Stochastic Analysis of Graph Transformation Systems

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Motivation

1. Quantitative analysis of systems with dynamic reconfigurations ...

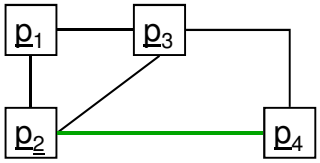
2. ... controlled by quantitative properties

Example 1: P2P Networks

Problem:

- no central infrastructure
- unreliable components

➔ removing nodes may disconnect network

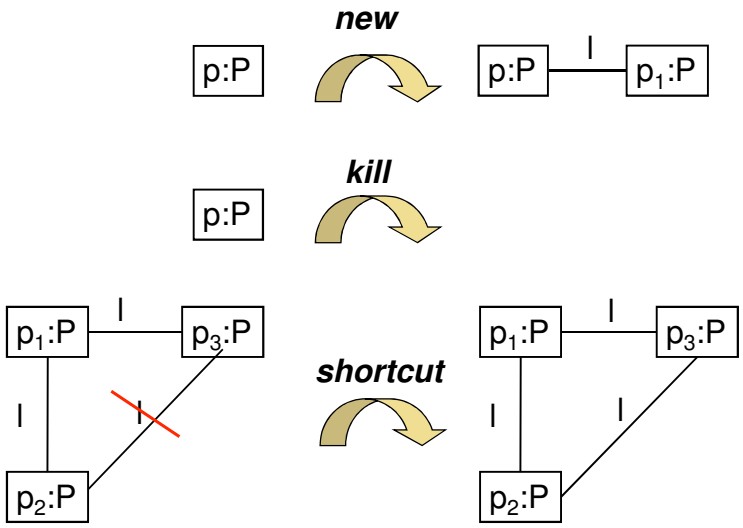


Solution: introduce redundancy!

Question: Which links should be added to guarantee a certain level of reliability ?

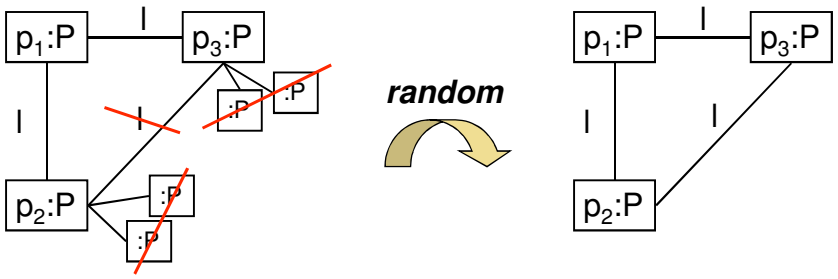
a) at random, up to a limit of **n** links
b) so that deletion of node does not increase distance

Modelling Change in the Network: A Graph Transformation System



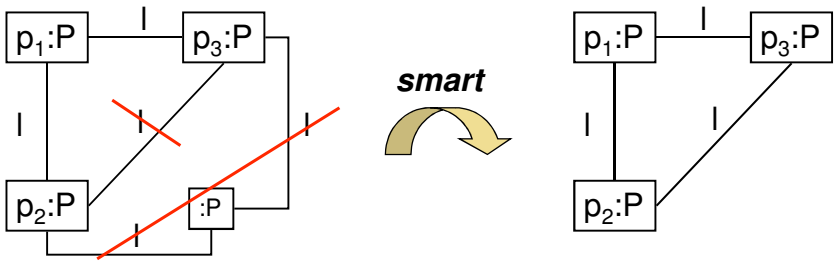
Which shortcuts?

a) At random (limit here: **n = 3** links)



Which shortcuts?

b) So that deletion of node does not increase distance



L. Mariani. Fault-tolerant routing for p2p systems with unstructured topology. Proc. International Symposium on Applications and the Internet (SAINT 2005), Trento, Italy.

Stochastic Graph Transformation (SGT)

- associate rate **ρ(p)** with every rule **p**
- **1/ρ(p)** average delay of p, once enables

$SG_{random, x}$

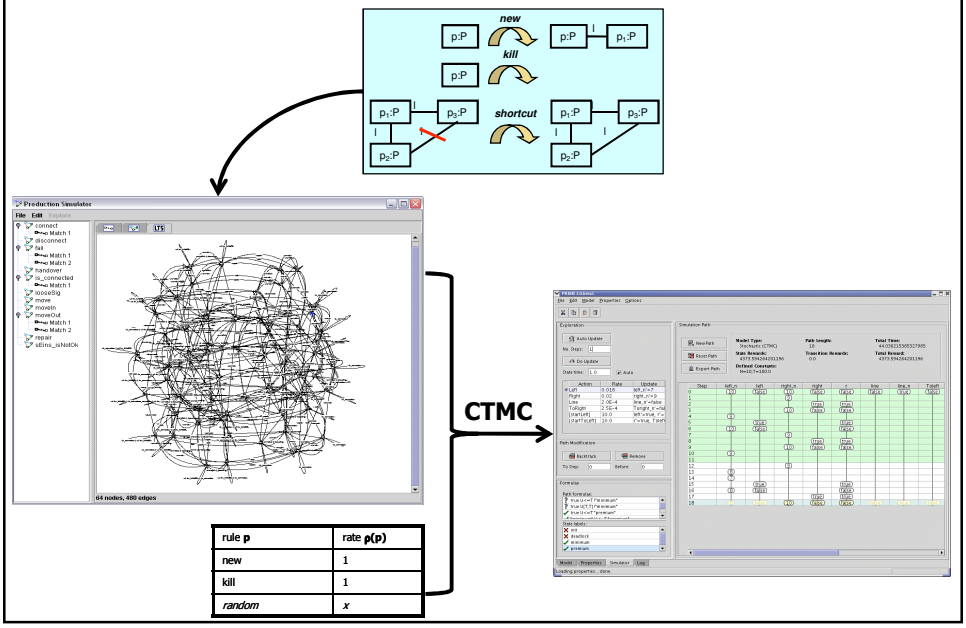
rule **p**	rate **ρ(p)**
new	1
kill	1
random	*x*

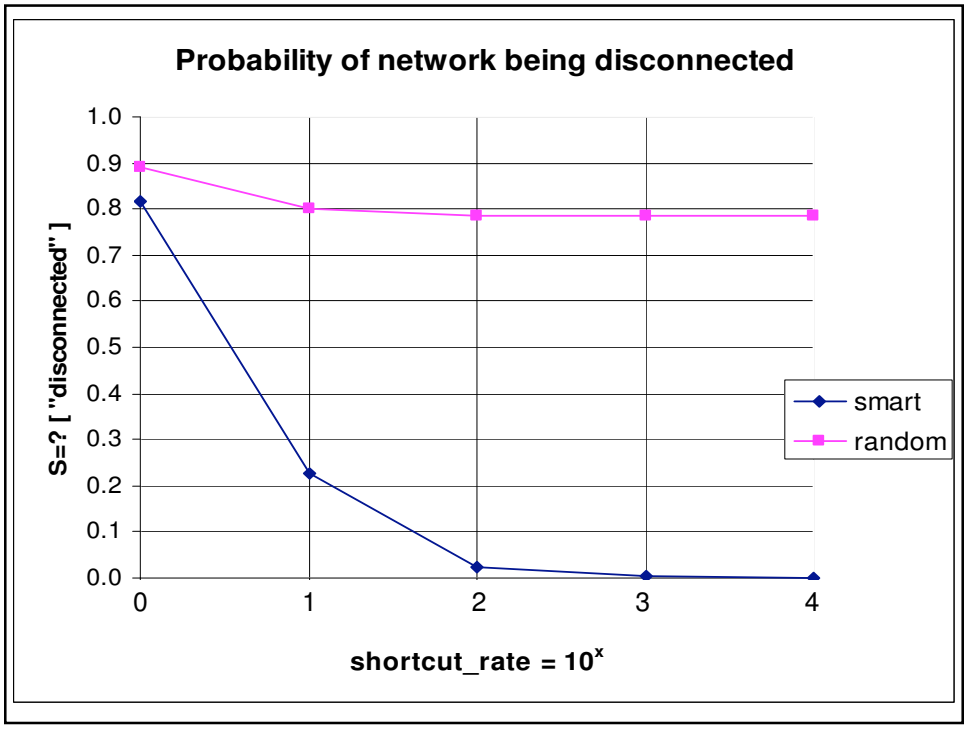
$SG_{smart, x}$

rule **p**	rate **ρ(p)**
new	1
kill	1
smart	*x*

x times as fast as new or kill

Tools for Querying the Model





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  ➔ Stochastic graph transformation (→ CTMC)
  - ✓ separation of struct. and stochastic aspect
  - ✓ simple tool chain for model checking

2. ... controlled by quantitative properties

Example 2: VoIP Services

Problem:

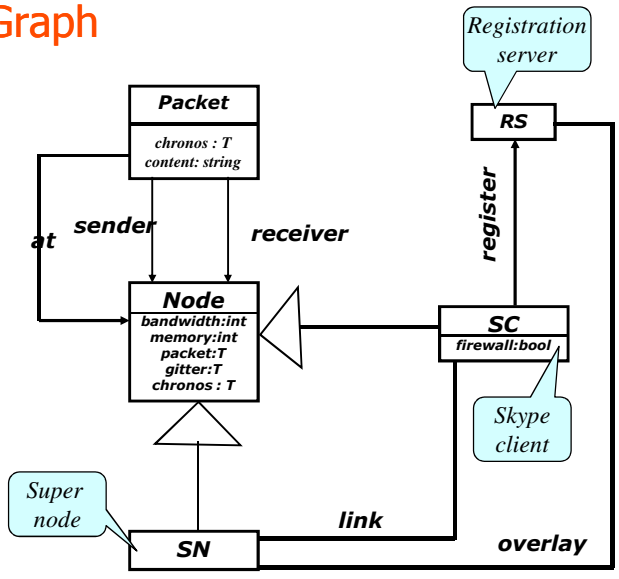
- no central infrastructure
- unreliable components

➔ fluctuations in audio quality caused by packet delay and jitter

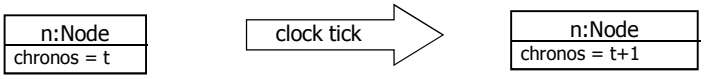
Solution: monitor quality attributes and trigger reconfiguration if required levels are not met

Challenge: modelling time

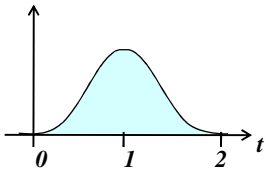
Type Graph



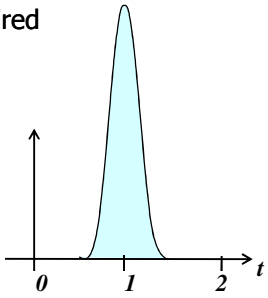
Modelling Time ➔ Generalised SGT



- timing governed by Gaussian (normal) distributions
  - mean = 1ms
  - variance – depends on accuracy required



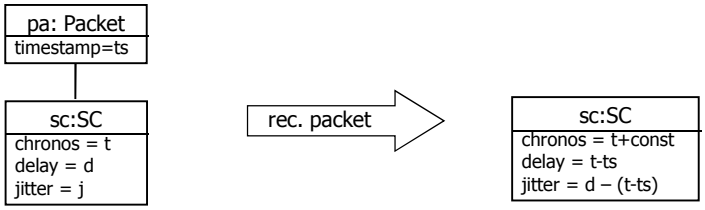
- very poor clock



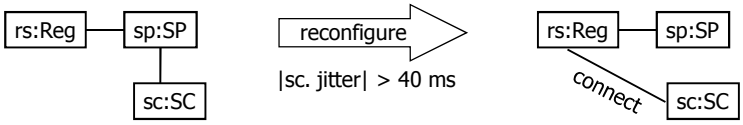
slightly better clock

Jitter and Reconfiguration

- Jitter = variation of packet delay



- Reconfiguration triggered by loss of quality



Generalised Markov Processes

- Transitions do not depend on *history prior to the current state*

$$\begin{aligned}\mathcal{P} = \langle\ & S : State\ set \\ & E : Event\ set \\ & \Gamma : State \to \wp(Event\ set) \\ & \Sigma : State \times Event \to State \\ & \Delta : Event \to (\mathbb{R} \to [1,0]) \\ & s_0 : State\ \rangle\end{aligned}$$

Simulation of Generalised SGT

Initially

- compute all enabled events (rule, match)
- for each event determine randomly (using appropriate distribution) their delay

Repeatedly

- select next event and apply (rule, match)
- update enabled events and remaining delays

Implementation based on

- GT tool to find and maintain collection of all matches for all rules in each state ➔ VIATRA
- simulation environment to compute random numbers, perform statistics, and visualise results ➔ SSJ

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  - ➔ Stochastic graph transformation (→ CTMC)
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    - ✓ simple tool chain for model checking
- ✓ ... controlled by quantitative properties
  - ➔ Generalised stochastic GT (→ GMP)
    - ✓ general distributions
    - ✓ stochastic simulation ongoing

Scalability problem with local clocks

- ✖ Assume
  - one clock per peer
  - 1000 ticks per second
  - 100 peers
- ✖ How many applications of *clock tick* for simulating one minute ... ?

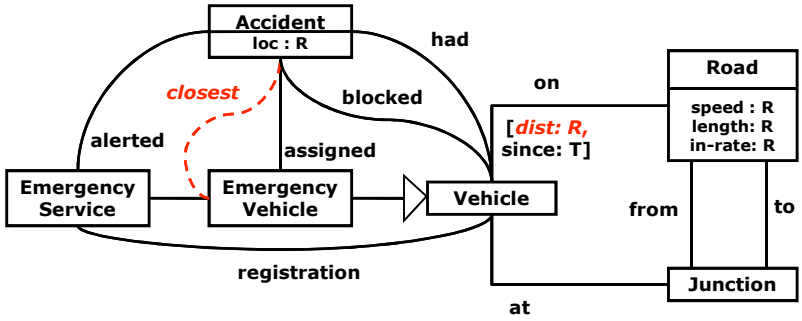
Try global time ...

Another Example: Mobile Services

- Cars are moving along roads. If an accident happens, cars travelling behind the one affected will be stopped or slowed down.
- Emergency services are alerted. They assign the accident to the nearest emergency vehicle.
- In case of shortage, accidents are served based on perceived urgency, like number of cars caught up in tailback, etc.
- Road can be blocked by switching traffic lights, etc.

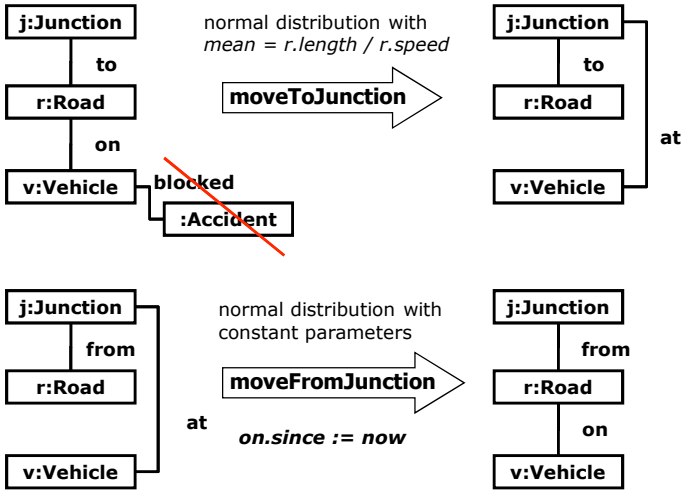
Question: What is a good strategy for serving accidents, blocking off roads, etc.

Type Graph

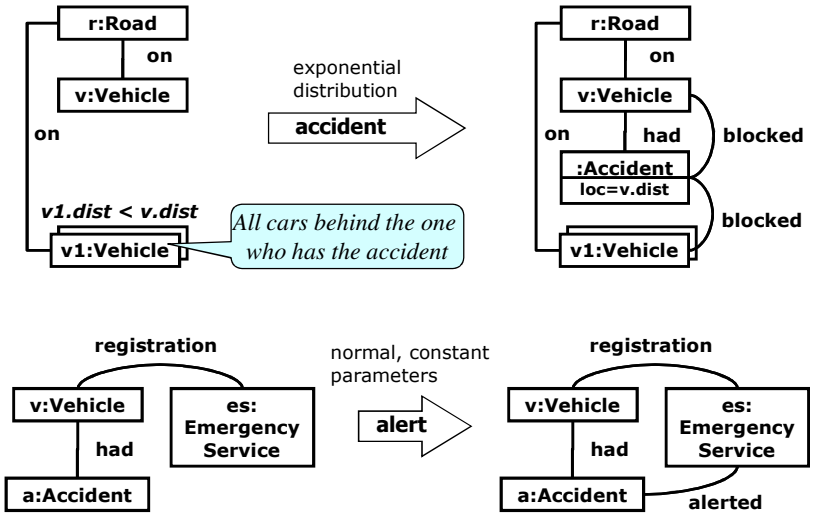


- derived attributes and edges, lazy evaluation
  - *on::dist := Road.speed * (now – since)*
  - *Accident::closest* ... points to available emergency vehicle with the quickest route to the accident
- constant *now* to access global simulation time

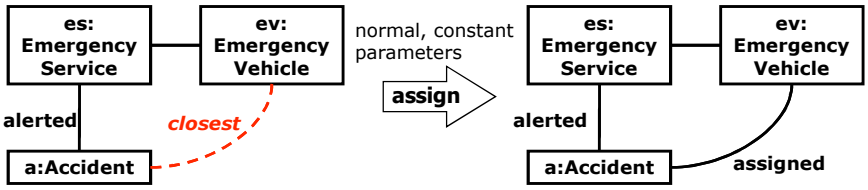
Moving to and fro



Accident happens, gets reported ...



And someone will take care of it, etc.



Advanced GT concepts

- negative conditions
- multi-objects
- derived attributes and edges

Closer Integration: Simulation → GT

- inject current simulation time
- extract attributes for computing distribution parameters

Challenges of Stochastic Modelling and Simulation

- Modelling of time in stochastic systems
  - CTMCs not aware of current time
  - Global (simulation) time vs. local stochastic clocks
- Allowing complex computations without slicing time / generating intermediate states
  - cf. auxiliary functions, equations vs rewrites in RL, etc.
  - derived attributes and edges provide visual abstraction
  - available or easily implementable in GT tools like VIATRA