Airy phenomena and analytic combinatorics of connected graphs" [175].

In the second part of the talk, we present with more details some "Analytic variations on the Airy distribution" [165] (related to area below the Brownian excursion), and its moments of fractional and negative order, for which the Mellin transform (another big love of Philippe!) is a key tool.

Alfredo Viola (Universitad de la República, Montevideo, Uruguay). What do we Learn from the Analysis of Hashing Algorithms?

The idea of hashing seems to have been originated by Luhn, in an internal IBM memorandum in January 1953. The first major paper published in the area is the classic article by Peterson in 1956, where he defines open addressing in general and gives empirical statistics about linear probing hashing. He also notices the degradation in performance when lazy deletions are presented. Nevertheless, as noted by Knuth, the word "hashing" to identifiy this technique appears for the first time in the literature in the survey of Morris in 1968, although it had been in common usage for several years.

On the other side one of the first mathematical challenges in the early 60's of the nascent Computer Science was the design of models to understand and predict the practical behaviour of access methods to data. These methods had shown to have very good empirical complexity. In 1962 Knuth presents a solution for Linear Probing Hashing, and this milestone is considered to be the the first algorithm ever analyzed as well as the origin of the Analysis of Algorithms.

Philippe Flajolet was fascinated with the mathematical analysis of hashing algorithms. Besides this historical reason, the mathematical properties behind the analysis of these algorithms have seemed emerged from a box full of surprises!

His first work in 1983 with Steyaert, analyzes the performance and evaluation of extendible hashing [36, 37]. This problem has been shown to be closely related with branching processes in trie searching and polynomial factorization. In 1992 [73, 100], with Gardy and Thimonier, Flajolet presents deep conections between hashing and random allocation problems. More specifically the analysis of generalizations of the birthday paradox, coupon collectors, caching algorithms and self-organizing search are performed for very general probabilistic settings. Moreover, in 2000 [159] with Mahmoud, Jacquet and Régnier, he has used tools already presented in previous analysis of hashing algorithms to study analytic variations on bucket selection and sorting.

Nevertheless, Flajolet was specially interested in the analysis of linear probing hashing. The combinatorial properties of several problems related with linear probing are extremely rich. In 1995, Philippe [121] together with Grabner, Kirschenhofer, and Prodinger he presents an analysis of Ramanujan's Q-function. This function appears in Knuth's first analysis in 1962, as well as in several important combinatorial problems. As an historical note, this paper was dedicated to D. E. Knuth on occasion of the 30th anniversary of his first analysis of an algorithm. To conclude this succint survey, in 1998 [142] with Poblete and Viola presents moment analyses and characterizations of limit distributions for the construction cost of hash tables under the linear probing strategy. For full tables, the construction cost has a limit law of the Airy type. Moreover, combinatorial relations with other problems leading to Airy phenomena (like graph connectivity, tree inversions, tree path length, or area under excursions) are also briefly discussed. All these, and other connections, are clearly presented in a wonderful survey together with Chassaing in 2003 [175] oriented to french students.

One of the main concerns of Philippe Flajolet, has been the developement of methodological tools for the analysis of algorithms. Most of his papers, besides the mathematical analysis of the problems at hand, are textbook examples that illustrate the use of general methods of analysis. Even more, we find sometimes final discussions about different potential methods to use. Every time we read again each of his papers (even if we are coauthors!), there is always room for learning something new, or better understand some important idea. In this talk I invite you to discover together part of the light that Philippe Flajolet gives us behind his deep and beautiful contributions presented in his analysis of hashing algorithms. The rest is up to each of us to discover. Philippe Flajolet's work will always be a source of inspiration and light in our everyday research.

Friday, 16:00-18:00

Philippe Dumas (INRIA Rocquencourt). Dr Flajolet's Elixir.

The Mellin transform, akin to Laplace transform or Fourier transform, originates from analytic number theory. Philippe Flajolet has popularized its use in the average case analysis of algorithms through numerous examples like register allocation, sorting, digital trees, divide-and-conquer strategy, cardinal estimate. Moreover he has developped mathematical tools to deal with sums which occur in that domains of study. The basic and salient property of the Mellin transform which explains its use is the strong link between the asymptotic behaviour of the original and the singularities of the image. We will elucidate this correspondance (in a very formal style, à la Flajolet) through examples excerpted from his works [41, 58, 49, 55, 52, 57, 61, 101, 120, 124, 125, 148, 193, 200, 204].

Mordecai Golin (Hong Kong University of Science & Technology, Hong Kong). Divide & Conquer Recurrences and The Mellin-Perron Formula.

Most computer scientists know of the "Master Theorem" for Divide & Conquer Recurrences. It provides a quick way of deriving first order asymptotics.

Very often, the solutions to Divide & Conquer recurrences contain periodic terms; sometimes as coefficients of first order asymptotics, sometimes as coefficients of lower order terms. Standard tools for analyzing these recurrences, such as the Master Theorem, often miss these periodic terms.

In this talk we describe a general technique developed and popularized by Philippe Flajolet for solving these and related functions [105, 115, 116, 199]. This technique easily derives the periodic terms. It is based on the Mellin-Perron formula, one of the galaxy of methods related to Mellin transform analysis. As in many Mellin transform analysis based techniques, the final step of the method transforms the problem into the calculation of the singularities of appropriate functions.

Pierre Nicodème (LIPN, Villetaneuse). Motif Statistics in the Work of Philippe Flajolet.

Philippe Flajolet considered different forms of Motif Statistics. His [74, 76] articles with P. Kirschenhofer and R.F. Tichy study the *simultaneous* counts of *all* words of length k in texts of length n when n tends to infinity and k is almost $\log(n)$. In the [151, 174] articles with P. Nicodème and B. Salvy, he counts the number of *matching positions* of any regular expression in random texts. Next, in the [164, 191] articles with Y. Guivarc'h, W. Szpankowski and B. Vallée, he counts in a random text the number of *occurrences* of a *hidden* word, where an occurrence is a subsequence of the text matching the word (there is an overlap with [151, 174] in the case of fully constrained motifs). The *sequences discrepancy* articles study the "normality" of strings generated under a Bernoulli uniform model. In the [74] article is proved that, asymptotically, almost all binary strings of length n contains all patterns of length $(1-\epsilon)\log_2(n)$ a close to uniform number of times. The [76] article proves weaker results, but for any finite alphabet. The [151, 174] articles counting the matches of regular expressions use an automaton construction while the [164, 191] number of hidden words is computed by language decomposition. The methods used are numerous: de Bruijn graphs [74, 76, 164, 191], automata theory [151, 174]; combinatorics of language [74, 164, 191] ; generating functions and Cauchy integrals in all articles with the exception of [76], which is purely probabilistic; Perron-Frobenius theory [151, 174, 164, 191]; Hwang's quasi-power theorem [151, 174, 164, 191] ; convergence of moments, graph theory and dynamical programming [164, 191].

Julien Clément (GREYC, Caen) and Mark Ward (Purdue University, USA). The Digital Tree Process.

The analysis of trees was integrated in Philippe Flajolet's writings and research throughout his life. In particular, he had a very keen interest in the *digital tree process* [34, 187]. The most pervasive kind of digital tree is the re*trie*val tree, named by Fredkin in 1960, and usually shortened to "trie".

Due to their generality, tries are one of the most widely-known and greatly-studied data structures for representing a set of words. The structure of tries is recursive. A partitioning of the data items—often using a sorting or classification by types—takes place at the root node. The tree is built recursively, according to subsequent bits or digits of the data. The children of the root are sorted further into subtrees and are thus partitioned more finely. The data items (also known as keys) eventually require no more sorting and are ultimately stored in leaves of the trie.

The applications of the digital tree process are abundant and are found practically anywhere that data is classified or sorted. The abstract data structure has given rise to many algorithmic variants, including PATRICIA tries, digital search trees, ternary search trees, LC-tries, etc. Some of these variants have been precisely analysed by Philippe Flajolet [61, 140, 161]. Moreover, the structure of a trie can be used to model or analyze the behavior of both deterministic and also stochastic algorithms in computer science. Tries are especially relevant to branching and sorting processes. So it is not surprising that the digital tree process has ramifications in the management of large databases (dynamic hashing [50], probabilistic counting [37]), in communication protocols [65] (for instance for leader election), data compression (Lempel-Ziv, suffix trees), but also, rather unexpectedly, in computational geometry (for exact comparison of rationals [157]). The digital tree process is elegant and simple. In its algorithmic form, it is intuitive to implement and utilize. What was certainly appealing to Philippe Flajolet is that the analysis of tries leads to challenging mathematical problems. Using tries, he made many fruitful connections between domains such as polynomial factorization [36], algebraic methods [53], differential equations [101], and random number generation [42].

Philippe Flajolet was especially concerned with the average case analysis of many parameters and properties of digital trees. Although these structures are very efficient on average (and, indeed, can compete with the best known data structures in many applications), the worst case complexity is infinite! This helps explain why the algorithmic, analytic and probabilistic aspects of digital trees are fundamental in both theoretical and applied domains in computer science. Philippe had a lifelong interest in their average case analysis. He sharpened and generalized several analytic tools by working with digital trees. Amongst these many techniques used by Philippe and his co-authors to analyze digital trees and their variants [39, 53, 101, 120], we emphasize his pioneering work to systematically apply generating functions, the symbolic method, singularity analysis, the saddle point method, the Mellin transform, and Poissonization. In the later stages of his career, he paid particular attention to developing and analyzing general probabilistic frameworks and tools [161, 202, 208], characterizing the stochastic generation of words and strings which are inserted in digital trees. The transfer operators of dynamical systems theory are a key example of these stochastic applications.

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