Formally Proving and Enhancing a Self-Stabilising Distributed Algorithm

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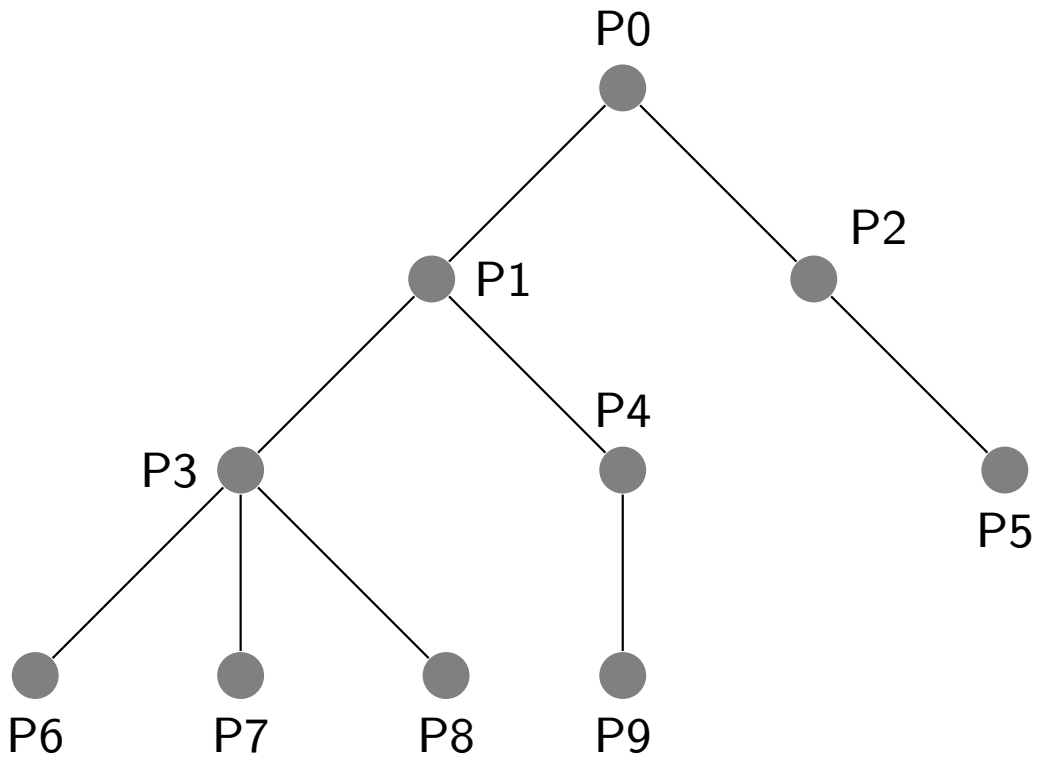
Roadmap

The algorithm



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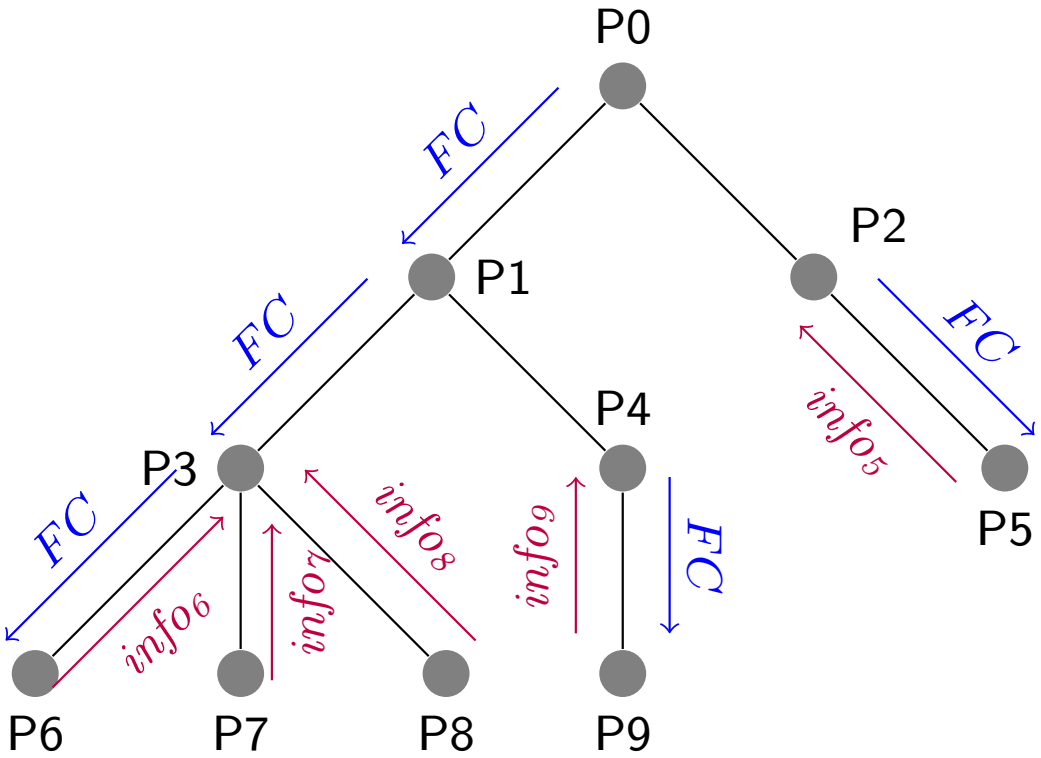

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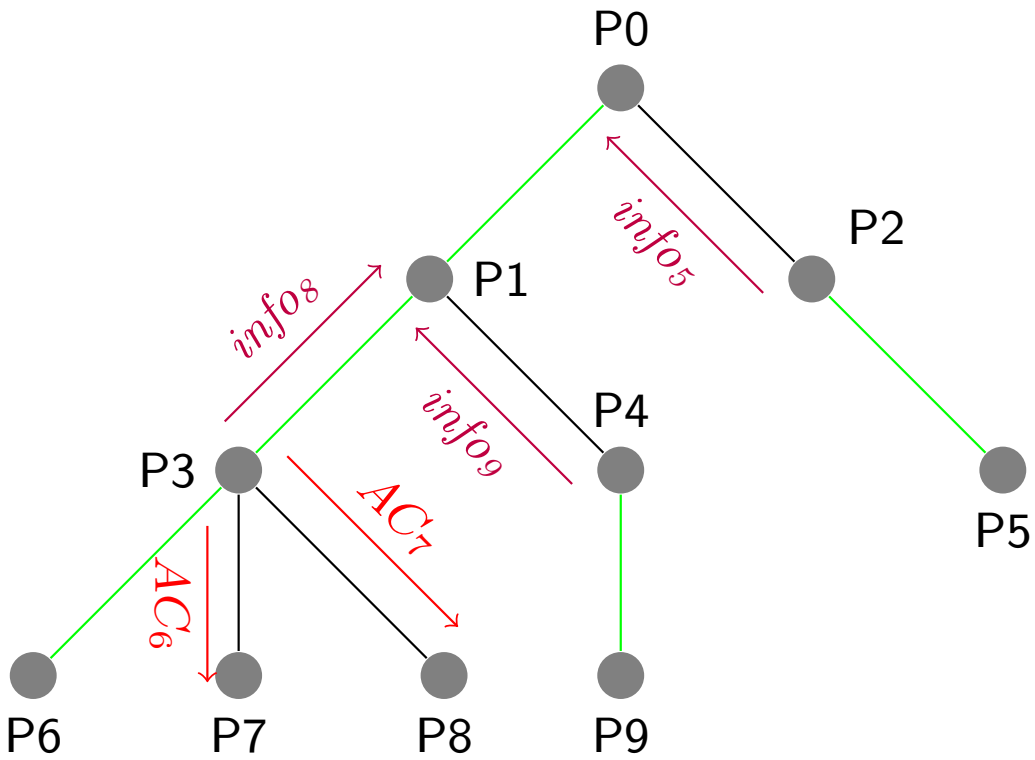

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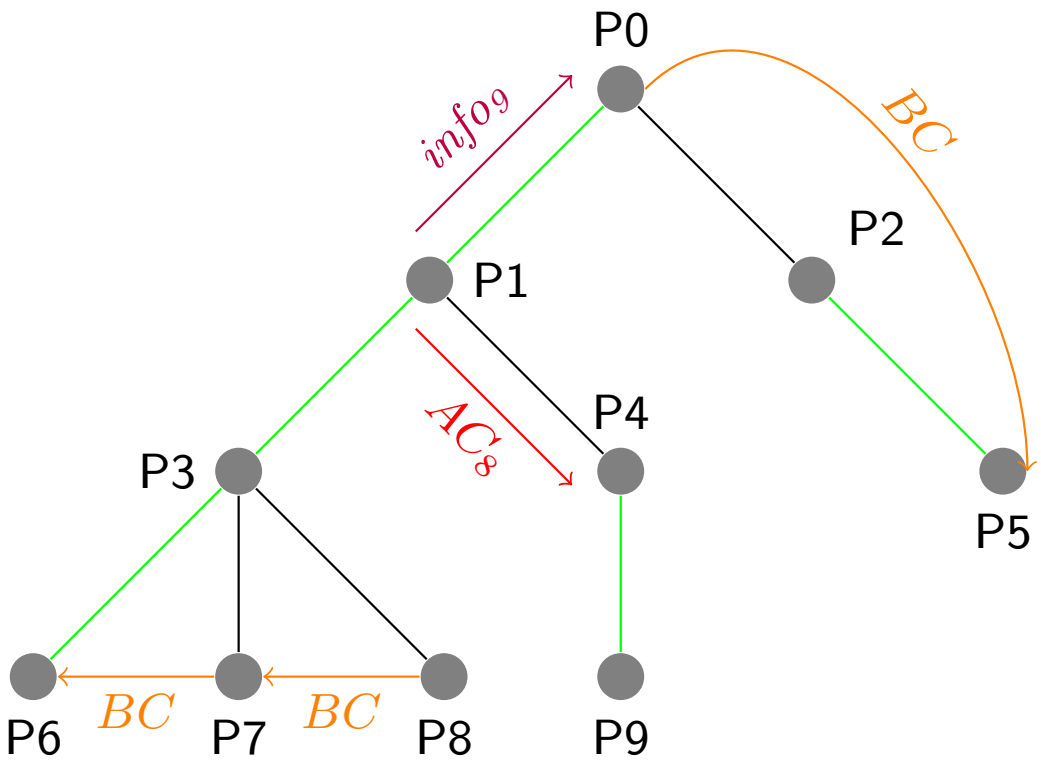

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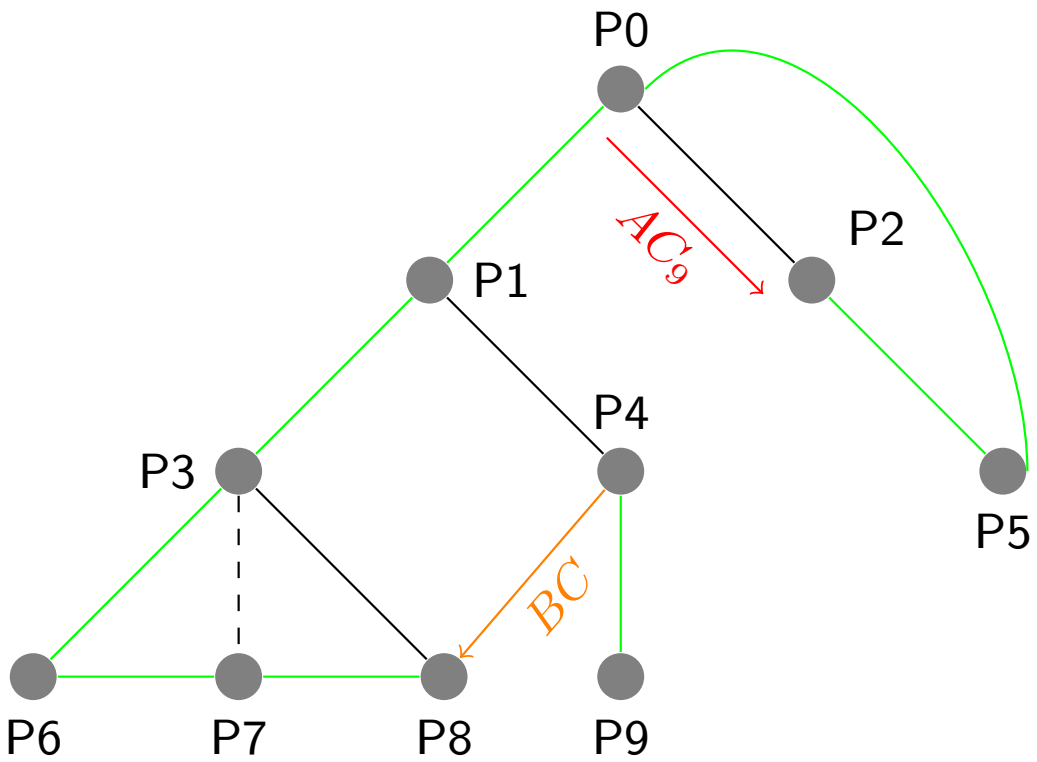

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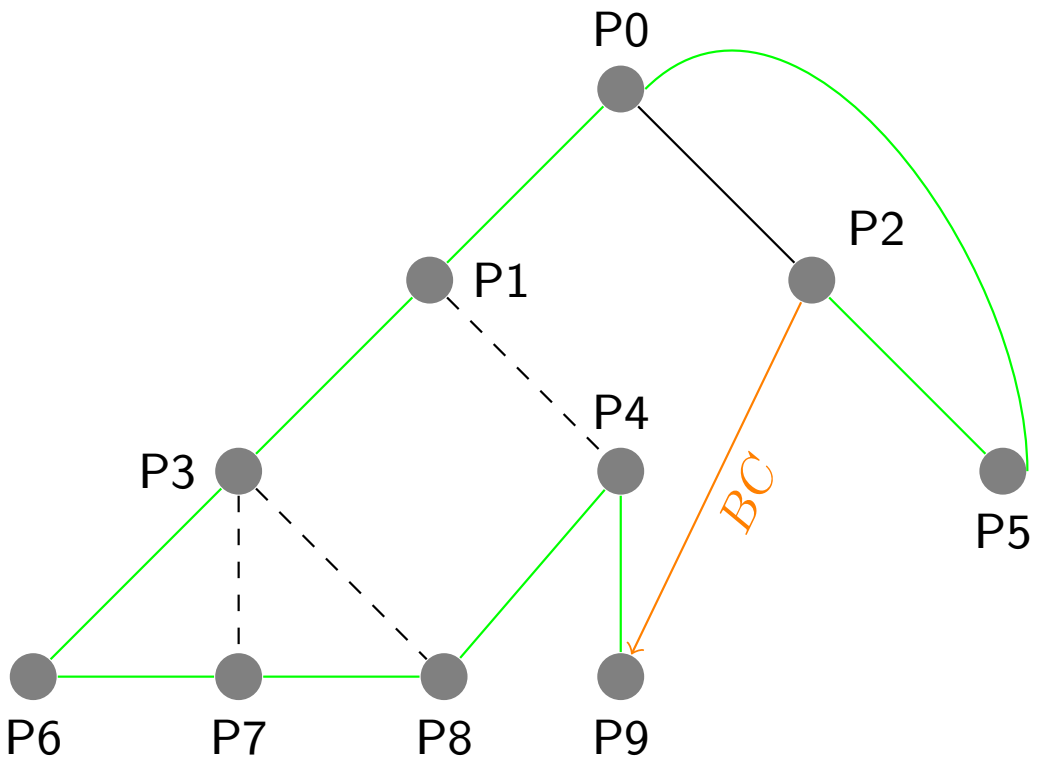

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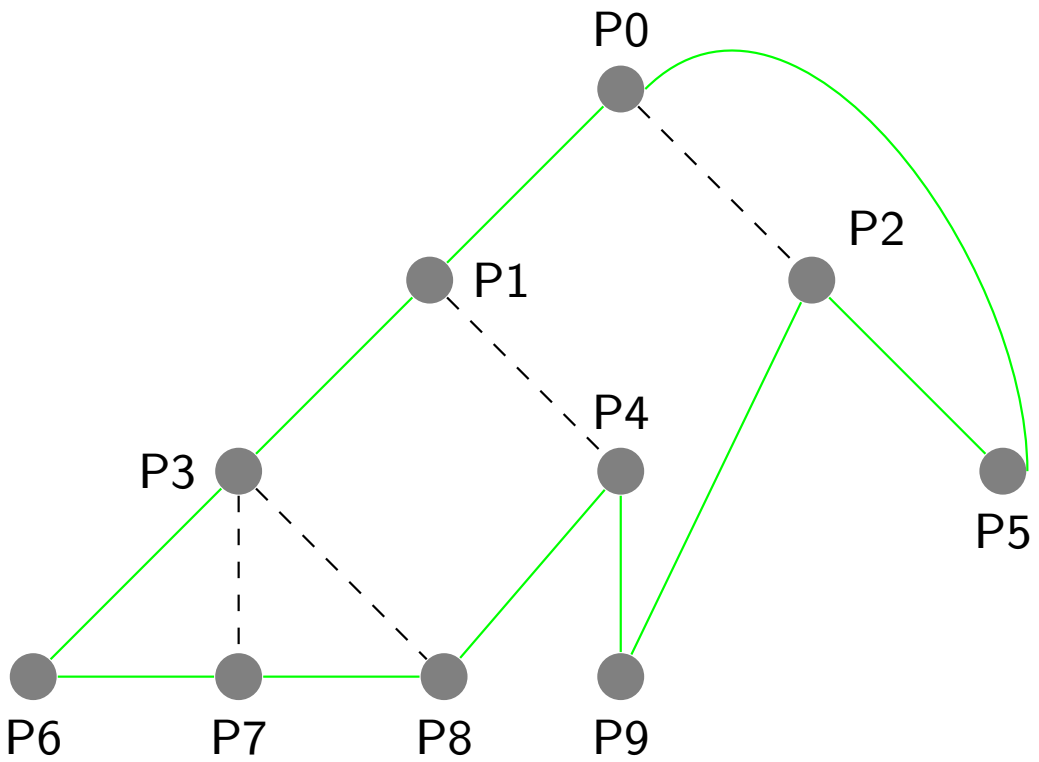

Properties of the algorithm

Transforms a tree topology to a ring

- Self-stabilising
- From *any* initial configuration (*ie* any initial tree), reaches a *legitimate* configuration
  - Initial configuration: tree, result of a process spawning
  - Legitimate configuration: ring
- When a legitimate configuration is reached, the system stays in this configuration
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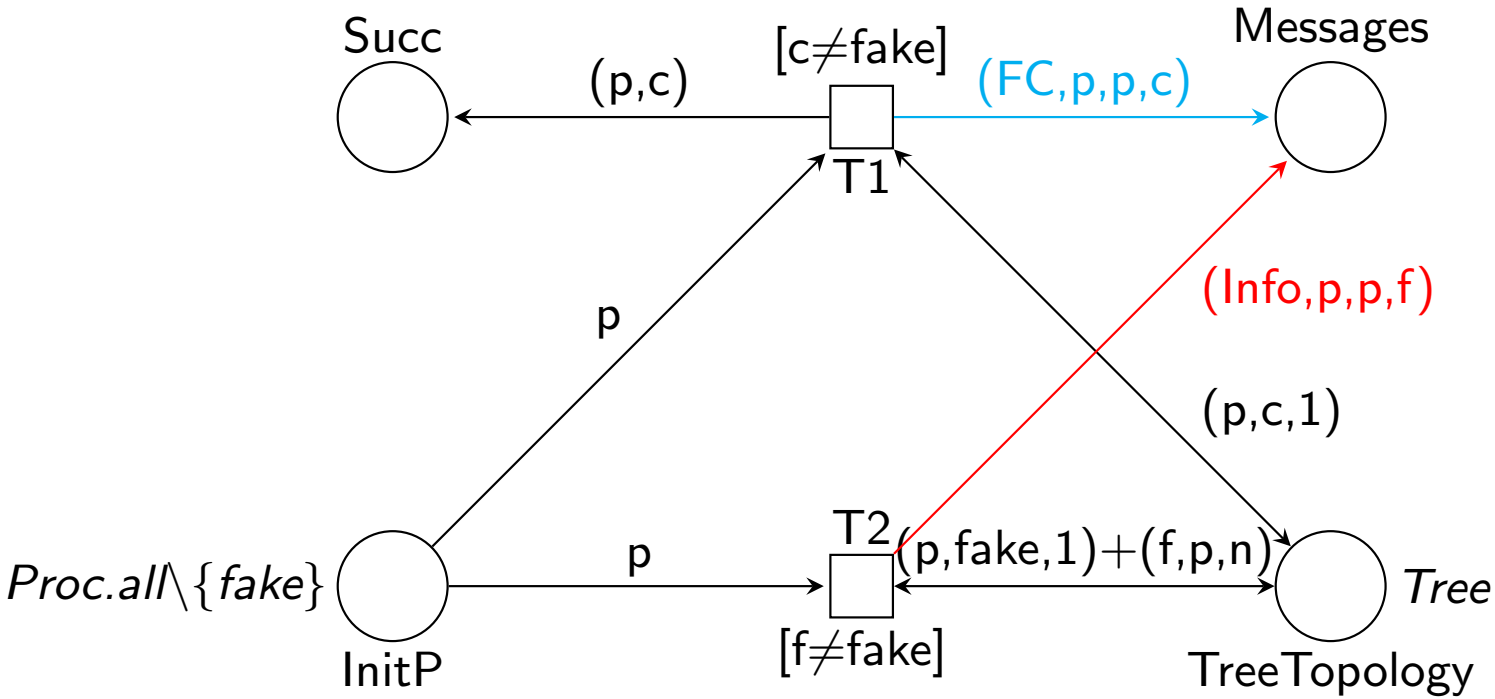
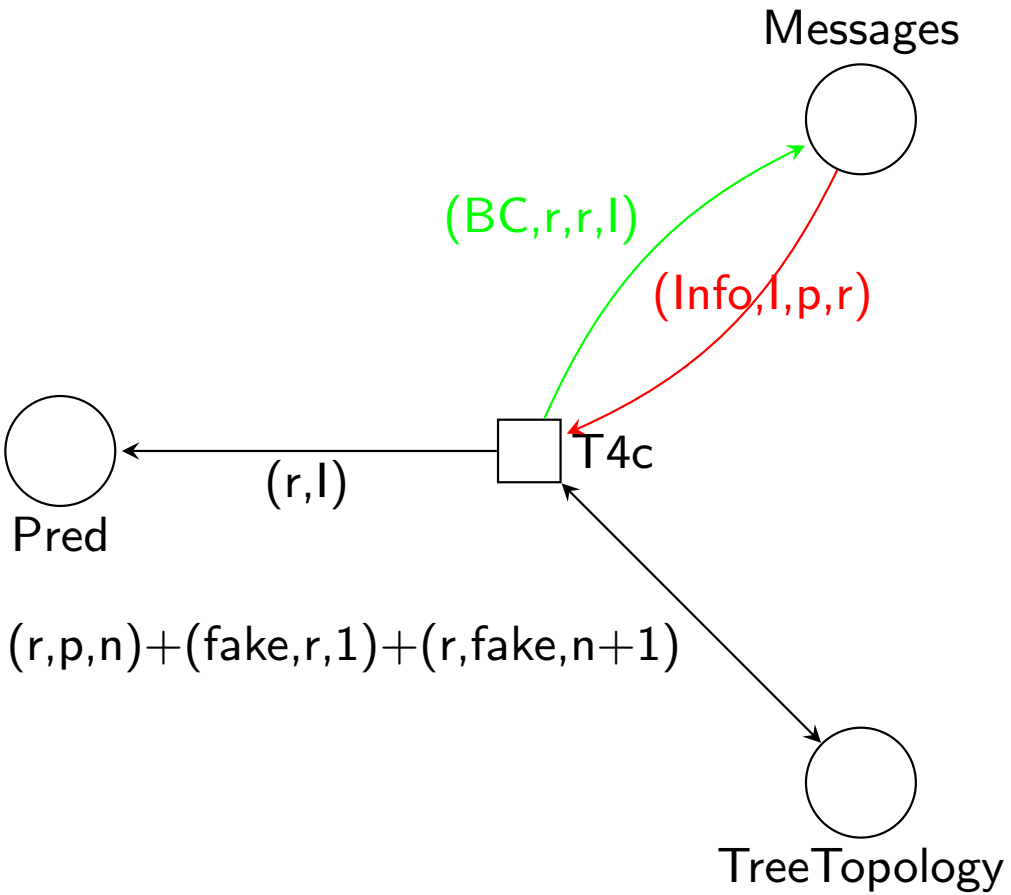
First step of the construction of a **Binomial Graph**

- Many interesting properties, making it very suitable for overlay networks supporting the run-time environment of distributed applications
- The ring-to-BMG construction is trivially self-stabilising
- Combination of two self-stabilising algorithms = self-stabilising

Coloured Petri Net Model: Intialisation



Coloured Petri Net Model: Main Part



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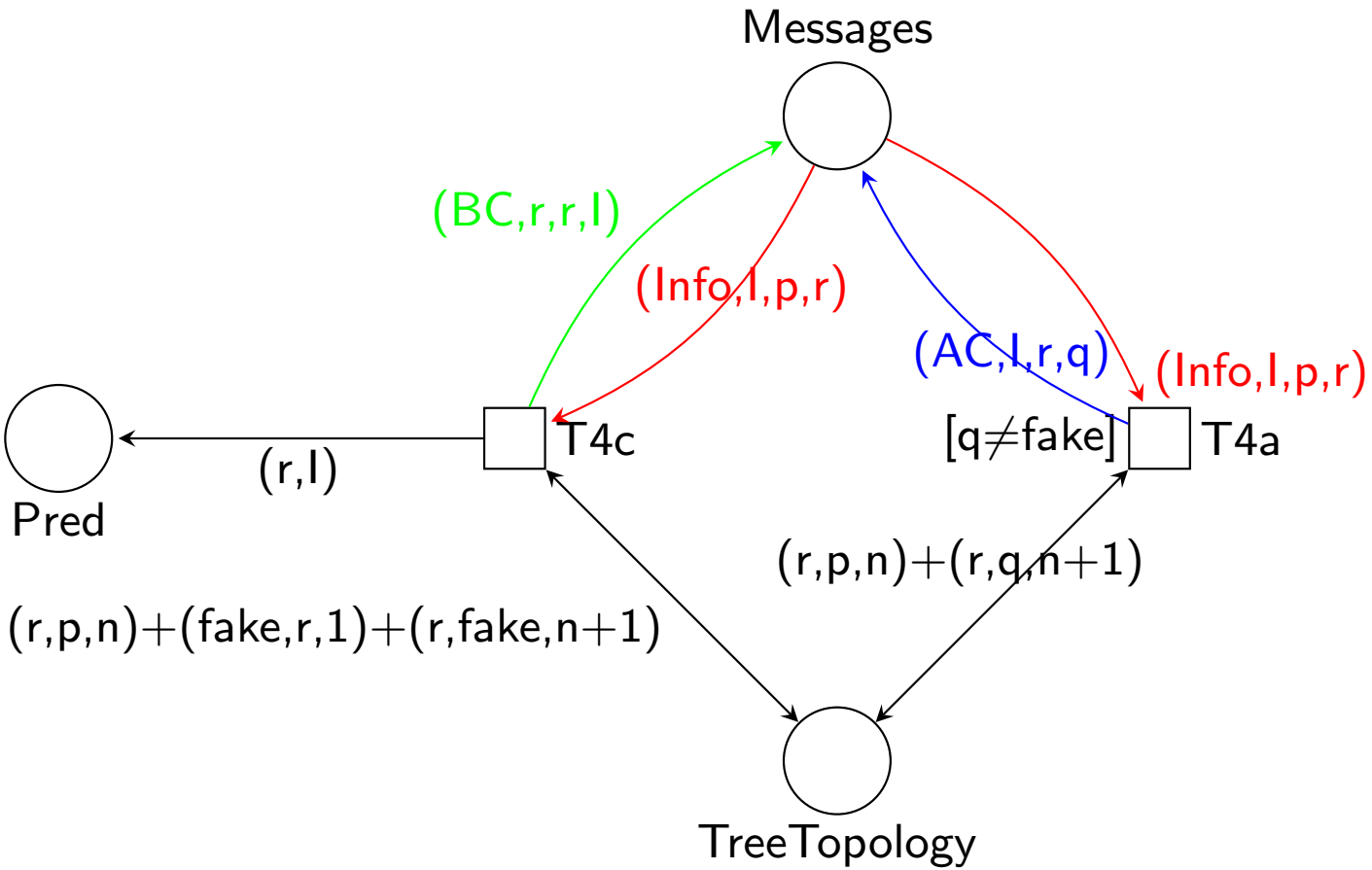

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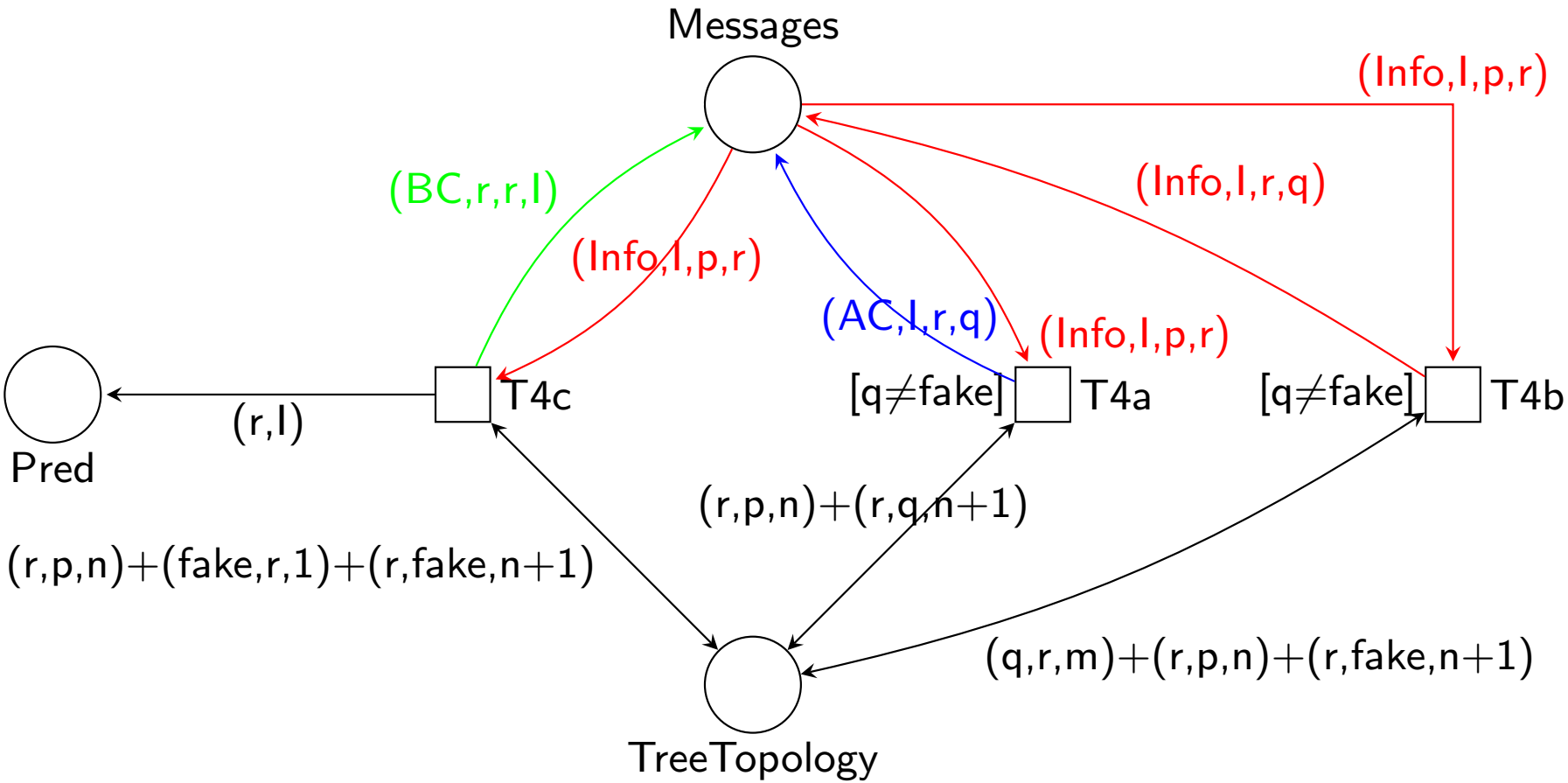

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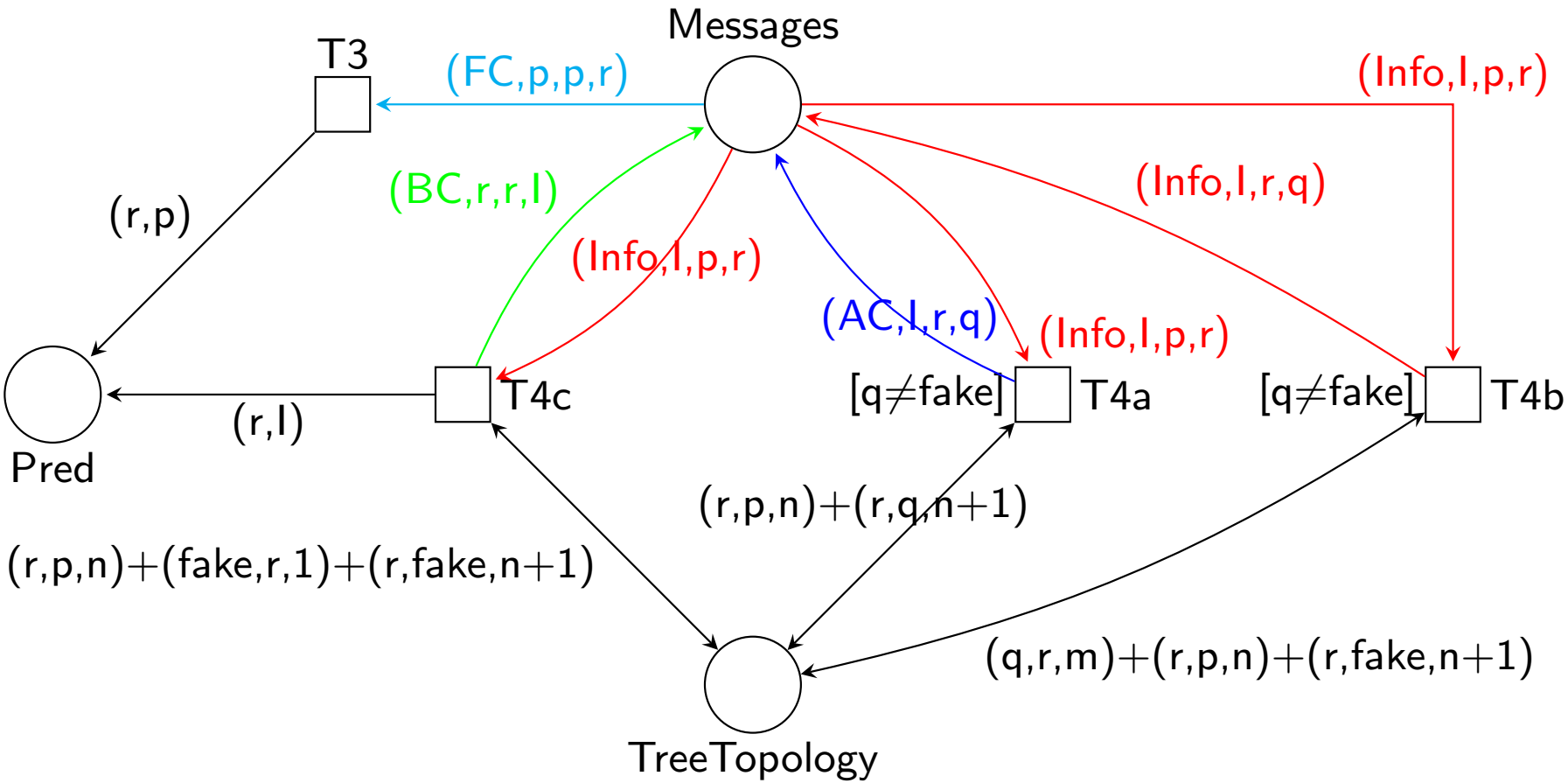
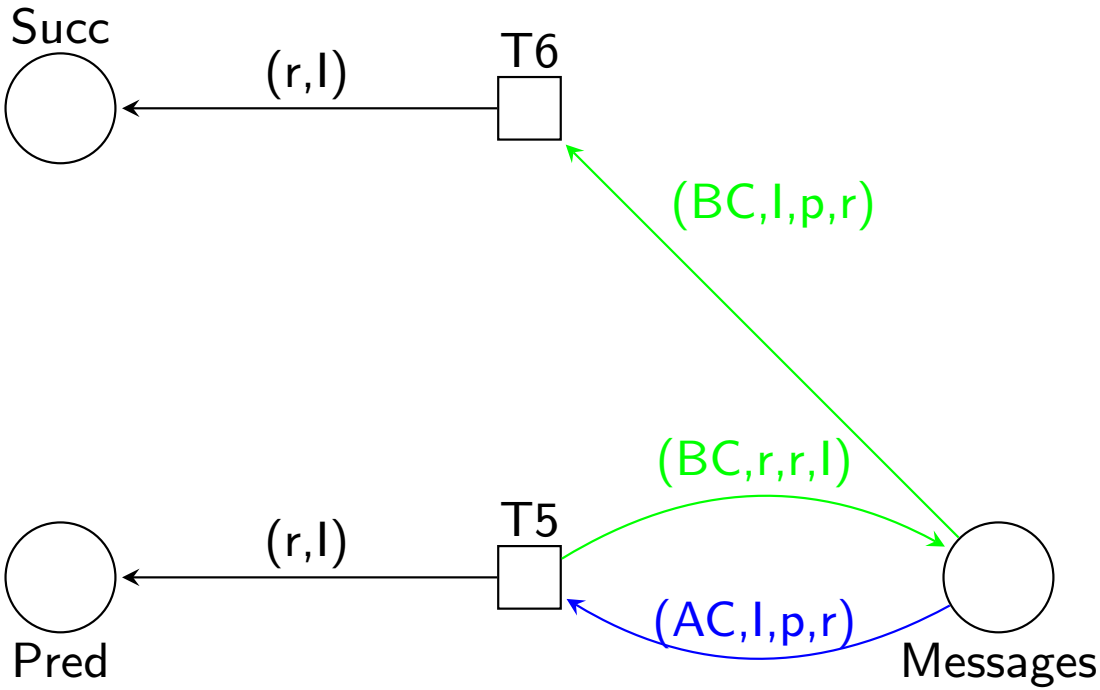


Coloured Petri Net Model: End



Formal Analysis of the Algorithm Properties

Invariants

- $\texttt{InitP} + \pi_1(\texttt{Succ}) + \pi_2(Messages(\texttt{Info})) + \pi_2(Messages(\texttt{AC})) + \pi_4(Messages(\texttt{BC})) = \texttt{Proc} \setminus \{\texttt{fake}\}$
- $\texttt{Succ} + \pi_{4,2}(Messages(\texttt{BC})) = \pi_{3,4}(Messages(\texttt{FC})) + \pi_{2,1}(\texttt{Pred})$

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Liveness properties

- For any tree with at least two nodes, either transition `T1` or transition `T2` can fire for every node. Thus, `InitP` will eventually be empty.
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Termination

The algorithm terminates and is self-stabilising.

Improving the Model to Improve the Algorithm

$Pred$ and $Succ$ are mirror images

Only one of them is necessary:

- Remove $Pred$ and FC messages (better when a lot of leaves in the tree)
- or remove $Succ$ and BC messages

Conclusion & Perspectives

We have:

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Further work will include:

- generalising the approach to self-stabilisation of other protocols
- designing a *self-healing algorithm*, based on the expertise gained

Self-stabilisation

Technique to **tolerate failures**

- From **any** initial configuration, reach a legitimate configuration
- Example:
  1. Start from a tree
  2. Build a ring
  3. A process fails $\rightarrow$ the ring is broken, equiv. to a degenerated tree
  4. A new process may or may not be respawned: we still have a (degenerated) tree
  5. Run the algorithm again to build a new ring

Difference with **self-healing**

- Self-stabilisation: restart the algorithm starting from the new configuration
- Self-healing: run a local protocol to recover from the failure
  - *e.g. re-knit the ring*

Self-stabilisation

Property: self-stabilising algorithms are **silent**

- Once the algorithm has stabilised, the communications are **fixed**
  - The processes can always send the same message to their neighbours
  - ... or no message at all.

Example: Bellman-Ford (used in RIP and BGP dynamic routing protocols)

- Builds a collector tree on each router of the network
  - Eventually each router knows the shortest path to the other neworks
- Communicate the length of the shortest path to each (known) network
- Once the system has stabilised:
  - Each router sends the length of the shortest paths to the networks it kows
  - Its neighbours do not make iny chage because they already know the shortest paths to these networks
  - Nothing changes
  - The system stays in the same configuration and the communications are fixed.

Failure detection

- Another problem on its own.
- Was proved impossible with asynchronous communications.
- In general: assume that we have an *eventually perfect failure* detector.