

Combinatorics and Arithmetic for Physics

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Abstracts.–

Cyril Banderier

Speaker: Cyril Banderier (CNRS/ Univ. Sorbonne Paris Nord)

Title: Phase transitions of composition schemes and their universal limit laws.

Abstract:

Multitudinous combinatorial structures are counted by generating functions satisfying a composition scheme $F(z) = G(H(z))$.

The corresponding asymptotic analysis becomes challenging when this scheme is critical (i.e., G and H are simultaneously singular). The singular exponents appearing in the Puiseux expansions of G and H then dictate the asymptotics.

Motivated by many examples (random mappings, planar maps, directed lattice paths), we consider a natural extension of this scheme, namely $F(z, u) = G(uH(z))M(z)$. We also consider a variant of this scheme, which allows us to analyse the number of H -components of a given size in F .

These two models lead to a rich world of limit laws, involving Mittag-Leffler distributions, stable distributions... We prove (double) phase transitions, additionally involving Boltzmann and mixed Poisson distributions, with a unified explanation of the associated thresholds. We explain why and when phase transitions involving a window of size $n^{1/3}$ are universal.

Applications are presented for random walks, trees (supertrees of trees, increasingly labelled trees, preferential attachment trees), and for some extension of works of Flajolet (on the Airy distribution for planar maps), of Pitman (on the Chinese restaurant process), of Janson (on triangular Pólya urns) ...

(Joint work with Markus Kuba and Michael Wallner.)

Keywords: analytic combinatorics, phase transitions, Mittag-Leffler distributions, stable laws, Boltzmann distributions.

Domains (Mathematics Subject Classification (MSC2020)):

60C05 (Combinatorial probability), 60E07 (Infinitely divisible distributions; stable distributions), 60G50 (Sums of independent random variables; random walks), 05A15 (Exact enumeration problems, generating functions).

Nicolas Behr

Speaker: Nicolas Behr (CNRS, Université Paris Cité, IRIF)

Title: Categorification of Rule Algebras.

Abstract:

Reporting on joint work in progress with P.-A. Melliès and N. Zeilberger, I will present a novel approach to formalize operations in compositional rewriting systems wherein the number of ways to apply a rewrite is of interest. The approach is based upon defining a suitable double category to capture individual rewriting steps as its 2-cells, requiring in addition certain fibrational properties to hold for the functors of vertical source and target as well as of horizontal composition of cells. Counting numbers of realizations of individual rewriting steps or sequences rewrites is then implemented via a presheaf calculus over 2-cells. I will demonstrate how the notion of rule algebra representations is captured in this calculus and how the rule algebras themselves are categorified via a categorical construction involving coends.

Gérard H. E. Duchamp

Speaker: Gérard H. E. Duchamp (Paris-North University, LIPN (UMR 7030), France)

Title: Strange gradings and elimination of generators.

Abstract: Elimination of generators (commutative or noncommutative) is linked to many combinatorial theories (Bisection and codes, Semidirect products of presented groups and Lie algebras, Strange gradings over semigroups, Lazard elimination). We will describe unifying (categorical) links between some of these theories and give general results allowing to get semidirect decompositions at first sight.

(Joint work with Paul-André Melliès and Vu Nguyen Dinh.)

Vladimir Fock

Speaker: Vladimir Fock (Strasbourg University).

Title: Cluster duality and non-holomorphic spectral curves.

Abstract:

Cluster duality is a correspondence between tropical points of a cluster A -variety and a canonical basis of functions on the corresponding X -variety. (It is a generalization of duality between integers and the multiplicative group.) In the talk we will suggest related geometric interpretations of the tropical points of the A -variety for local system of the curve. On one hand it can be considered as a class of graphs on the surfaces colored by generators of the affine Weyl group. This is a generalization of the notion of a measured lamina. On the other hand it can be interpreted as a class of Lagrangian coverings in the cotangent bundle to

the curve representing integer classes of homology. Finally they are related to the "cells" of the space of local system on the curve with values in the affine group.
(Joint with A.Thomas and V.Tatitscheff)

Keywords: Cluster varieties, Character varieties.

Nihar Gargava

Speaker: Nihar Gargava (EPFL, Switzerland).

Title: Random lattices as sphere packings.

Abstract:

In 1945, Siegel showed that the expected value of the lattice-sums of a function over all the lattices of unit covolume in an n -dimensional real vector space is equal to the integral of the function. In 2012, Venkatesh restricted the lattice-sum function to a collection of lattices that had a cyclic group of symmetries and proved a similar mean value theorem. Using this approach, new lower bounds on the most optimal sphere packing density in n dimensions were established for infinitely many n .

In the talk, we will outline some analogues of Siegel's mean value theorem over lattices. This approach has modestly improved some of the best known lattice packing bounds in many dimensions. We will also show how such results can be made effective and talk of some variations.

(Joint work with Vlad Serban.)

Keywords: Sphere packing problem, Codes, Division rings, Lattices

MSC: 11H06, 52C07.

Volker Genz

Speaker: Volker Genz (IBS center for Geometry and Physics)

Title: Reddening sequences for Cluster Algebras.

Abstract: While cluster algebras generally are not finitely generated, reddening sequences offer a more relaxed notion of finiteness. The existence of a reddening sequence has far reaching consequences for a cluster algebra (generic finite dimensionality of the Jacobian, numeric Donaldson-Thomas invariants, canonical bases). While it is not clear how to determine if a cluster algebra admits a reddening sequence, in this talk we discuss some cases in which reddening sequences have been established.

Darij Grinberg

Speaker: Darij Grinberg (Drexel University)

Title: The one-sided cycle shuffles in the symmetric group algebra.

Abstract: We study a new family of elements in the group ring of a symmetric group – or, equivalently, a class of ways to shuffle a deck of cards.

Fix a positive integer n . Consider the symmetric group S_n . For each $1 \leq \ell \leq n$, we define an element

$$t_\ell := \text{cyc}_\ell + \text{cyc}_{\ell, \ell+1} + \text{cyc}_{\ell, \ell+1, \ell+2} + \cdots + \text{cyc}_{\ell, \ell+1, \dots, n}$$

of the group ring $\mathbb{R}[S_n]$, where $\text{cyc}_{i_1, i_2, \dots, i_k}$ denotes the cycle that rotates through the given elements i_1, i_2, \dots, i_k . We refer to these n elements t_1, t_2, \dots, t_n as the somewhere-to-below shuffles, since the standard interpretation of elements of $\mathbb{R}[S_n]$ in terms of card shuffling allows us to view them as shuffling operators. Note that t_1 is the well-known top-to-random shuffle studied by Diaconis, Fill, Pitman and others, whereas $t_n = \text{id}$.

Similar families of elements of $\mathbb{R}[S_n]$ include the Young-Jucys-Murphy elements, the Reiner-Saliola-Welker elements, and the Diaconis-Fill-Pitman elements. Unlike the latter three families, the somewhere-to-below shuffles t_1, t_2, \dots, t_n do not commute. However, they come close to commuting: There is a basis of $\mathbb{R}[S_n]$ on which they all act as upper-triangular matrices; thus, they generate an algebra whose semisimple quotient is commutative (which entails, in particular, that their commutators are nilpotent).

This basis can in fact be constructed combinatorially, and bears several unexpected connections, most strikingly to the Fibonacci sequence. One of the consequences is that any \mathbb{R} -linear combination $\lambda_1 t_1 + \lambda_2 t_2 + \cdots + \lambda_n t_n$ (with $\lambda_1, \lambda_2, \dots, \lambda_n \in \mathbb{R}$) can be triangularized and its eigenvalues explicitly computed (along with their multiplicities); the number of distinct eigenvalues is at most the Fibonacci number f_{n+1} . If all these f_{n+1} eigenvalues are indeed distinct, then the matrix is diagonalizable.

While we have been working over \mathbb{R} for illustrative purposes, all our proofs hold over any commutative ring (or, for the diagonalizability claim, over any field). Several open questions remain (joint work with Nadia Lafrenière).

Keywords: Symmetric group, symmetric group algebra, shuffles, representation theory, probability, Markov chains.

Sasha Gorsky

Speaker: Sasha Gorsky (IITP RAS)

Title: Statistical models on random regular graphs.

Abstract: Using the matrix-forest theorem and the Parisi-Sourlas trick we formulate and solve a one-matrix model with non-polynomial potential which provides perturbation theory for massive spinless fermions on dynamical planar graphs. This is a version of 2d quantum gravity discretized via RRG coupled to massive spinless fermions. Our model equivalently describes the ensemble of spanning forests on the same graph. The solution is formulated in terms of an elliptic curve. We then focus on a near-critical scaling limit when both the graphs and the trees in the forests are macroscopically large. In this limit we obtain universal one-point scaling functions (condensates), parameterized in terms of the Lambert function.

Our results provide a rare example where one can explore the flow between two gravity models – in this case, the theories of conformal matter coupled to 2d gravity with $c=-2$ (large trees regime) and $c=0$ (small trees regime). We shall also present the results of numerical simulations concerning phase transitions in RRG ensemble and their relation with Anderson localization.

Dimitry Gurevich

Speaker: Dimitry Gurevich (IITP, Moscow)

Title: New applications of the Reflection Equation Algebras.

Abstract: The REA are treated to be q -analogs of the enveloping algebras $U(\mathfrak{gl}(N))$. In particular, each of them has a representation category similar to that of $U(\mathfrak{gl}(N))$.

I plan to exhibit new applications of these algebras:

1. q -analog of Schur-Weyl duality
2. q -Capelli formula
3. q -Frobenius formula

Richard Kerner

Speaker: Richard Kerner (LPTMC, Sorbonne-Université - CNRS UMR 7600)

Title: Geometry and physics of covalent network glasses.

Abstract: Glasses are characterized by the absence of long-range order which defines crystalline materials. However, they possess a rich and varied array of short to medium range order, which originates from chemical bonding and related interactions. whereas covalent systems (mostly chalcogenides like As-Se, Ge-As-Se systems) or oxides (borate, boro-silicate and silicate glasses), have sparsely packed, strongly bound network structures, like tetrahedral SiO_2 units or B_3O_3 boroxol rings. These very different structures results in different physical properties and applications.

We present a simple mathematical model of glass transition based on the analysis of molecular agglomeration in overcooled liquids. The model uses the space of probabilities of appearance of given local structures, and their slow time evolution during annealing from a liquid melt. The evolution of probabilities is described as action of an appropriate stochastic matrix. The glass transition is defined as a fixed point resulting from the requirement of maximal homogeneity. With simple assumptions concerning local configurations and their bonding energies, and with elementary combinatorics we are able to derive the dependence of the glass transition temperature T_g on chemical composition in non-organic covalent glasses. Numerous examples are shown to confirm the validity of the stochastic agglomeration model.

Arthemy Kiselev

Speaker: Arthemy Kiselev (University of Groningen, IHÉS)

Title: Kontsevich's star-product up to order seven for affine Poisson brackets, or: Where are the Riemann zeta values?

Abstract:

Noncommutative associative star-products are deformations of the usual product of functions on smooth manifolds; in every star-product, its leading deformation term is a Poisson bracket. Kontsevich's star-products on finite-dimensional affine Poisson manifolds are encoded using weighted graphs with ordering of directed edges. The associativity is then obstructed only by the Jacobiator (and its differential consequences) for the bi-vector which starts the deformation. Finding the real coefficients of graphs in Kontsevich's star-product expansion is hard in practice; conjecturally irrational Riemann zeta values appear from the fifth order onwards.

In a joint work with R. Buring (arXiv:2209.14438 [q-alg]) we obtain the seventh order formula of Kontsevich's star-product for affine Poisson brackets (in particular, for linear brackets on the duals of Lie algebras). We discover that all the graphs near the Riemann "zetas of concern" assimilate into differential consequences of the Jacobi identity, so that all the coefficients in the star-product formula are rational for every affine Poisson bracket. Thirdly, we explore the mechanism of associativity for Kontsevich's star-product for generic or affine Poisson brackets (and with harmonic propagators from the original formula for the graph weights): here, we contrast the work of this mechanism up to order six with the way associativity works in terms of graphs for orders seven and higher.

Keywords: Deformation quantization, Kontsevich's star-product, diagrammatic algebra, affine Poisson bracket, Riemann zeta.

Maxim Kontsevich

Speaker: Maxim Kontsevich (IHÉS)

Title: Introduction to resurgence.

Abstract: I will explain the phenomenon of resurgence in a (apparently) new example related to Stirling formula, and its generalization to quantum dilogarithm. Let us define rational Stirling numbers $(St_k) = (1, 1/12, 1/288, \dots)$ as coefficients in the asymptotic expansion of the normalized factorial:

$n! \sim \sqrt{2\pi n} n^n e^{-n} (1 + \frac{1}{12n} + \frac{1}{288n^2} - \frac{139}{51840n^3} + \dots)$. Then the asymptotic behavior of St_k for large even k is controlled by numbers St_k for small odd k , and vice versa. In the case of quantum dilogarithm one deforms Stirling numbers to Euler polynomials.

Bea de Laporte

Speaker: Bea de Laporte (University of Cologne)

Title: Landau-Ginzburg potentials via projective representations.

Abstract:

Many interesting spaces arise as partial compactifications of Fock-Goncharov's cluster varieties, among them (affine cones over) flag varieties which are important objects in representation theory of algebraic groups. Due to a construction of Gross-Hacking-Keel-Kontsevich those partial compactifications give rise to Landau-Ginzburg potentials on the dual cluster varieties whose tropicalizations define interesting polyhedral cones parametrizing the theta basis on the ring of regular functions on the cluster varieties. In this talk, after explaining the background, we give an interpretation of these Landau-Ginzburg potentials as F-polynomials of projective representations of Jacobian algebras.

This is joint work with Daniel Labardini-Fragoso.

Paul-André Melliès

Speaker: Paul-André Melliès (CNRS, Université Paris Cité, IRIF)

Title: A gentle introduction to template games and linear logic.

Abstract:

Game semantics is the art of interpreting formulas (or types) as games and proofs (or programs) as strategies. In order to reflect the interactive behaviour of programs, strategies are required to follow specific scheduling policies. Typically, in the case of a sequential programming language, the program (Player) and its environment (Opponent) play one after the other, in a strictly alternating way. On the other hand, in the case of a concurrent language, Player and Opponent are allowed to play several moves in a row, in a non alternating way. In the two cases, the scheduling policy is designed very carefully in order to ensure that the strategies synchronise properly and compose well when plugged together. A longstanding conceptual problem has been to understand when and why a given scheduling policy works and is compositional in that sense. In this talk, I will introduce the notion of template game and exhibit a number of simple and fundamental combinatorial properties which ensure that a given scheduling policy defines (indeed) a monoidal closed bicategory of games, strategies and simulations. The notion of template game will be illustrated by constructing two game models of linear logic with different flavours (alternating and asynchronous) using the same categorical combinatorics, performed in the category of small 2-categories.

Frédéric Patras

Speaker: Frédéric Patras (CNRS/Université Côte d'Azur)

Title: Substitutions in non-commutative multivariate power series.

Abstract:

We describe a group law on formal power series in non-commuting variables induced by their interpretation as linear forms on a Hopf algebra of sentences. We study the corresponding structures and show how they can be used to recast in a group theoretic form various identities and transformations on formal power series that have been central in the context of non-commutative probability theory. Based on a joint work with K. Ebrahimi-Fard, N. Tapia and L. Zambotti.

Karol A. Penson

Speaker: Karol A. Penson* (LPTMC, CNRS UMR 7600, Sorbonne Université, Paris, France)

Title: Hausdorff moment problems for combinatorial numbers: heuristics via Meijer G-functions.

Abstract:

We report on further investigations of combinatorial sequences in form of integral ratios of factorials. We conceive these integers as Hausdorff power moments for weights $W(x)$, concentrated on the support $x \in (0, R)$, and we solve this moment problem by furnishing the exact expressions for $W(x)$'s. In many instances we can formally prove that the sequences are positive definite. We considered a large set of families of such sequences including: formulas of Tutte et al. for enumerations of planar maps, several generalizations of Catalan numbers such as Fuss-Catalan and Raney numbers, the constellation numbers, and the ratios of multiple factorials, such as the iconic Kontsevich $(\frac{(6n)!n!}{(3n)![(2n)!]^2})$ and Chebyshev $(\frac{(30n)!n!}{(6n)!(10n)!(15n)!})$ sequences. Furthermore, we provide the exact solutions for all three parametrized families of Bober ratios (2009) of factorials, as well as for the "sporadic" ratios, for all of which the ordinary generating functions (ogf) are algebraic. Finally, in the same spirit, we studied the sequences recently constructed by Rodriguez Villegas (2019-2022), including $\frac{(63n)!(8n)!(2n)!}{n!(4n)!(16n)!(21n)!(31n)!}$. In all the cases listed above we have identified a precisely defined and persistent pattern relating the Meijer G-encodings of appropriate ogf $G(z)$ and of $W(x)$. In fact, it appears that in the language of Meijer G-functions, the solutions $W(x)$ are practically automatically obtained by reshuffling of data characterizing the ogf $G(z)$ only, i.e. the parameter lists and its radius of convergence R^{-1} . We attempt to categorize these observations and try to find the criteria for moments which would allow for such an automatisation. It is also intriguing that the counterexamples can be found, which clearly point to the limits of this procedure. Finding the precise criteria for moments which would permit for such a speedy method, is still a challenging open problem.

* Collaboration at various stages of this work with:

N. Behr, G. H. E. Duchamp, K. Górska, M. Kontsevich, and G. Koshevoy.

Travis Scrimshaw

Speaker: Travis Scrimshaw (Hokkaido University).

Title: Canonical Grothendieck polynomials with free fermions.

Abstract:

A now classical method to construct the Schur functions is constructing matrix elements using half vertex operators associated to the classical boson-fermion correspondence. This construction is known as using free fermions. Schur functions are also known to be polynomial representatives of cohomology classes of Schubert varieties in the Grassmannian. By instead using K-theory, the representatives become the (symmetric) Grothendieck polynomials. A recent generalization was given by Hwang et al. called the (refined) canonical Grothendieck polynomials based on the work of Galashin–Grinberg–Liu and Yeliussizov. In this talk, we take the Jacobi–Trudi formulas of Hwang et al. as our definition and use Wick’s theorem to give a presentation for the canonical Grothendieck polynomials and their dual basis using free fermions. This generalizing the recent work of Iwao. Using this, we derive many known identities, as well as some new ones, through simple computations. This is based on joint work with Shinsuke Iwao and Kohei Motegi (arXiv: 2211.05002).

Keywords: Grothendieck polynomial; free-fermion.

MSC Classification: 05E05, 82B23, 14M15, 05A19

Thomas Simon

Speaker: Thomas Simon (Paul Painlevé Lq. (UMR 8524), Université de Lille)

Title: On the positivity of Meijer G-functions.

Abstract:

I will discuss some results and conjectures on the positivity of Meijer G-functions on the positive half-line, in the non-trivial case where the underlying random variable is not an independent product of quotients of beta random variables. Some emphasis will be put on the notions of logarithmic infinite divisibility and quasi infinite divisibility. If time permits, I will discuss some extensions to Fox H-functions.

Lauren Williams

Speaker: Lauren Williams (Harvard University, Department of Mathematics, Cambridge, USA)

Title: Combinatorics of the amplituhedron.

Abstract:

The amplituhedron is the image of the positive Grassmannian under a map induced by a totally positive matrix. It was introduced by Arkani-Hamed and Trnka to compute scattering amplitudes in $N=4$ super Yang Mills. I’ll give a gentle

introduction to the amplituhedron, surveying its connections to cluster algebras, matroids, and combinatorics (Eulerian numbers, Narayana numbers, etc).

Keywords: Positive Grassmannian, cluster algebra.

Sergey Yurkevich

Speaker: Sergey Yurkevich (University of Vienna & Inria, Saclay)

Title: Creative Telescoping for the Canham model in genus 1.

Abstract:

The algorithmic method of Creative Telescoping turns out to be an extremely useful tool in experimental mathematics, when dealing with concrete mathematical problems. As striking examples, it can be used to compute and prove automatically: a recurrence satisfied by any binomial sum (like the Apéry numbers), the equality of two period functions (in the sense of Kontsevich and Zagier), or a recurrence for the moments of a measure.

In this talk, I will explain some theory behind Creative Telescoping, and show how it can be applied in practice on a problem originating from biological physics. The problem concerns the shape of biomembranes, such as blood cells, and examines the uniqueness of the variational Helfrich problem in the case of genus 1 with a prescribed isoperimetric ratio. This question boils down to computing the surface area and volume of a projection of the Clifford torus in terms of Gaussian hypergeometric functions. We tackle this using Creative Telescoping, and then prove that the rescaled ratio of these functions is monotonically increasing. The talk will be based on joint work with Alin Bostan and Thomas Yu.

Domains: Special functions (33-XX), Differential geometry (53-XX).

Keywords: Symbolic Integration, creative telescoping, period functions, Clifford torus, isoperimetric ratio, hypergeometric functions.