Abstracts

Nicolas Behr

Speaker: Nicolas Behr (Université de Paris, IRIF)
Title: Experimental combinatorics via rule-algebraic methods.
Abstract:
A computational approach to combinatorics will be presented that is based upon techniques from the realm of rewriting theories over finitary adhesive and extensive categories. In the scenario of a given combinatorial structure being specified in terms of a generator and an initial configuration, the approach permits to perform ”experiments” on the combinatorial structure in the sense that equations describing pattern count distributions of various kinds may be derived in a principled manner. The core mathematical structure of our methodology is the so-called rule algebra, an associative unital algebra encoding sequential compositions of rewriting steps. The representation theory of the rule algebras as well as elements of the stochastic mechanics framework for rewriting-based continuous-time Markov chains form additional building blocks of our framework, revealing en passant close conceptual relationships between problems in combinatorics and in dynamical systems. After providing the general theory, some illustrative results for the application example of planar binary rooted trees generated by the so-called Rémy generator will be presented (joint work with N. Zeilberger), including the discovery of a certain form of bisimulation structure for the evolution equations of pattern count distributions.

Marc Bellon

Speaker: Marc Bellon (LPTHE, Sorbonne Université, Paris and CNRS)
Title: Relating Taylor expansions and sums over poles: a source of poyzeta identities.
Abstract:
The evaluation of Feynman diagrams with arbitrary powers of the propagators gives rise to quite special meromorphic functions of many variables. In a the simplest case of the one loop diagram, I show how the Taylor expansion and the sum over poles can be related through a collection of poyzeta identities best expressed in terms of generating functions.

Bérénice Delcroix-Oger

Speaker: Bérénice Delcroix-Oger (Université de Paris, IRIF)
Title: On the (non)-freeness of operads considered as pre-Lie algebras.
Abstract:
Operads can be naturally endowed with a structure of pre-Lie algebra. We present here a work in progress with E. Burgunder and D. Manchon on the freeness of operads endowed with this structure. After introducing the basic notions on operads ans pre-Lie algebras, we will explain why there exists no operads which would be a free pre-Lie algebra. We moreover present a bi-pre-Lie structure on the linear span of planar rooted trees, linked with the magmatic operad, which is not free but generated by only two elements.

Gérard H. E. Duchamp

Speaker: Gérard Duchamp (IHP and LIPN, Univ. Paris XIII)
Title: A localization principle for the Basic Triangle Theorem.
Abstract:
The Basic Triangle Theorem, in its classical form, provides a necessary and sufficient condition for coefficients (local coordinates in the group-like case) of solutions of a NCDE (Non Commutative Differential Equation) to be linearly free with respect to a subfield. As this primitive form requires a differential subfield it is, in practice, not easy to handle. In the realm of analytic functions, for instance, one has to cope with germs. In the one of C^∞ functions, it is even worse : a blunt localization kills almost all interesting phenomena. In this talk we give a *ready to use* form of this theorem with applications the polylogarithms and diverse subrings of interest.
This is a joint work with Hoang Ngoc Minh (LIPN, Paris) and Nihar Gargava (EPFL, Lausanne).

Thomas Fernique

Speaker: Thomas Fernique (CNRS and LIPN, Univ. Paris XIII)
Title: Compact packings of spheres.
Abstract:
It is well known that the best way to pack oranges in a (very large) box is to place them on a face-centered cubic lattice (also known as checkerboard), although this has been formally demonstrated only in 1998 (with difficulty). This talk focuses

on what happens when the dimension or number of different spheres change. In particular, so-called compact packings (the term will be defined properly) seem good candidates to maximize density. In this tallk, we propose a survey of the known mathematical results and a discussion of possible applications in chemistry, including self-assembly of supercrystals..

Stéphane Gaubert

Speaker: Stéphane Gaubert (INRIA and CMAP, École Polytechnique)
Title: Tropical convexity, mean payoff games and nonarchimedean convex programming.
Abstract:
Convex sets can be defined over ordered fields with a non-archimedean valuation. Then, tropical convex sets arise as images by the valuation of non-archimedean convex sets. The tropicalization of polyhedra and spectrahedra are of special interest, since they can be described in terms of deterministic and stochastic games with mean payoff. In that way, one gets a correspondence between classes of zero-sum games, with an unsettled complexity, and classes of semilagebraic convex optimization problems over non-archimedean fields. We shall present some applications of this correspondence (including a counter example in linear programming obtained by game methods), and give an overview of the geometry of tropical convex sets, with aspects related to Hilbert's metric geometry and alcoved polyhedra. This is based on work with Allamigeon, Benchimol, Joswig and Skomra, and on recent work with Akian and Vannucci.

Dima Grigoryev

Speaker: Dimitri Grigoryev (CNRS Painlevé Lab, Univ. Lille)
Title: On a tropical version of the jacobian conjecture.
Abstract:
We prove for a tropical rational map that if for any point the convex hull of Jacobian matrices at smooth points in a neighborhood of the point does not contain singular matrices then the map is an isomorphism. We also show that a tropical polynomial map on the plane is an isomorphism if all the Jacobians have the same sign (positive or negative). In addition, for a tropical rational map we prove that if the Jacobians have the same sign and if its preimage is a singleton at least at one regular point then the map is an isomorphism.
This is a joint work with Danylo Radchenko, ETH (Zürich).

Dmitry Gurevich

Speaker: Dmitry Gurevich (Valenciennes University, France)
Title: Quantum determinants.
Abstract:
I'll discuss different approaches to defining analogs of determinants of matrices with entries belonging to some non-commutative algebras. In the first turn I am interested in the so-called quantum matrix algebras, Yangians-like algebras and similar ones, related to braidings (solutions to braid relation). I'll also consider determinants of super-matrices and compare them with Berezinians.

Richard Kerner

Speaker: Richard Kerner (LPTMC, Sorbonne Université, Paris)
Title: $\mathbb{Z}_3$-graded extension of Lorentz-Poincaré Algebra.
Abstract:
We propose a modification of standard QCD description of the colour triplet of quarks by introducing a 12-component colour generalization of Dirac spinor, withbuilt-in $\mathbb{Z}_3$ grading playing an important algebraic role in quark confinement.

In "colour Dirac equations" the SU(3) colour symmetry is entangled with the $\mathbb{Z}_3$-graded generalization of Lorentz symmetry, containing three 6-parameter sectors related by $\mathbb{Z}_3$ maps.

The generalized Lorentz covariance requires simultaneous presence of 12 colour Dirac multiplets which lead to the description of all internal symmetries of quarks: besides SU(3) x SU(2) x U(1), the flavour symmetries and three quark families.

Arthemy Kiselev

Speaker: Arthemy Kiselev (University of Groningen/IHES)
Title: Poisson bracket deformations using (un)oriented graphs: open problems.
Abstract:
We formulate several open problems from the theory of universal - by using the calculus of (un)oriented graphs - infinitesimal symmetries of Poisson brackets on arbitrary finite-dimensional affine manifolds.

- One class of problems concerns the (extremal) combinatorial properties of those unoriented graph cocycles from which all known proper nonlinear deformations are built by the orientation morphism.

- The other class of open problems is about combinatorial and topological properties of visibly non-generic oriented graphs that arise from the corresponding Poisson cocycle factorization equations, and about the newly discovered topological identities in the spaces of Leibniz graphs which balance such equations.

- Thirdly, an open question is whether (or why not) the universal flows are nontrivial in the Poisson cohomology modulo improper terms (which vanish by force of the Jacobi identity).

- Independently, the action of known symmetries on classes of "nearly generic" Poisson brackets yields a set of open problems from the standard geometry of PDE and integrable systems.

Based on the recent work [arXiv:1910.05844] (cf. references therein), the talk is aimed to provide an overview of these research topics.

Maxim Kontsevich

Speaker: Maxim Kontsevich (IHES)
Title: Duality of hypergeometric functions.

Gleb Koshevoy

Speaker: Gleb Koshevoy (ISCP, Moscow)
Title: Schur positivity, cluster monomials and lattice polytopes.

Léon Mazurel

Speaker: Léon Mazurel (LPTMC, Sorbonne Université, Paris)
Title: Evolution model of cell interactions.
(joint work with
Abstract:
In this talk we present some results concerning cellular interactions within a pathogenic dynamics governed by integro-differential evolution equations in the space of probability densities.

Vincel Hoang Ngoc Minh

Speaker: Vincel Hoang Ngoc Minh (Univ. Lille and LIPN-Paris XIII)
Title: Towards a noncommutative Picard-Vessiot theory
(with simple applications).
Abstract:
We are constructing the first steps of a noncommutative Picard-Vessiot theory and illustrate this theory with the study of independences of a family of eulerian Gamma functions.

Gabriel Morgado

Speaker: Gabriel Morgado (LPTMC, Sorbonne Université, Paris)
Title: Wave fronts with cross-diffusion.
(joint work with
Abstract:
In this talk, we present some results concerning the propagation of a chemical wave front ruled out by partial differential (evolution) equations in the space of concentrations.

Frédéric Patras

Speaker: Frédéric Patras (LJAD, Univ. Côte d'Azur and CNRS)
Title: Operads of partitions, interacting bialgebras, and planar QFT.
(joint work with Kurusch Ebrahimi-Fard, Loic Foissy and Joachim Kock.)
Abstract:
There are (at least) two approaches to relations between full and connected Green's functions in planar QFT: a combinatorial one, based on the lattice of noncrossing partitions, an algebraic one, based on shuffle products, exponentials and logarithms. We establish and explore a relationship between two approaches. We achieve this by exhibiting two operad structures on (noncrossing) partitions, different in nature: one is an ordinary, non-symmetric operad whose composition law is given by insertion into gaps between elements, the other is a coloured, symmetric operad with composition law expressing refinement of blocks. We show that these operad structures interact so as to make the corresponding incidence bialgebra of the former a comodule bialgebra for the latter. Furthermore, this interaction is compatible with the shuffle structure and thus unveils how the two approaches

are intertwined. Moreover, the constructions and results are general enough to extend to ordinary set partitions.

Karol A. Penson

Speaker: Karol A. Penson (LPTMC, Sorbonne Université, Paris)
Title: Lévy-Stable Distributions: Mathematical Properties and Explicit Representations.
(joint work with K. Gorska and A. Horzela, both at IFJ, PAN, Cracow and G. Dattoli at ENEA, Frascati, Roma.)
Abstract:
We shall present a concise overview of salient properties of one-dimensional Lévy-stable probability distributions W(x). We will focus on their symmetry, asymptotics, unimodality, unicity and convolution properties, in the so-called one-sided $(x > 0)$ case. The combined use of integral transforms (Laplace, Fourier, Mellin) will allow one to produce their explicit representations for rational values of parameters involved. An extensive use of computer algebra systems permits to visualise these distributions graphically. A path will be traced towards the use of Lévy-stable distributions in statistical theories involving fractional derivatives and pseudodifferential operators.
References - K. Gorska, K. A. Penson et al.: PRL 105, 210604 (2010), Phys. Rev. E 83, 061125 (2011), JMP 53 053302 (2012),
J.Phys. A 49, 065201 (2016), Ann. Phys. (Berlin) 1700374 (2017), JMP 58, 063510 (2017).

Vincent Rivasseau

Speaker: Vincent Rivasseau (Laboratoire de Physique Théorique, Université de Paris-Sud, 91400 Orsay.)
Title: Constructive Matrix Theory for Single Trace Interactions.
(joint work with T. Krajewski and V. Sazonov.)
Abstract:
We review a new method (loop vertex representation) which proves a uniform (in N) domain of analyticity (in the coupling constant) of the free energy of N by N matrix models with interaction $Tr(M^{2p})$ and p any integer. Previously only the quartic case (p=2) was known.

Alan Sokal

Speaker: Alan Sokal (University College London and New York University)
Title: Coefficientwise Hankel-total positivity.
(joint work with Mathias Pétréolle and Bao-Xuan Zhu)

A matrix M of real numbers is called totally positive if every minor of M is nonnegative. Gantmakher and Krein showed in 1937 that a Hankel matrix $H = (a_{i+j})_{i,j\geq 0}$ of real numbers is totally positive if and only if the underlying sequence $(a_n)_{n\geq 0}$ is a Stieltjes moment sequence. Moreover, this holds if and only if the ordinary generating function $\sum_{n=0}^{\infty} a_n t^n$ can be expanded as a Stieltjes-type continued fraction with nonnegative coefficients:

$$\sum_{n=0}^{\infty} a_n t^n \;=\; \cfrac{\alpha_0}{1 - \cfrac{\alpha_1 t}{1 - \cfrac{\alpha_2 t}{1 - \cfrac{\alpha_3 t}{1 - \cdots}}}}$$

(in the sense of formal power series) with all $\alpha_i \geq 0$. So, totally positive Hankel matrices are closely connected with the Stieltjes moment problem and with continued fractions.

Here I will introduce a generalization: a matrix M of polynomials (in some set of indeterminates) will be called coefficientwise totally positive if every minor of M is a polynomial with nonnegative coefficients. And a sequence $(a_n)_{n\geq 0}$ of polynomials will be called coefficientwise Hankel-totally positive if the Hankel matrix $H = (a_{i+j})_{i,j\geq 0}$ associated to (a_n) is coefficientwise totally positive. It turns out that many sequences of polynomials arising naturally in enumerative combinatorics are (empirically) coefficientwise Hankel-totally positive. In some cases this can be proven using continued fractions, by either combinatorial or algebraic methods; I will sketch how this is done. In many other cases it remains an open problem.

One of the more recent advances in this research is perhaps of independent interest: we have found branched continued fractions for ratios of contiguous hypergeometric series ${}_rF_s$ for arbitrary r and s, which generalize Gauss' continued fraction for ratios of contiguous ${}_2F_1$. For the cases $s = 0$ we can use these branched continued fractions to prove coefficientwise Hankel-total positivity.

One conjectured but unproven example of coefficientwise Hankel-total positivity concerns the inversion enumerators for trees $I_n(y)$, which are closely related

to the generating polynomials of connected graphs $C_n(v)$, and thus to the Potts model on the complete graph in the limit $q \to 0$.
Reference: Mathias Pétréolle, Alan D. Sokal and Bao-Xuan Zhu, arXiv:1807.03271

Pierre Vanhove

Speaker: Pierre Vanhove (IPhT CEA/Saclay and HSE)
Title: The Calabi-Yau geometry of the sunset Feynman graphs.
(joint work with Charles Doran and Andrey Novoseltsev)
Abstract:
In this talk we will discuss the algebraic and transcendental features of the computation of multiloop sunset Feynman integrals.

Starting from the realization of arbitrary Feynman graph hypersurfaces as (generalized) determinantal varieties, we describe the Calabi-Yau sub-varieties of permutohedral varieties that arise from the multiloop sunset Feynman graphs and some key features of their geometry and moduli.

We will explain how the "creative telescoping" algorithm allows to derive the inhomogeneous differential equation when the standard Griffiths-Dwork algorithm fails.
We will show how the results can be understood by understanding the geometry and the moduli of the Calabi-Yau varieties of the sunset graph. In particular the how specialization of physical parameters leads to rank jump.
We will explain the realization of Calabi-Yau pencils as Landau-Ginzburg models mirror to weak Fano varieties.