

DERIVING LTS

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(joint work with Julian Rathke)

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PLAN

1. Introduction

2. Full asynchrony

3. Asynchrony

4. Synchrony

BACKGROUND

- Process calculi in the CCS/Pi tradition come with two semantics
 - **reduction semantics**
 - “closed” - how the program evolves
 - easy to define
 - contextual preorders/equivalences: reduction precongruence, reduction congruence, barbed congruence
 - **labelled semantics**
 - “open” - how the program interacts
 - harder to define and justify
 - simulation, bisimulation
- Basic underlying issues
 - soundness: eg. is bisimilarity included in contextual equivalence?
 - completeness: eg. is contextual equivalence included in bisimilarity?

REDUCTION SEMANTICS

- Structural congruence $(P \parallel Q) \parallel R \equiv P \parallel (Q \parallel R)$

$$P \parallel Q \equiv Q \parallel P$$

$$P \parallel 0 \equiv P$$

- Think of a process as a “chemical soup”

- Reduction TS is usually defined with

- a number of parametric rules, eg $a!P \parallel a?Q \rightarrow P \parallel Q$

- a set of “reactive” contexts

$$\frac{P \rightarrow P'}{P \parallel Q \rightarrow P' \parallel Q}$$

- closed under structural congruence $\frac{Q \equiv P \quad P \rightarrow P' \quad P' \equiv Q'}{Q \rightarrow Q'}$

RELATIVE PUSHOUTS (RPO)

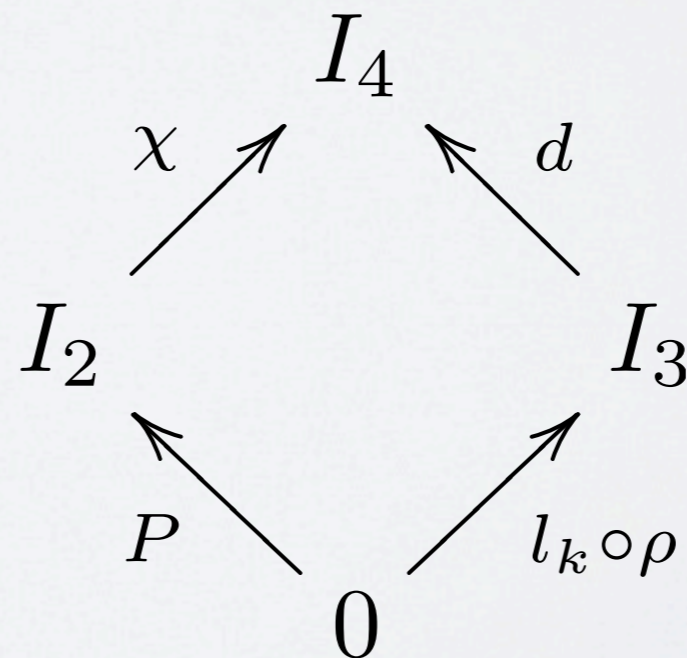
(J. LEIFER R. MILNER, DERIVING BISIMULATION CONGRUENCES FOR REACTIVE SYSTEMS, CONCUR '00)
(P. SEWELL, FROM REWRITE RULES TO BISIMULATION CONGRUENCES, CONCUR '98)

- Passing from
 - “internal” reduction semantics (what processes do) to
 - “external” labelled semantics (how processes interact)

$\chi[P] \rightarrow P'$ & χ is the smallest such context



$$P \xrightarrow{\chi} P'$$



WHERE RPOS GO WRONG

- The derivation process is global
 - no compositional, inductive presentation (SOS)
 - joint work with Julian Rathke on how to derive SOS
- Often give the wrong equivalences
 - eg. restricting to asynchronous subcalculus still gives synchronous lts
 - problem and solution illustrated in this talk

SOS LABELLED SEMANTICS

- Semantics of a term completely determined by semantics of its subterms
- No structural congruence rule, this is real syntax
- Our rules are always SOS
- a set of (positive) SOS rules defines a monotonic function on relations, let Φ be the lfp

$$\frac{P \xrightarrow{a} P'}{P \parallel Q \xrightarrow{a} P' \parallel Q}$$

$$\Phi : \mathcal{P}(P \times L \times P) \rightarrow \mathcal{P}(P \times L \times P)$$

- **the** LTS defined by a set of rules is $\mathcal{C} \stackrel{\text{def}}{=} \Phi(\emptyset)$

CONTEXTUAL EQUIVALENCE

(K. Honda, N. Yoshida, On reduction-based process semantics, TCS 152(2):436-486, 1995)

- Suppose that reductions cause “changes in heat”
- Observer can
 - introduce new ingredients
 - measure changes in heat
- Reduction precongruence
 - largest precongruence \lesssim that satisfies $P \lesssim Q$ & $P \rightarrow P'$ implies there exists Q' with $Q \rightarrow Q'$ and $P' \lesssim Q'$
- Reduction congruence - symmetric version

LTS AND OBSERVABILITY

- What is the meaning of a labelled transition in an LTS?
 - indication of a possible interaction
- A labelled transition is observable if there exists a contextual characterisation of the label
 - ie $P \xrightarrow{\alpha} P'$ iff there exists context χ_{α} s.t. ...
 - the ... should be preserved by contextual equivalence

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FULL ASYNCHRONY

“SOUP OF INTERACTING MOLECULES”

Syntax

$$P ::= 0 \mid a! \mid a? \mid P \parallel Q \mid \tau$$

Reduction semantics

$$\tau \rightarrow 0$$

$$a! \parallel a? \rightarrow 0$$

Structural congruence

$$(P \parallel Q) \parallel R \equiv P \parallel (Q \parallel R)$$

$$P \parallel Q \equiv Q \parallel P$$

$$P \parallel 0 \equiv P$$

$$P \rightarrow P'$$

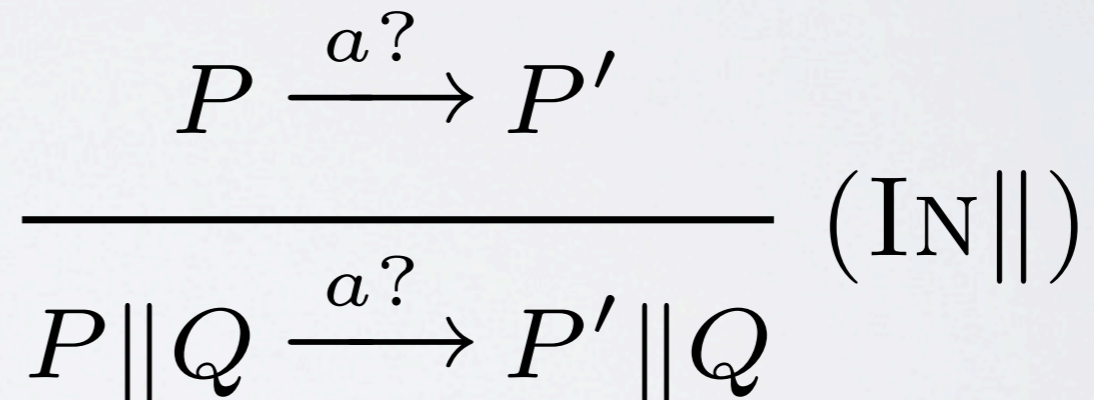
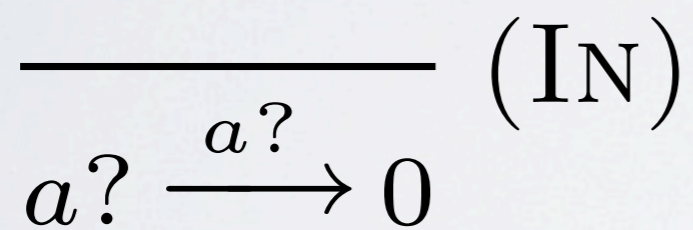
$$P \parallel Q \rightarrow P' \parallel Q$$

$$\frac{Q \equiv P \quad P \rightarrow P' \quad P' \equiv Q'}{Q \rightarrow Q'}$$

$$Q \rightarrow Q'$$

EXPERIMENT I - INPUT

- labelled transition = “log of experiment”
- input experiment - observe change in heat after adding an output ($a!$)



+ symmetric rule

EXPERIMENT 2 - OUTPUT

- output experiment - observe change in heat after adding an input ($a?$)

$$\frac{\quad}{a! \xrightarrow{a!} 0} \text{ (OUT)} \quad \frac{P \xrightarrow{a!} P'}{P \parallel Q \xrightarrow{a!} P' \parallel Q} \text{ (OUT||)}$$

+ symmetric

EXPERIMENT 3 - TAU

- tau experiment - we observe heat but we haven't added anything

$$\frac{}{\tau \xrightarrow{\tau} 0} \text{ (TAU)} \quad \frac{P \xrightarrow{\tau} P'}{P \parallel Q \xrightarrow{\tau} P' \parallel Q} \text{ (TAU||)}$$

$$\frac{P \xrightarrow{a?} P' \quad Q \xrightarrow{a!} Q'}{P \parallel Q \xrightarrow{\tau} P' \parallel Q'} \text{ (COMM)}$$

THE LTS

$$\begin{array}{c}
 \frac{}{a? \xrightarrow{a?} 0} \text{ (IN)} \qquad \frac{P \xrightarrow{a?} P'}{P \parallel Q \xrightarrow{a?} P' \parallel Q} \text{ (IN||)} \\
 \\
 \frac{}{a! \xrightarrow{a!} 0} \text{ (OUT)} \qquad \frac{P \xrightarrow{a!} P'}{P \parallel Q \xrightarrow{a!} P' \parallel Q} \text{ (OUT||)} \qquad \frac{P \xrightarrow{a?} P' \quad Q \xrightarrow{a!} Q'}{P \parallel Q \xrightarrow{\tau} P' \parallel Q'} \text{ (COMM)} \\
 \\
 \frac{}{\tau \xrightarrow{\tau} 0} \text{ (TAU)} \qquad \frac{P \xrightarrow{\tau} P'}{P \parallel Q \xrightarrow{\tau} P' \parallel Q} \text{ (TAU||)}
 \end{array}$$

- Inductive presentation of RPO LTS
- Context lemma:

$$\text{Let } \chi_{a!} = a? \quad \chi_{a?} = a! \quad \chi_{\tau} = 0$$

$$P \xrightarrow{\alpha} P' \quad \Rightarrow \quad P \parallel \chi_{\alpha} \rightarrow P'$$

SOUNDNESS

- similarity is contained in reduction precongruence
- bisimilarity is contained in reduction congruence
- **Proof:** tau-labelled transitions agree with reductions and (bi) similarity is a (pre)congruence
- What about completeness?

EXPERIMENT MISMATCH

$$P_1 \stackrel{\text{def}}{=} a? \parallel a! \qquad P_2 \stackrel{\text{def}}{=} \tau$$

$$P_1 \lesssim P_2 \text{ but } P_1 \not\lesssim_c P_2$$

(in fact $P_1 \simeq P_2$)

so completeness does not hold...

- Cause of problem: no account of “unsuccessful” experiments

HONDA TOKORO RULES

(K. Honda & M. Tokoro, An object calculus for asynchronous communication, ECOOP '91)

$$\frac{P \xrightarrow{\tau} P'}{P \xrightarrow{a?} P' || a!} \text{ (INHT)} \quad \frac{P \xrightarrow{\tau} P'}{P \xrightarrow{a!} P' || a?} \text{ (OUTHT)}$$

- Rules may appear only at the last place in the derivation
 - ie we are looking at the LTS $\mathcal{HT} \stackrel{\text{def}}{=} \Psi \mathcal{C}$ where

$$\Psi \stackrel{\text{def}}{=} \{(\text{INHT}), (\text{OUTHT})\}$$

- With these rules we have both soundness and completeness

PROOF

- for soundness, enough to show simulation a precongruence
 - $\{(P \parallel R, Q \parallel R) \mid P \lesssim Q\}$ is a simulation
 - case $P \parallel R \xrightarrow{\tau} P' \parallel R'$ where $P \xrightarrow{a!} P'$, $R \xrightarrow{a?} R'$
 - matched by “real output” $Q \xrightarrow{a!}_c Q'$ $P' \lesssim Q'$
 - matched by Honda-Tokoro transition
$$Q' = Q'' \parallel a? \quad Q \xrightarrow{\tau} Q'' \quad P' \lesssim Q'$$
$$Q \parallel R \xrightarrow{\tau} Q'' \parallel R = Q'' \parallel a? \parallel R' = Q' \parallel R'$$
- for completeness:
 - easy to show: $P \xrightarrow{\alpha}_{\mathcal{HT}} P'$ iff $P \parallel \chi_{\alpha} \rightarrow P'$
 - this implies that reduction precongruence is a simulation

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ASYNCHRONY

Syntax

$$P ::= 0 \mid a! \mid a?P \mid P \parallel Q \mid \tau P$$

Structural congruence

$$(P \parallel Q) \parallel R \equiv P \parallel (Q \parallel R)$$

$$P \parallel Q \equiv Q \parallel P$$

$$P \parallel 0 \equiv P$$

Reduction semantics

$$\tau P \rightarrow P$$

$$a! \parallel a?P \rightarrow P$$

$$\frac{P \rightarrow P'}{\quad}$$

$$P \parallel Q \rightarrow P' \parallel Q$$

$$\frac{Q \equiv P \quad P \rightarrow P' \quad P' \equiv Q'}{\quad}$$

$$Q \rightarrow Q'$$

ASYNCHRONOUS EXPERIMENTS

$$\frac{}{a?P \xrightarrow{a?} P} \text{ (IN)} \quad \frac{P \xrightarrow{a?} P'}{P \parallel Q \xrightarrow{a?} P' \parallel Q} \text{ (IN||)} \quad \frac{}{\tau \xrightarrow{\tau} 0} \text{ (TAU)}$$

$$\frac{}{a! \xrightarrow{a! \downarrow R} R} \text{ (OUT)} \quad \frac{P \xrightarrow{a! \downarrow R} P'}{P \parallel Q \xrightarrow{a! \downarrow R} P' \parallel Q} \text{ (OUT||)} \quad \frac{P \xrightarrow{\tau} P'}{P \parallel Q \xrightarrow{\tau} P' \parallel Q} \text{ (TAU||)}$$

$$\frac{P \xrightarrow{a?} P' \quad Q \xrightarrow{a! \downarrow 0} Q'}{P \parallel Q \xrightarrow{\tau} P' \parallel Q'} \text{ (COMM)}$$

$$\Phi \stackrel{\text{def}}{=} \{(\text{TAU}), (\text{TAU||}), (\text{IN}), (\text{IN||}), (\text{OUT}), (\text{OUT||}), (\text{COMM})\}$$

$$\mathcal{C}_a \stackrel{\text{def}}{=} \Phi_a(\emptyset)$$

CONTEXT LEMMA & SOUNDNESS

- Context lemma

$$\chi_{a!\downarrow R} = a?R \quad \chi_{a?} = a! \quad \chi_{\tau} = 0$$

$$P \xrightarrow{\alpha} P' \Rightarrow P \parallel \chi_{\alpha} \rightarrow P'$$

- Soundness wrt contextual equivalence

EXPERIMENT MISMATCH

$$P_1 \stackrel{\text{def}}{=} a?a! \quad P_2 \stackrel{\text{def}}{=} \tau$$

$$P_1 \lesssim P_2 \quad P_1 \not\lesssim_c P_2$$

- This means that some of our observations (labels) are morally unobservable. Which ones?

COMPLETENESS

$$\frac{P \xrightarrow{\tau} P'}{P \xrightarrow{a?} P' \parallel a!} \text{ (INHT)} \quad \frac{P \xrightarrow{\tau} P'}{P \xrightarrow{a! \downarrow R} P' \parallel a?R} \text{ (OUTHT)}$$

- The resulting LTS is sound and complete

PROOF

- Soundness: $\{(P \parallel R, Q \parallel R) \mid P \lesssim Q\}$ is a simulation

- case $P \parallel R \xrightarrow{\tau} P' \parallel R'$

$$P \xrightarrow{a! \downarrow 0} c P' \quad R \xrightarrow{a?} c R$$

$$R = R'' \parallel a?S \quad R' = R'' \parallel S$$

$$P \xrightarrow{a! \downarrow S} c P' \parallel S$$

$$Q \xrightarrow{\tau} Q' \quad Q \xrightarrow{a! \downarrow S} \mathcal{HT} Q' \parallel a?S$$

$$P' \parallel S \lesssim Q' \parallel a?S$$

$$Q \parallel R \xrightarrow{\tau} Q' \parallel R = Q' \parallel a?S \parallel R''$$

$$P' \parallel R' = P' \parallel S \parallel R''$$

- Completeness: $P \xrightarrow{\alpha} \mathcal{HT} P'$ iff $P \parallel \chi_{\alpha} \rightarrow P'$

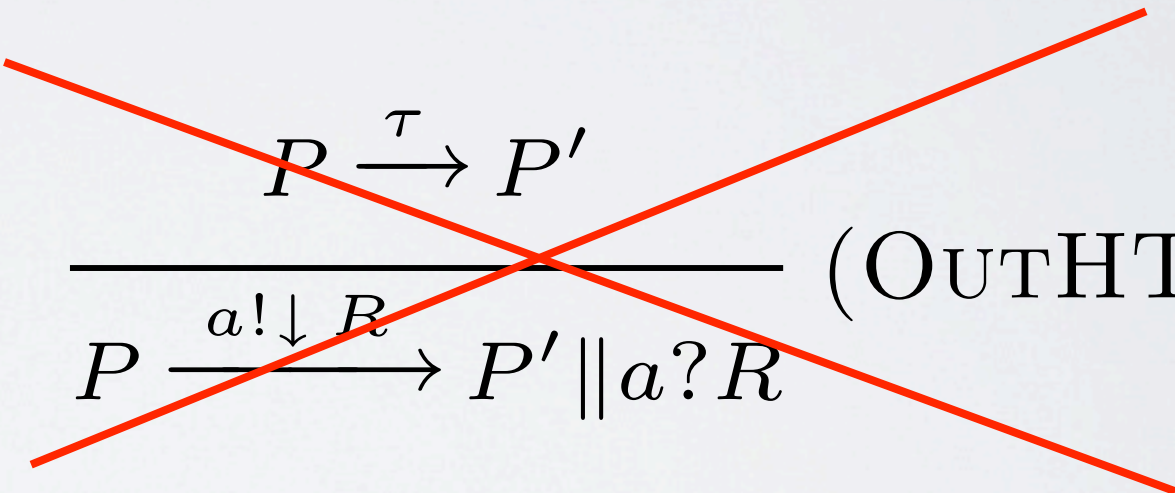
REFINING

- But here outputs are observable!

$$a! \parallel Q \simeq R \quad \Rightarrow \quad R = a! \parallel R'$$

proving this is surprisingly tricky

$$\frac{P \xrightarrow{\tau} P'}{P \xrightarrow{a?} P' \parallel a!} \quad (\text{INHT})$$


$$\frac{P \xrightarrow{\tau} P'}{P \xrightarrow{a! \downarrow R} P' \parallel a? R} \quad (\text{OUTHT})$$

- It is good to get rid of HT rules when they are not necessary
 - LTS are smaller
 - bisimulations are easier to construct

ASYNCHRONOUS BISIMULATION

(R. AMADIO, I. CASTELLANI, D. SANGIORGI, ON BISIMULATIONS FOR THE ASYNCHRONOUS PI CALCULUS, TCS 195(2):291-324, 1998)

PRQ & $P \xrightarrow{a?} P'$ then either $Q \xrightarrow{a?} Q'$ & $P'RQ'$ or
 $Q \xrightarrow{\tau} Q'$ & $P'R(Q' \parallel a!)$

- Putting facts about observability into the definition of equivalence
- We don't like this
 - need to reprove basic facts about bisimilarity
 - not clear exactly what is "asynchronous" about the bisimilarity
- We like the principle of getting the "right" labelled transitions into the LTS

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SYNCHRONY

Syntax

$$P ::= 0 \mid a!P \mid a?P \mid P \parallel Q \mid \tau P$$

Reduction semantics

$$\begin{array}{l} \tau P \rightarrow P \\ a!P \parallel a?Q \rightarrow P \parallel Q \end{array}$$

Structural congruence

$$(P \parallel Q) \parallel R \equiv P \parallel (Q \parallel R)$$

$$P \parallel Q \equiv Q \parallel P$$

$$P \parallel 0 \equiv P$$

$$\begin{array}{c} \frac{P \rightarrow P'}{P \parallel Q \rightarrow P' \parallel Q} \\ \frac{Q \equiv P \quad P \rightarrow P' \quad P' \equiv Q'}{Q \rightarrow Q'} \end{array}$$

SOS

$$\frac{}{a?P \xrightarrow{a?\downarrow R} P\parallel R} \text{ (IN)} \quad \frac{P \xrightarrow{a?\downarrow R} P'}{P\parallel Q \xrightarrow{a?\downarrow R} P'\parallel Q} \text{ (IN\parallel)}$$

$$\frac{}{\tau \xrightarrow{\tau} 0} \text{ (TAU)}$$

$$\frac{}{a! \xrightarrow{a!\downarrow R} R} \text{ (OUT)} \quad \frac{P \xrightarrow{a!\downarrow R} P'}{P\parallel Q \xrightarrow{a!\downarrow R} P'\parallel Q} \text{ (OUT\parallel)}$$

$$\frac{P \xrightarrow{\tau} P'}{P\parallel Q \xrightarrow{\tau} P'\parallel Q} \text{ (TAU\parallel)}$$

$$\frac{P \xrightarrow{a?\downarrow 0} P' \quad Q \xrightarrow{a!\downarrow 0} Q'}{P\parallel Q \xrightarrow{\tau} P'\parallel Q'} \text{ (COMM)}$$

- Context lemma & soundness

HONDA TOKORO

$$\frac{P \xrightarrow{\tau} P'}{P \xrightarrow{a? \downarrow R} P' \parallel a!R} \text{ (INHHT)} \qquad \frac{P \xrightarrow{\tau} P'}{P \xrightarrow{a! \downarrow R} P' \parallel a?R} \text{ (OUTHT)}$$

- Again, the LTS completed with HT rules is sound and complete
- This time, both the actions are observable and so both Honda Tokoro rules are unnecessary
 - ie the SOS on the previous slide is sound and complete for contextual equivalence

GENERAL HT RULE FORM

- Suppose that α is an action with an associated context χ_α

$$P \xrightarrow{\alpha} P' \Rightarrow \chi_\alpha(P) \rightarrow P'$$

$$\frac{P \xrightarrow{\tau} P'}{P \xrightarrow{\alpha} \chi_\alpha(P')} \quad (\alpha\text{HT})$$

MORALS OF THE STORY

- Labelled transitions are used to
 1. generate the reduction relation inductively
 2. give a proof method for reasoning about contextually defined process equivalence
- The first (choosing experiment) can be done systematically, starting from reductions
- Morally non-observable labels can then be made unobservable using Honda-Tokoro rules, characterising contextual equivalence
 - observability is a calculus-specific notion