Toward an algebraic characterization of substitutive multidimensional Sturmian sequences

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Limit points of substitutions on words are of special interest in symbolic dynamical systems. Tilings or multidimensional sequences generalize words to higher dimensions, while hierarchical properties are intended to generalize the notion of limit points of a substitution. However, there is no canonical hierarchy for tilings or multidimensional sequences, and the ones defined do not admit characterizations as complete as for limit points of substitutions on words.

Here, we first briefly review existing hierarchies for words, tilings and multidimensional sequences and the corresponding known characterizations. We then focus on particular multidimensional sequences, the *Sturmian* ones, and give a partial algebraic characterization of the ones which are fixed points of a *generalized substitution*. This result is a first step to a multidimensional generalization of a result on words: a Sturmian word of slope α is a fixed point of a substitution if and only if its slope has a periodic continued fraction expansion.