Methods for Partitioning Data to Improve Parallel Execution Time for Sorting on Heterogeneous Clusters

C. Cérin[1] J.-C. Dubacq[1] J.-L. Roch[2]

[1]LIPN
Université de Paris Nord

[2]ID-IMAG
Université Joseph Fourier, Grenoble

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Outline

1. Motivation
   - The partitioning problem
   - Splitting data

2. Contribution
   - General exact analytic approach
   - Dynamic evaluation of complexity function
   - Non uniformly related processors
   - Experiments

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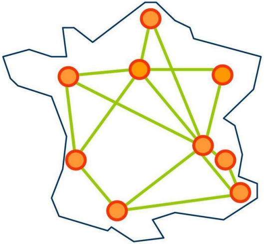
Observation

With fixed p, the computation-intensive part is step 2.

Context: Grid'5000, heterogeneous clusters



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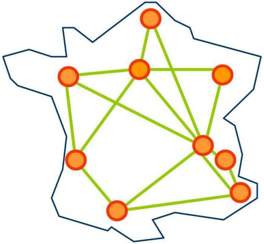

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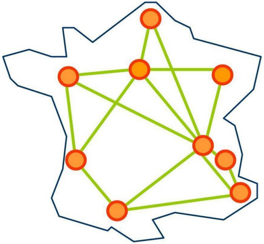

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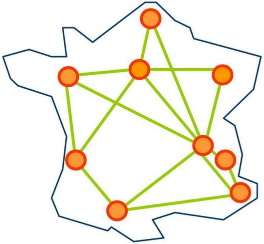

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Heterogeneity

Clusters have different processors, same family-processors have different clock speeds.

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We have N objects to transmit and transform using p nodes. We want all computation to end at exactly the same time. Final merging is not relevant.

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We define the relative speed k_i of a node i as the quantity of operations it can do by unit of time compared to a reference node, and $K = \sum_j k_j$.

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Naïve algorithm uses chunks of size $\frac{k_i}{K}N$ and yields inadequate computation time.

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$$\begin{array}{llll} \text{Node 1} & k_1 = 1 & n_1 = \frac{N}{3} & T_1 = n_1 \log n_1 \\ \text{Node 2} & k_2 = 2 & n_2 = \frac{2N}{3} & T_2 = \frac{n_2 \log n_2}{k_2} \end{array}$$

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Theorem (Cérin,Koskas,Jemni,Fkaier)

For large N, optimal chunk size is

$$n_i = \frac{k_i}{K}N + \epsilon_i, \quad (1 \leq i \leq p) \text{ where } \epsilon_i = \frac{N}{\ln N}\left[\frac{k_i}{K^2}\sum_{j=1}^{p} k_j \ln\left(\frac{k_j}{k_i}\right)\right]$$

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We use $\tilde{f}$ as the complexity function ($T_i = \tilde{f}(n_1)/k_i$).

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Only one unknown variable left!

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Proof: $\tilde{f}$ is multiplicative.

$$\begin{aligned} \sum_{i=1}^{p} \tilde{f}^{-1}(T.k_i) = N \Longrightarrow \qquad & N = \tilde{f}^{-1}(T) \sum_{i=1}^{p} \tilde{f}^{-1}(k_i) \\ \Longrightarrow \qquad & T = \tilde{f}\left(\frac{N}{\sum_{i=1}^{p} \tilde{f}^{-1}(k_i)} \right) \end{aligned}$$

$\square$

The polylog case

Theorem

Initial values of n_i can be asymptotically computed by

$$\sum_{i=1}^{p} \frac{Tk_i + Tk_i \ln\ln(Tk_i)}{(\ln(Tk_i))^2} = N \textit{ and } n_i = \frac{Tk_i + Tk_i \ln\ln(Tk_i)}{(\ln(Tk_i))^2}$$

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A well known approximation is $W(x) = \ln x - \ln\ln(x) + o(1)$. $\square$

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- A piecewise representation of the complexity function is built, and missing values are interpolated.

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4. A new batch can begin.

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Theorem

Computation of optimal partition is done in $\mathcal{O}(N^2 p)$ time.

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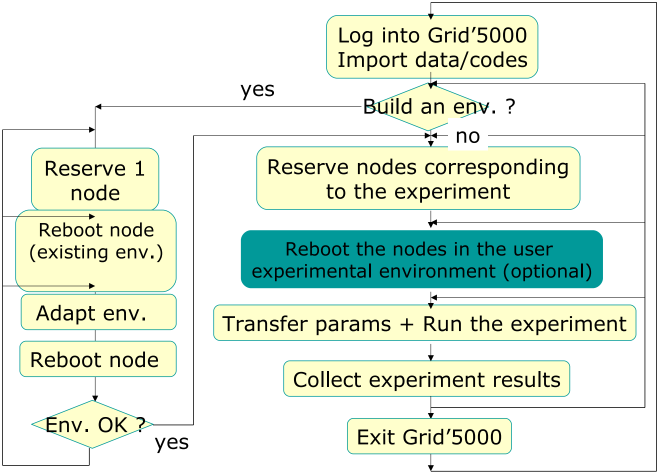
Experiment workflow



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naive algo	partitioning	partitioning (2 threads)
125.4s	112.7s	69.4s

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- Future work
  - Limited bandwidth models and heterogeneous network links.
  - Non-linear computation time models.
  - Global optimisation.