Quasiperiodic Tilings and Quasicrystals

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Université Paris 13

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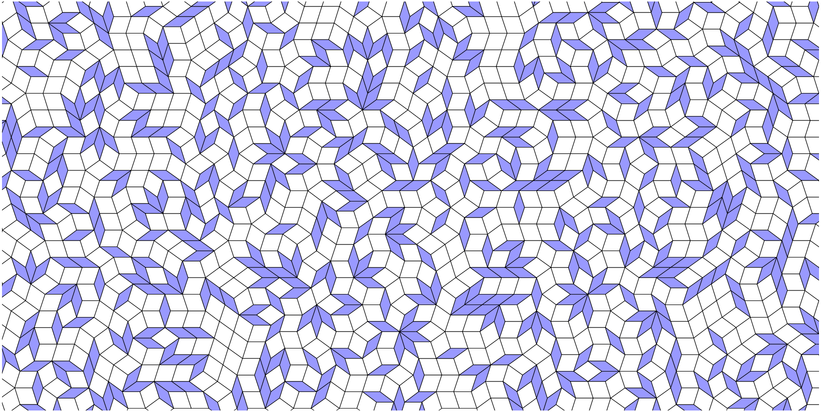
1 Quasiperiodic tilings

2 Local rules

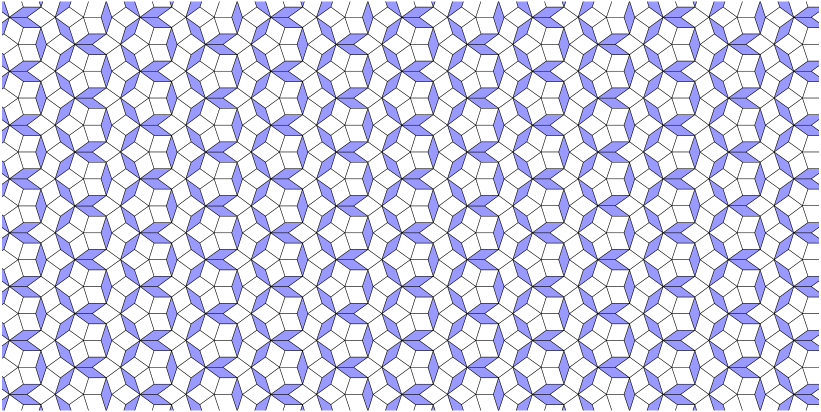
3 Quasicrystals

Tiling



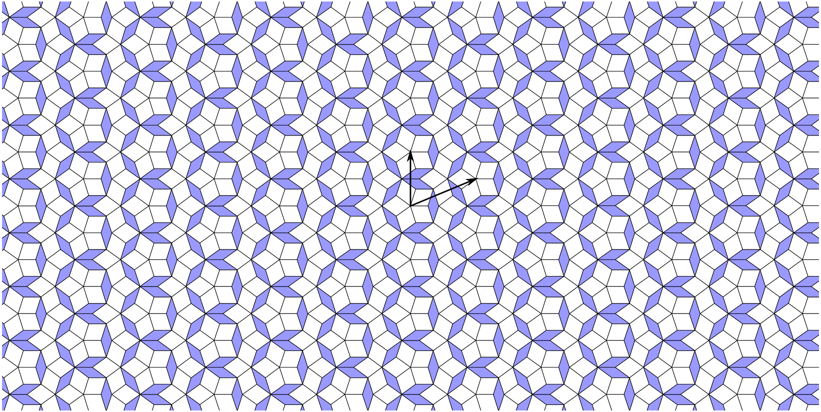
Overlap-free covering of the plane by compact sets called *tiles*.

Periodic tiling



Invariant by two independant translations.

Periodic tiling



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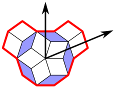
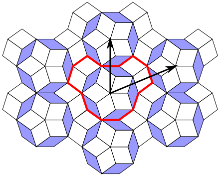


Invariant by two independant translations. Simple description!

Periodic tiling



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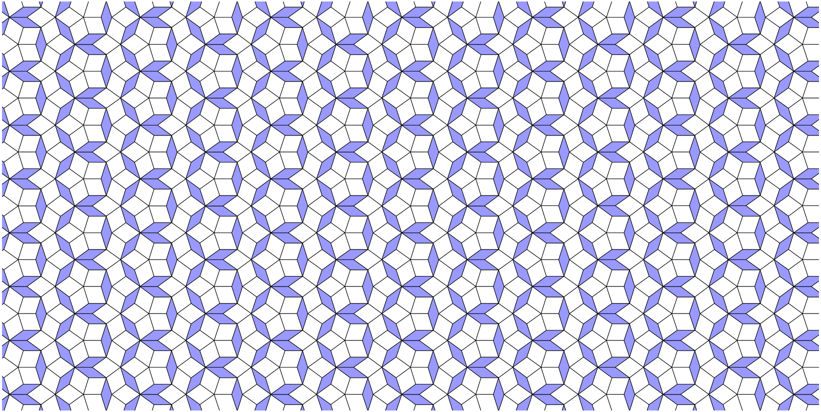

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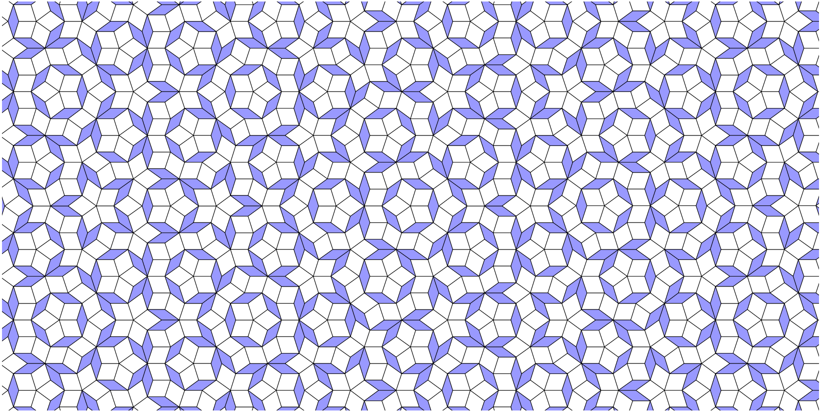
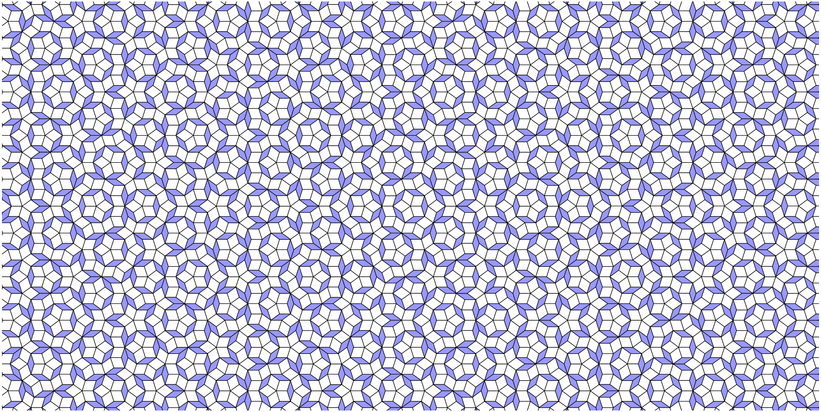
Quasiperiodic tiling



Any pattern P reoccurs at distance at most $d(P)$ from any point.

Quasiperiodic tiling



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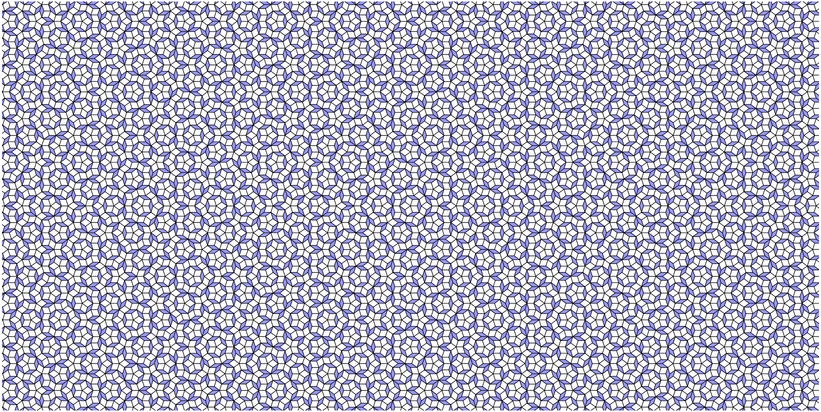

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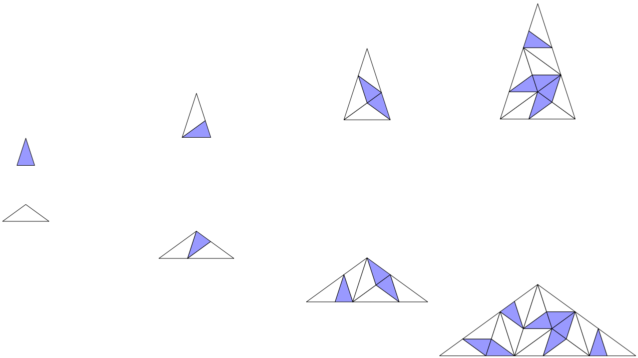
Substitutions



Inflate the tiles and subdivide each one into original tiles. Iterate.

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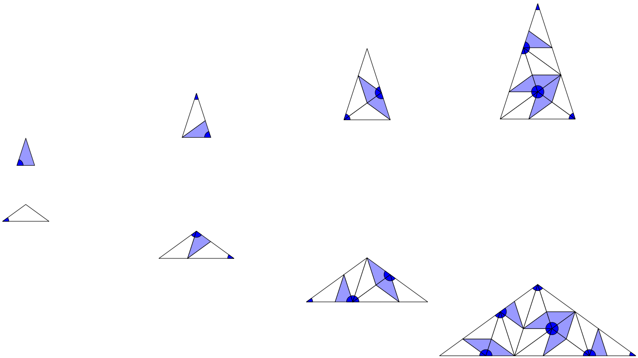

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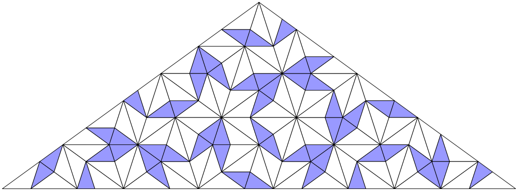

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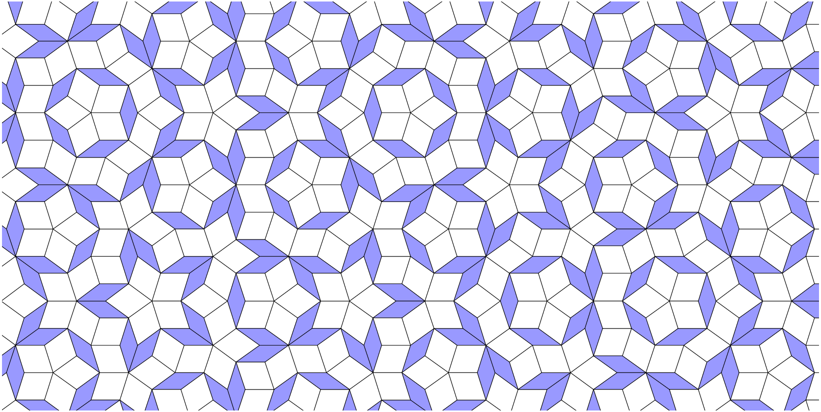

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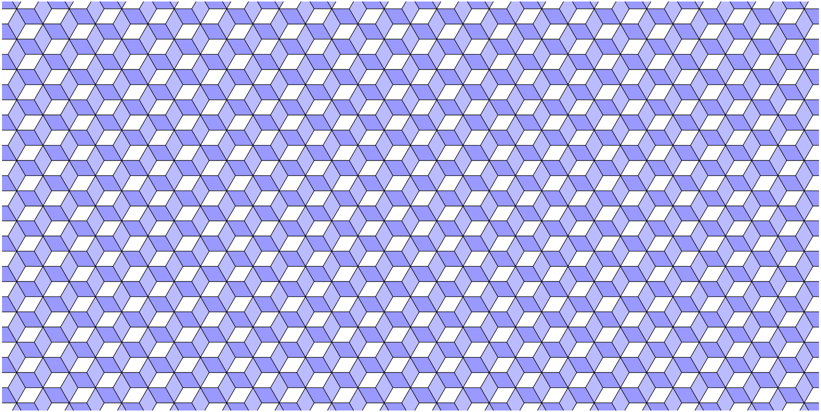
Cut and projection (DeBruijn 1981)



Select the 2D unit faces of $\mathbb{Z}^n$ lying into $E + [0,1]^n$. Project on E.

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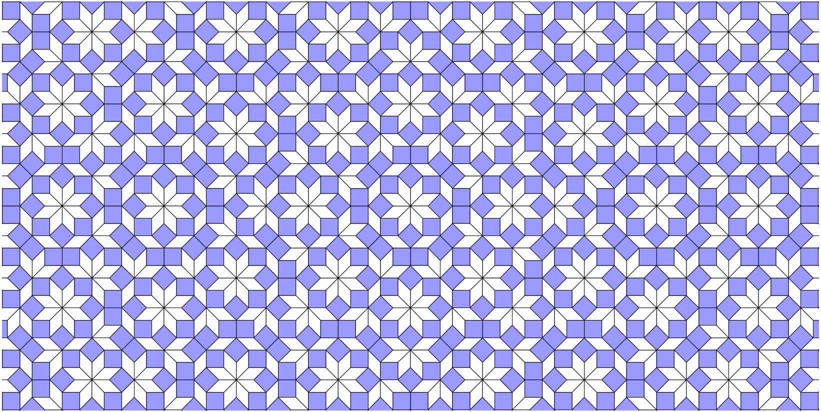

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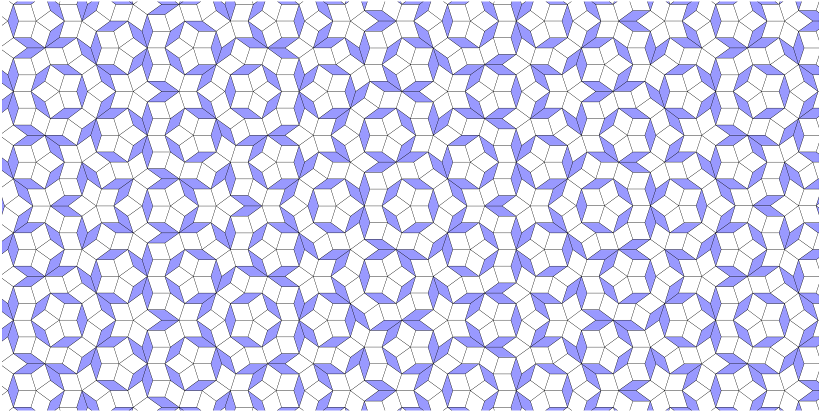

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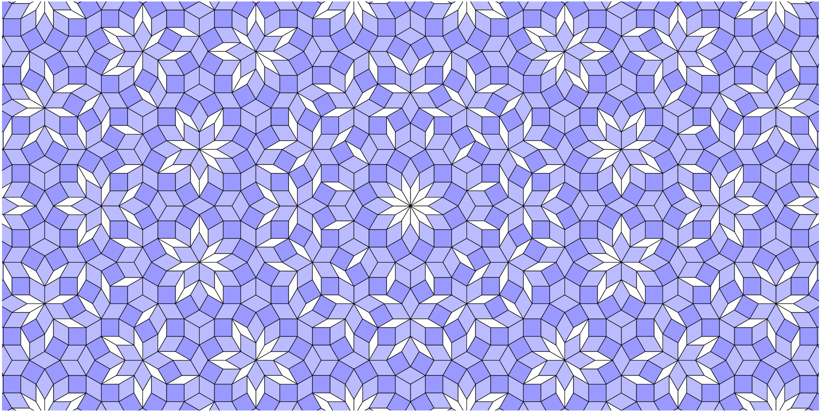

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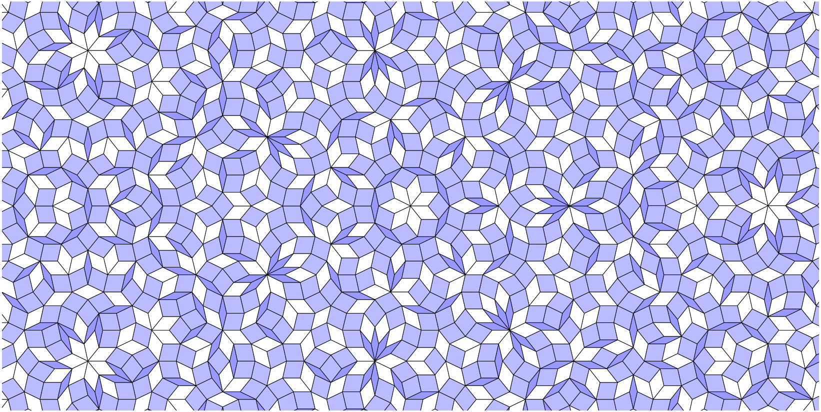

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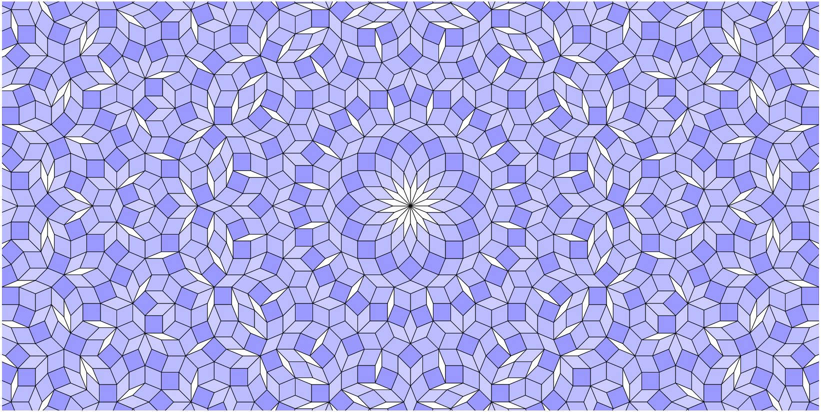
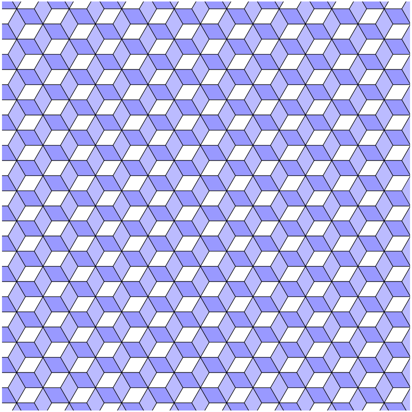
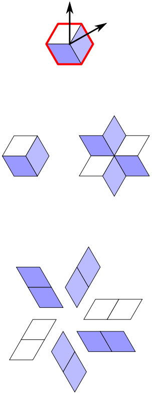


Select the 2D unit faces of $\mathbb{Z}^n$ lying into $E + [0,1]^n$. Project on E.

1. Quasiperiodic tilings
2. Local rules
3. Quasicrystals

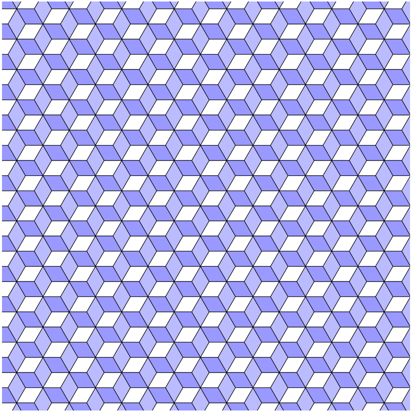
Periodic tilings

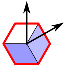


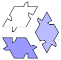


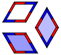
Alternative description by finitely many allowed/forbidden patterns.

Periodic tilings



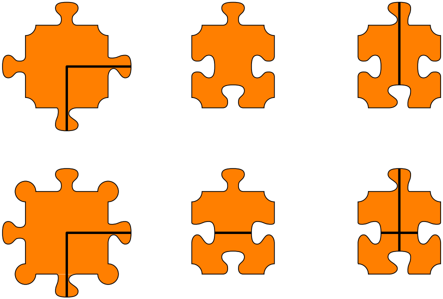


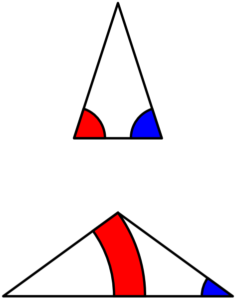




Alternative description by finitely many notched/decorated tiles.

Non-periodic tiling

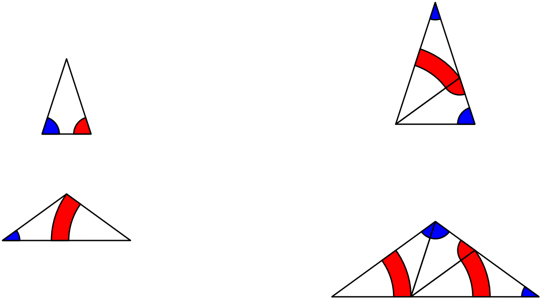




Theorem (Berger 1964)

Decorated tile sets that form only non-periodic tilings do exist.

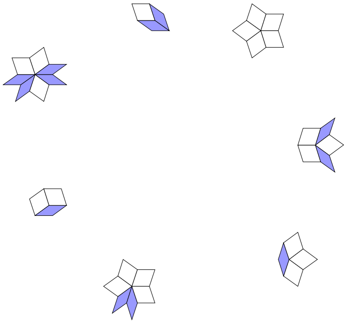
Substitutive tiling



Theorem (Mozes 1990, Goodmann-Strauss 1998, F.-Ollinger 2010)

Substitutive tilings can (generally) be enforced by decorated tiles.

Cut and project tiling (1/2)



Cuts enforced by allowed patterns are algebraic.

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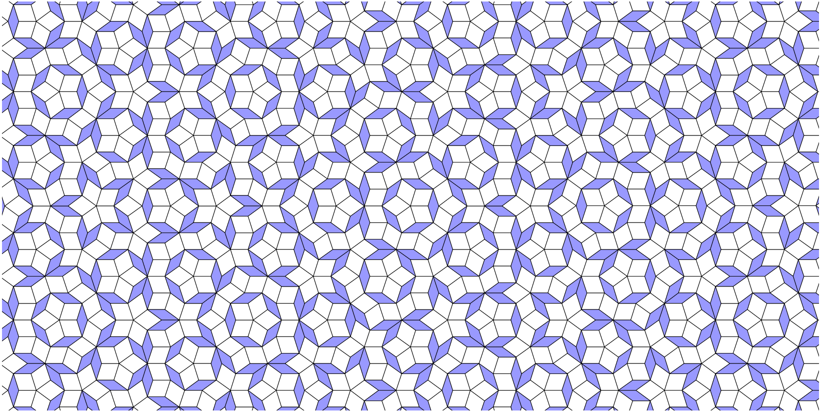

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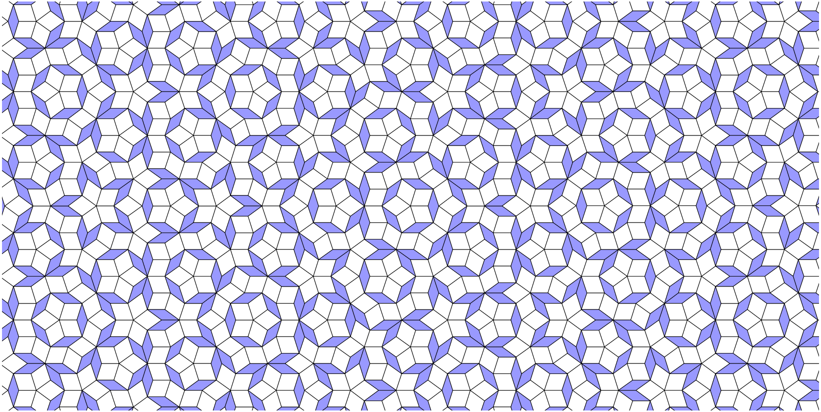
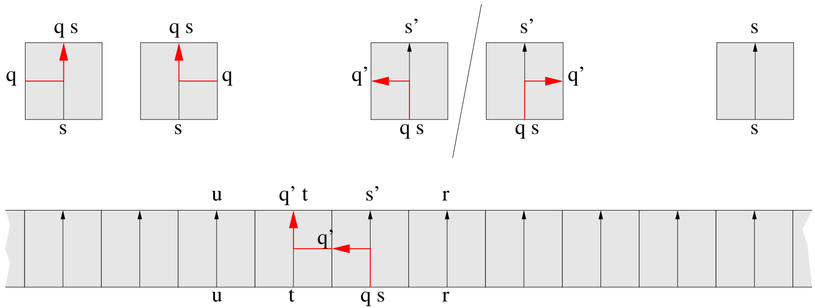


Cuts enforced by allowed patterns are algebraic. More precisely?

Cut and project tiling (2/2)



Theorem (F.-Sablik 2012)

A cut is enforced by decorated tiles if and only if it is computable.

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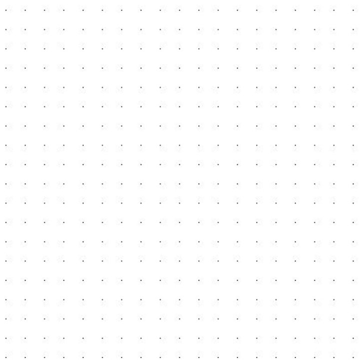
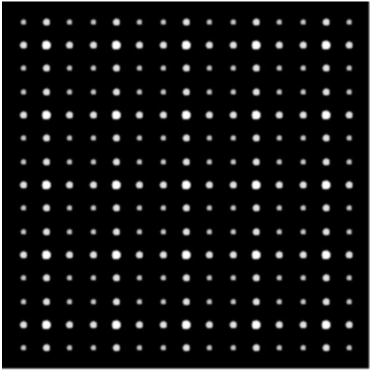
Crystals (18th century)





Periodic atom arrangement $\rightsquigarrow$ sharp peaks diffraction. Conversely?

The crystallographic restriction

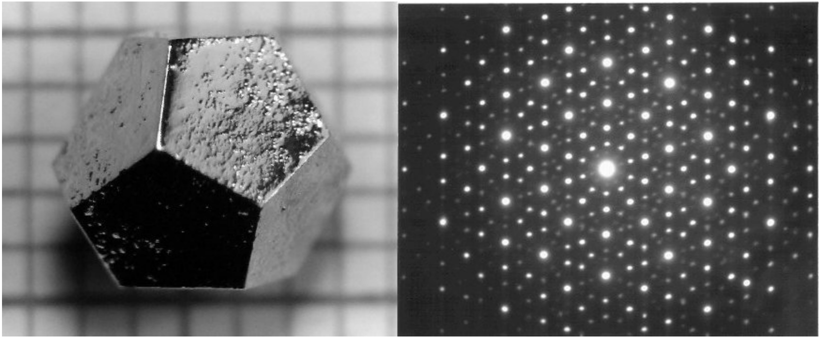
Theorem

If a lattice is invariant under a rotation by $\frac{2\pi}{n}$, then $n \in \{2, 3, 4, 6\}$.

Proof:

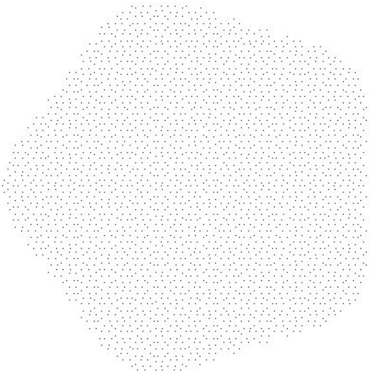
- the rotation has an integer matrix in any lattice base;
- the trace of a rotation by θ is $2\cos(\theta)$, in any base;
- $2\cos(\theta) \in \mathbb{Z} \Rightarrow \theta \in \{0, \frac{2\pi}{2}, \frac{2\pi}{3}, \frac{2\pi}{4}, \frac{2\pi}{6}\}$.

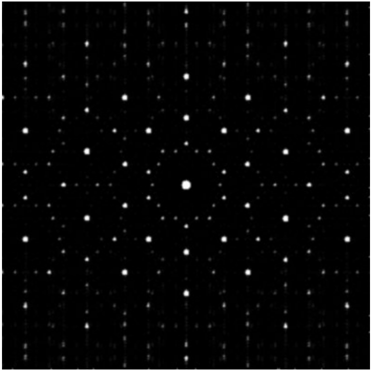
Shechtmanite (1982)



Sharp peaks diffraction but non-periodic rotational symmetry!!?

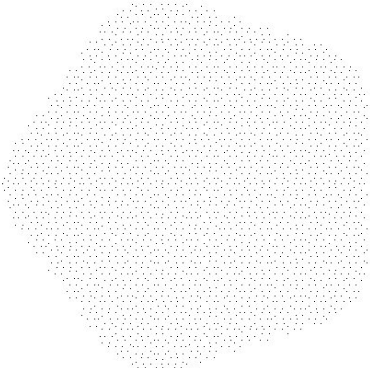
Quasicrystals (1992)

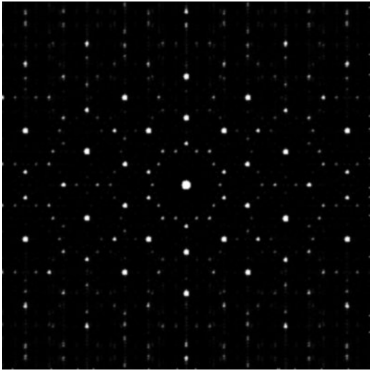




As quasiperiodic tilings!

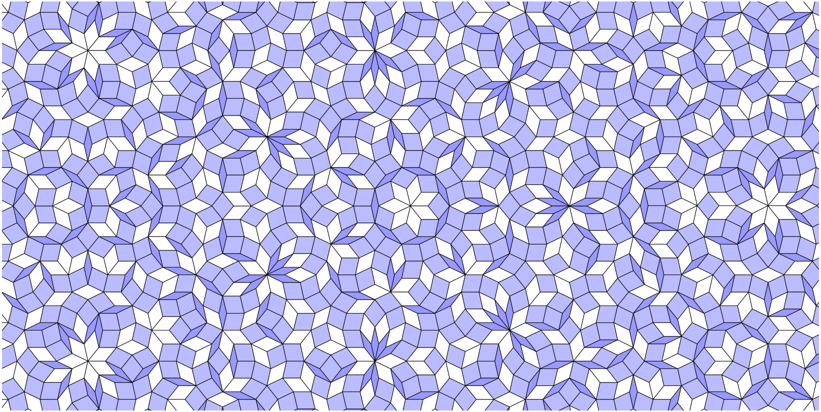
Quasicrystals (1992)





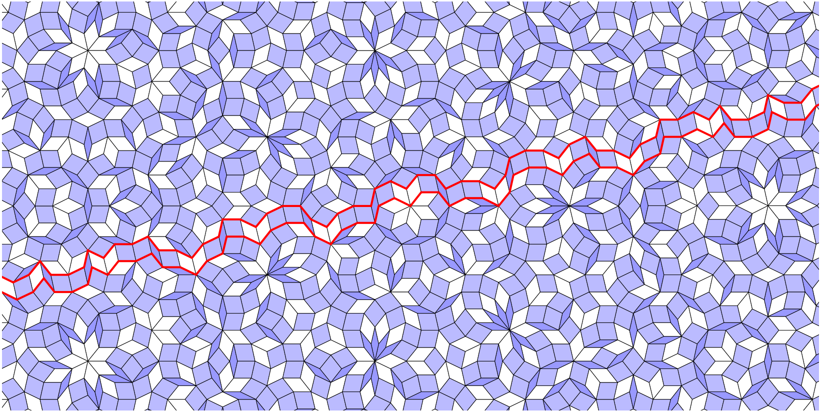
As quasiperiodic tilings! With local rules aiming to explain stability.

Open questions



The 7-fold tilings admit local rules.

Open questions



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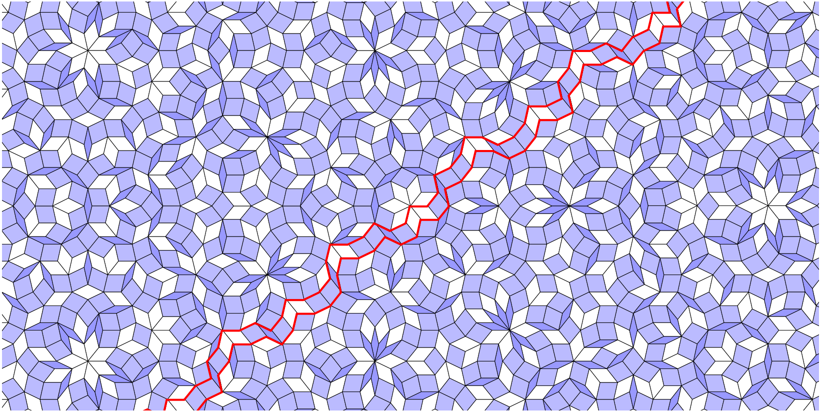

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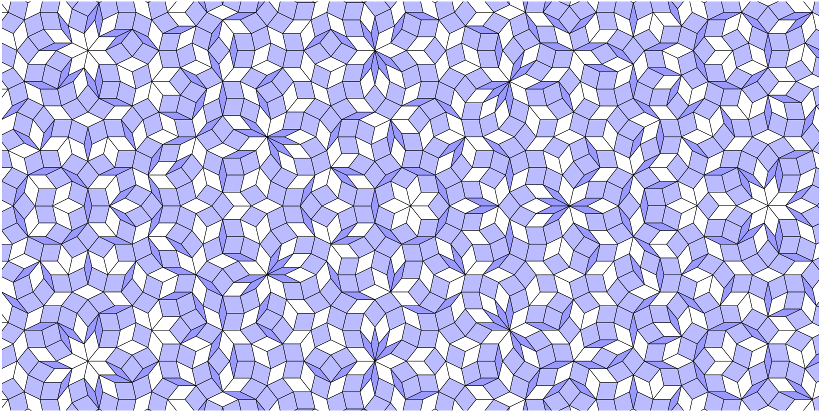

The 7-fold tilings admit local rules. Where are 7-fold quasicrystals?