Combinatorics and Arithmetic for Physics
IHES, 30 Nov. to 02 Dec. 2021

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Abstracts.–

Cyril Banderier

Speaker: Cyril Banderier (CNRS & LIPN, Paris-North University)
Title: Enumeration and generation of Young tableaux with walls: the density method.
Abstract:
We consider a generalization of Young tableaux in which we allow some consecutive pairs of cells with decreasing labels, conveniently visualized by a "wall" between the corresponding cells. This leads to new classes of recurrences, and to a surprisingly rich zoo of generating functions (algebraic, hypergeometric, D-finite, differentially-algebraic). Some patterns lead to nice bijections with trees, lattice paths, or permutations. Our approach relies on the density method, a powerful way to perform both uniform random generation and enumeration. It finds its origins in number theory (values of the zeta function, with a Kontsevich-Zagier period point of view) and in poset theory (volume of polytopes). We also apply this approach to describe the asymptotic fluctuations of the limit surface of Young tableaux.
Based on several articles with Philippe Marchal and Michael Wallner.

Nicolas Behr

Speaker: Nicolas Behr (CNRS, Université de Paris, IRIF)
Title: Towards Executable Applied Category Theory in Coq.
Abstract:
This talk will present the "coreact.wiki" initiative, which aims to develop a novel form of wiki engine that will couple a database of human-readable mathematical knowledge with a database containing machine-readable and -executable representations of this knowledge in proof assistants such as Coq. For the concrete example of analytic combinatorics à la Flajolet and Sedgewick, I will provide an overview of the types of statements that can be efficiently formalized in Coq at

present and in the near future, and how we plan to provide an interactive web-based interface to the "coreact.wiki" platform based upon jsCoq to permit computations and formal proofs in a user-friendly fashion. Time permitting, I will also sketch the possibility of extracting prototypical reference algorithms from formalized categorical statements in Coq via the use of SMT solvers.

Thomas Fernique

Speaker: Thomas Fernique (CNRS & LIPN, Paris-North University)
Title: Maximally dense sphere packings.
Abstract:
It is well known that to cover the greatest proportion of the Euclidean plane with identical disks, we have to center these disks in a triangular grid. This problem can be generalized in two directions: in higher dimensions or with different sizes of disks. The first direction has been the most studied (for example, in dimension 3, the Kepler's conjecture was proved by Hales and Ferguson in 1998). In this talk, we will rather focus on the second direction, in particular on the cases of two or three disc sizes. We will survey recent results for a large audience.

Stéphane Gaubert

Speaker: Stéphane Gaubert (INRIA and CMAP Ecole Polytechnique)
Title: Ambitropical convexity, mean payoff games and nonarchimedean convex programming.
Abstract:
Convex sets can be defined over ordered fields with a non-archimedean valuation. Then, tropical convex sets arise as images by the valuation of non-archimedean convex sets. The tropicalization of polyhedra and spectrahedra can be described in terms of deterministic and stochastic games with mean payoff, being characterized in terms of sub or super-fixed point sets of Shapley operators, which determine the value of the game. This is motivated by open complexity issues in linear programming. We shall discuss here especially a generalization of tropical convexity: considering fixed point sets of Shapley operators, instead of sub or super-fixed points sets, leads to a richer "ambitropical" theory, which includes tropical convexity and its dual in a unified framework. We shall present several characterizations of ambitropical convex sets, with features related to normed spaces (nonexpansive retracts and hyperconvexity), lattice theory (order preserving retracts), or of a combinatorial nature (cell decompositions in alcoved polyhedra).

The results on ambitropical convexity is from a work with Akian and Vannucci; the ones on the tropicalization of nonarchimedean convex sets are from works with Allamigeon, Benchimol, Joswig and Skomra.

Volker Genz

Speaker: Volker Genz (IBS CGP)
Title: Crystal operators on Cluster Algebras.
Abstract:
Crystal operators on canonical bases as introduced by Kashiwara/Lusztig provide in particular a toolbox to compute within the category of finite dimensional representations of finite dimensional simple Lie algebras. Motivated by this we introduce certain operators on the lattice of tropical points of mirror dual A- and X-cluster spaces. In particular, this yields a crystal-like structure on the canonical basis due to Gross-Hacking-Keel-Kontsevich. We expect these operators to have a wider range of applications in the theory of cluster algebras and in physics. This is partially based on joint work with Gleb Koshevoy and Bea Schumann.

Vasily Golyshev

Speaker: Vasily Golyshev (IITP RAS Moscow, IHES)
Title: Modularity of conifold fibers in some hypergeometric families.
Abstract:
I will explain how the computational technique of fibered motives can be used to obtain modularity proofs for certain conifold fibers in Calabi-Yau families (joint with Don Zagier, and with Kilian Bönisch and Albrecht Klemm).

Dimitri Grigoryev

Speaker: Dimitri Grigoryev (CNRS Painlevé Lab, Univ. Lille)
Title: A tropical version of Hilbert polynomial.
Abstract:
We define Hilbert function of a semiring ideal of tropical polynomials in n variables. For $n = 1$ we prove that it is the sum of a linear function and a periodic function (for sufficiently large values). The leading coefficient of the linear function equals the tropical entropy of the ideal. For an arbitrary n we discuss a conjecture that the tropical Hilbert function of a radical ideal is a polynomial of degree at most $n - 1$ (for sufficiently large values). For $n = 1$ the conjecture is true, also we have proved it for zero- dimensional ideals and for planar tropical curves.

Darij Grinberg

Speaker: D. Grinberg (Drexel University, Philadelphia, PA, USA.)
Title: Noncommutative Birational Rowmotion on Rectangles.
Abstract:
The operation of birational rowmotion on a finite poset has been a mainstay in

dynamical algebraic combinatorics for the last 8 years.
Since 2015, it is known that for a rectangular poset of the form $[p] \times [q]$, this operation is periodic with period $p + q$. (This result, as has been observed by Max Glick, is equivalent to Zamolodchikov's periodicity conjecture in type AA, proved by Volkov.)
In this talk, I will outline a proof (joint work with Tom Roby) of a noncommutative generalization of this result. The generalization does not quite extend to the full generality one could hope for it covers noncommutative rings, but not semirings; however, the proof is novel and simpler than the original commutative one. Extending this to semirings and to other posets is work in progress.

Dimitry Gurevich

Speaker: Dimitry Gurevich (Valenciennes University, France)
Title: q-cut-and-join operators and q-Capelli identity on Reflection Equation algebras
Abstract:
There exists a way, based on the notion of Quantum Doubles, to introduce analogs of partial derivatives on the so-called Reflection Equation algebras. Analogously to the classical case it is possible to use these "q-derivatives" for different applications. I plan to explain their utility for constructing q-analogs of the Casimir operators, close to them cut-and-join operators, and the Capelli identity.

Richard Kerner

Speaker: Richard Kerner (LPTMC, Sorbonne-Université & CNRS URA 7600)
Title: Geometry, Matter and Physics.
Abstract:
We show how the fundamental statistical properties of quantum fields combined with the superposition principle lead to continuous symmetries including the $SL(2, \mathbf{C})$ group and the internal symmetry groups $SU(2)$ and $SU(3)$. The exact colour symmetry is related to ternary Z_3-graded generalization of the fermionic commutation relations for quarks. A Z_3-graded generalization of the Dirac equation is presented, and its invariance properties are analyzed. They lead to an enlarged Z_3-graded Lorentz group, operating in the Hilbert space of quark states including flavors and generations.

Maxim Kontsevich

Speaker: Maxim Kontsevich (IHES)
Title: An update on algebraic hypergeometric series.
Abstract:
Algebraic hypergeometric series in one variable were classified in 1989 by F.Beukers and G.Heckman, in terms of finite complex reflection groups. Recently, K.Penson observed that one of such series is a generating series of a probability density with compact support, given again by an algebraic function. Then together with N.Behr, G.Duchamp and G.Koshevoy, we found that this is a general phenomenon. The proof is an immediate application on an explicit integral by Bateman and Erdélyi.The probability density is a so called Meijer's G-function, which is the unique solution of the hypergeometric differential equation with the pure ramification at point 1. I will speak about it, and also on the genus zero property of the corresponding planar algebraic curve.

Paul-André Melliès

Speaker: Paul-André Melliès, (CNRS, Université de Paris)
Title: A functorial excursion between algebraic geometry and linear logic.
Abstract:
In this talk, I will use the functor of points approach to Algebraic Geometry to establish that every covariant presheaf X on the category of commutative rings — and in particular every scheme X — comes equipped "above it" with a symmetric monoidal closed category PshModX of presheaves of modules. This category PshModX defines moreover a model of intuitionistic linear logic, whose exponential modality is obtained by glueing together in an appropriate way the Sweedler dual construction on ring algebras. The purpose of this work is to explore the idea that linear logic is a logic of generalised vector bundles, in the same way as dependent type theory is understood today as a logic of spaces up to homotopy.

Hoang Ngoc Minh

Speaker: Hoang Ngoc Minh (LIPN, Paris-North University)
Title: On the solutions of Knizhnik-Zamolodchikov differential equations by noncommutative Picard-Vessiot theory.
Abstract:
In this talk, basing on the algebraic combinatorics on noncommutative formal power series with holomorphic coefficients and, on the other hand, a Picard-Vessiot theory of noncommutative differential equations, we give a recursive construction of solutions of the Knizhnik-Zamolodchikov equations satisfying asymptotic conditions.

Sergei Nechaev

Speaker: Sergei Nechaev (Interdisciplinary Scientific Center Poncelet (CNRS IRL 2615, Moscow, Russia))
Title: Anomalous statistics of extreme random processes.
Abstract:
I plan to discuss three problems of extremal statistics in which unusual (but related to each other) features arise:
a) statistics of two-dimensional "stretched" random walks above a semicircle,
b) spectral properties of sparse random matrices,
c) statistics of one-dimensional paths in the Poissonian field of traps. I will pay attention to the relationship of these problems with the Anderson localization in 1D, and with some number-theoretic properties of eta-Dedekind function.

Frédéric Patras

Speaker: Frédéric Patras (CNRS/Université Côte d'Azur)
Title: Noncommutative Wick polynomials.
Abstract:
Wick polynomials are at the foundations of QFT (they encode normal orderings) and probability (they encode chaos decompositions). In this lecture, we survey the construction and properties of noncommutative (or free) analogs using shuffle Hopf algebra techniques. Based on joint works with K. Ebrahimi-Fard, N. Tapia and L. Zambotti.

Lionel Pournin

Speaker: Lionel Pournin (LIPN, Paris-North University)
Title: Distance, strong convexity, flagness, and associahedra.
Abstract:
One can always transform a triangulation of a convex polygon into another by performing a sequence of edge flips, which amounts to follow a path in the graph G of the associahedron. The least number of flips required to do so is then a distance in that graph whose estimation is instrumental in a variety of contexts, as for instance in computational biology, in computer science, or in algebraic topology. On the other hand, it is known that paths in G correspond to a certain kind of 3-dimensional triangulation. This talk is about the recent proof that these 3-dimensional triangulations are flag when the corresponding path is a geodesic. This result, that provides a new powerful tool to study the geometry of G, can be thought of as a 3-dimensional analogue of a well-known strong convexity property of G. Several consequences on the computation of distances in G and on strong convexity in related graphs will be discussed. This talk is based on joint work with Zili Wang (Dartmouth College).

Léonard Ferdinand and Vincent Rivasseau

Speakers: Léonard Ferdinand and Vincent Rivasseau (Laboratoire de physique des deux infinis Irène Joliot-Curie, Université Paris-Saclay)
Title: Some new Taylor-BKAR formulas.
Abstract:
We here introduce some combinatorial and analytic tools, conceived to make possible to perform new expansions in the context of constructive field theory and multiscale analysis. These formulas generalize the idea of performing cluster expansion using a sum indexed by forest to the case of a Taylor expansion of order more than zero. They are expected to help construct new field theories of the matrices and tensors type.

Bea Schumann

Speaker: Bea Schumann (University of Cologne)
Title: String cones and cluster varieties.
Abstract:
We study defining inequalities of string cones via a potential function on a reduced double Bruhat cell. We give a necessary criterion for the potential function to provide a minimal set of inequalities via tropicalization and conjecture an equivalence. This is based on joint work with Gleb Koshevoy.

Alexandros Singh

Speaker: Alexandros Singh (LIPN, Paris-North University)
Title: Asymptotic Distribution of Parameters in Trivalent Maps and Linear Lambda Terms.
Abstract:
Structural properties of large random maps and lambda-terms may be gleaned by studying the limit distributions of various parameters of interest. In our work we focus on restricted classes of maps and their counterparts in the lambda-calculus, building on recent bijective connections between these two domains. In such cases, parameters in maps naturally correspond to parameters in lambda-terms and vice versa. By an interplay between lambda-terms and maps, we obtain various combinatorial specifications which allow us to access the distributions of pairs of related parameters such as: the number of bridges in rooted trivalent maps and of subterms in closed linear lambda-terms, the number of vertices of degree 1 in (1,3)-valent maps and of free variables in open linear lambda-terms etc. To analyse asymptotically these distributions, we introduce appropriate tools: a moment-pumping schema for differential equations and a composition schema inspired by Bender's theorem.
Joint work with Olivier Bodini and Noam Zeilberger.

Jim Propp

Speaker: Jim Propp (UMass Lowell)
Title: Overview of tensor models.
Abstract:
The work of Conway and Lagarias applying combinatorial group theory to packing problems suggests what we might mean by "domain-wall boundary conditions" for the trimer model on the infinite triangular lattice in which the permitted trimers are triangle trimers and three-in-a-line trimers. Looking at subregions of the lattice with those sorts of boundaries, we find intriguing numerology governing the number of trimer covers. This wealth of conjecture is in stark contrast with the paucity of mathematical tools that permit exact enumeration of trimer covers as compared to dimer covers.

Sergey Yurkevich

Speaker: Sergey Yurkevich (University of Vienna & Inria, Saclay.)
Title: How to conjecture and prove that the generating function of the Yang-Zagier numbers is algebraic.
Abstract:
In a recent paper Don Zagier mentions a mysterious integer sequence $(a_n)_{n\geq 0}$ which arises from a solution of a topological ODE discovered by Marco Bertola, Boris Dubrovin and Di Yang. In my talk I show how to conjecture, prove and even quantify that $(a_n)_{n\geq 0}$ actually admits an algebraic generating function which is therefore a very particular period. The methods are based on experimental mathematics and algorithmic ideas in differential Galois theory, which I will show in the interactive part of the talk. The presentation is based on joint work with A. Bostan and J.-A. Weil.