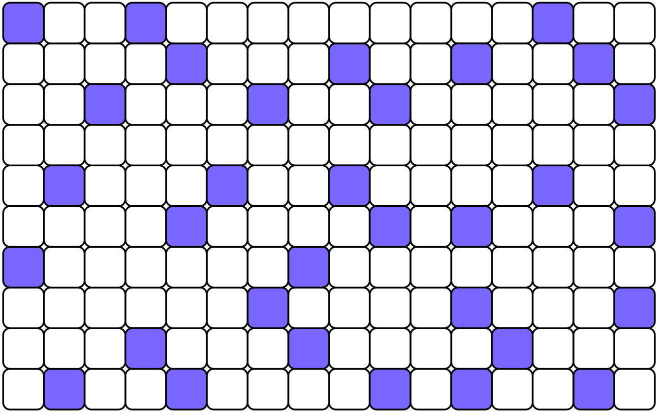
Pavages : que faire ?

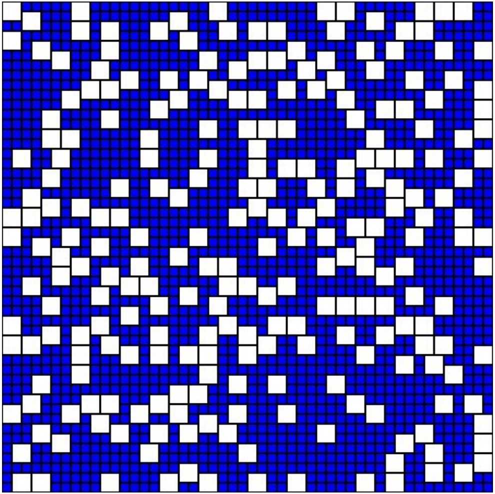
Thomas Fernique (LIPN)

Caen, 4 avril 2013

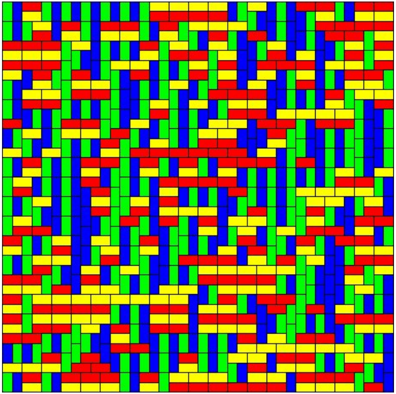
Pavages de Fibonnacci

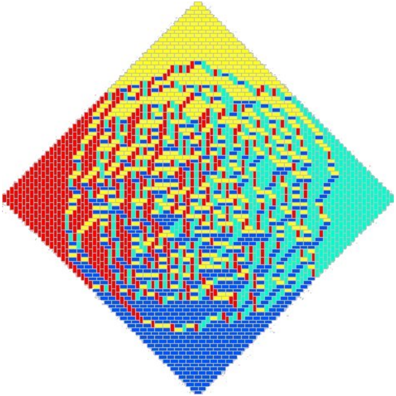


Pavages à grumeaux carrés

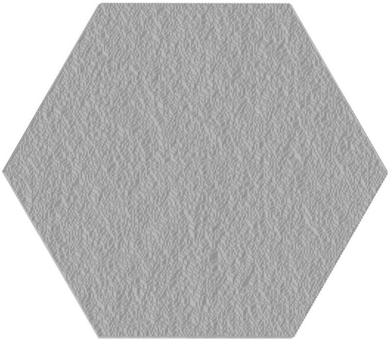


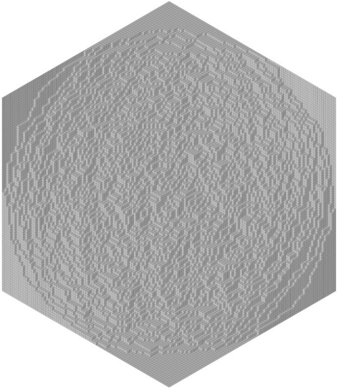
Pavages par dominos





Pavages par losanges





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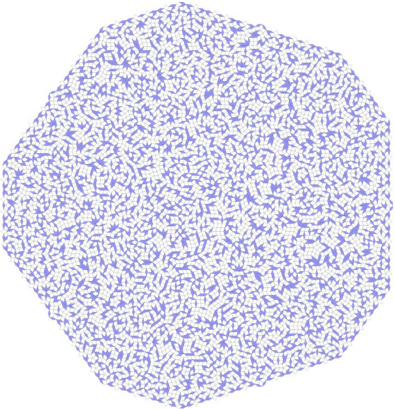

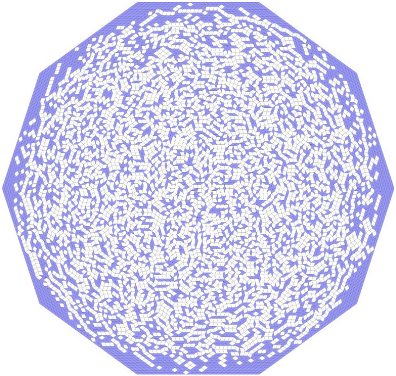

Des questions

- Spécification ?
- Comptage ?
- Entropie ?
- Structure ?
- Génération aléatoire ?
- Propriétés typiques ?
- Dimensions supérieures ?
- . . .

Des idées ?

- Approche par bande de largeur croissante ?
- Encadrements par “combinatoire injective” ?
- Guessing ?
- Couplage “from the past” à base d’enveloppe convexe ?
- ...

Des réponses

Essentiellement pour les dimères :

- Algorithme de shuffle et cercle arctique
- Comptage par déterminants
- Principe variationnel et courbes arctiques
- Couplage “from the past” et temps de mélange
- ...

Algorithme de shuffle et cercle arctique

W. Jockusch, J. Propp, P. Shor, *Random Domino Tilings and the Arctic Circle Theorem*, 1995.

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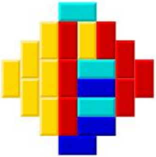

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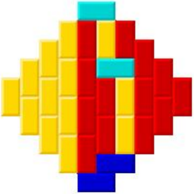

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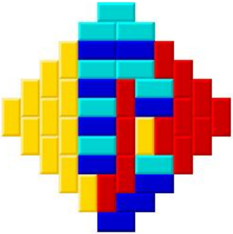

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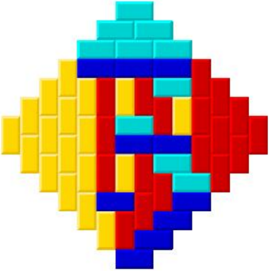

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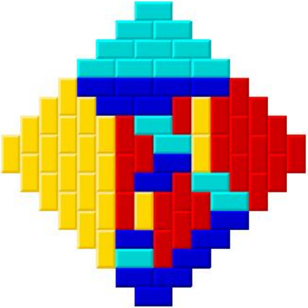

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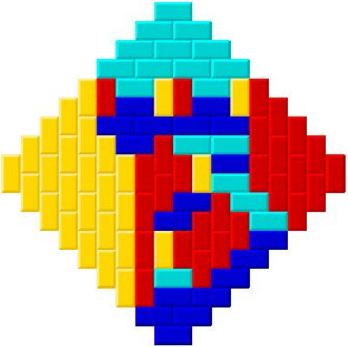

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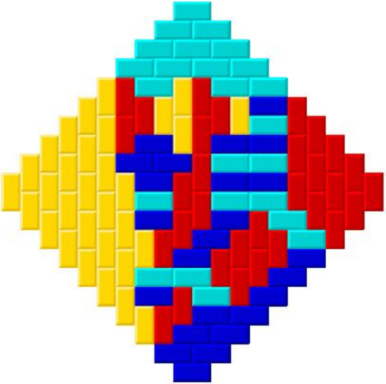

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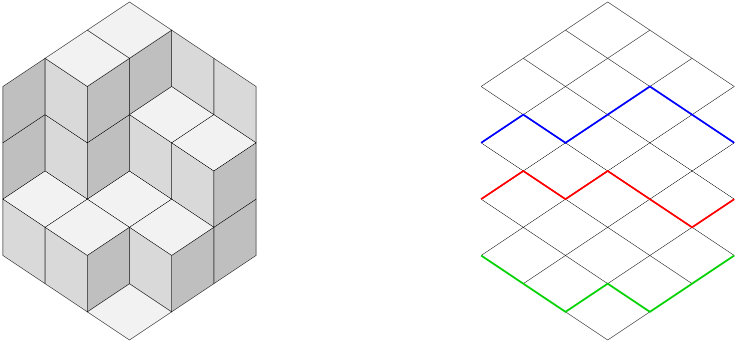
Comptage par déterminants



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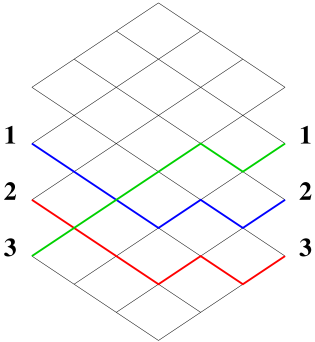

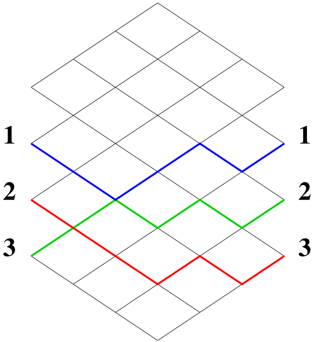

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Principe variationnel et courbes arctiques

Height function of a dimer tiling: distance to $x + y + z = 0$.

Theorem (H. Cohn, R. Kenyon, J. Propp, 2011)

Let $R \subset \mathbb{R}^2$ be bounded by a piecewise smooth simple closed curve. If, for $n \geq 0$, R_n is a tileable domain which approximates nR, then

$$\lim_{n\to\infty} s(R_n) = \sup_h \frac{1}{|R|} \iint_R \mathrm{ent}\left(\frac{\partial h}{\partial x}, \frac{\partial h}{\partial y}\right) dx\, dy,$$

where $\mathrm{ent} : \mathbb{R}^2 \to \mathbb{R}$ and h is any 2-Lipschitz real function on R.

Moreover, the normalized R_n's random height functions converge in probability (exponentially fast) towards the integral-maximizing h.

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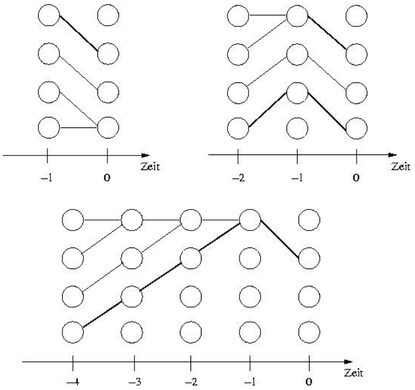
Frozen boundaries: algebraic curves (Kenyon-Okounkov, 2005).

Couplage “from the past” et temps de mélange



J. Propp, D. B. Wilson, *Exact sampling with coupled Markov chains and applications to statistical mechanics*, 1996.