Tilings of a polycell : algorithmic and structural aspects

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Outline

1. Basic notions
   - Tilings of a polycell
   - Tools : counters and height functions
2. Construction of a tiling
   - Construction of a counter in the bipartite case
   - From counter to binary counter
3. A distributive lattice over the tilings of a polycell
   - Definition through height functions
   - Flip-accessibility as a covering relation
4. Applications and examples
   - Enumeration and random sampling
   - Classical dimers tilings and perfect matchings

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Planar polycell

Definition

A *planar polycell* $(\mathcal{C}, I, v^*)$ is defined by :

- a set of circuits (called *cells*) $\mathcal{C} = \{C_1, C_2, \ldots, C_k\}$ such that the graph $\cup_i C_i$ is planar ;
- a subset I of the edges within $\mathcal{C}$ (called *inner edges*).
- any one distinguished vertex v^*.

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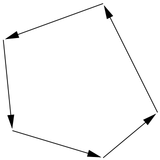

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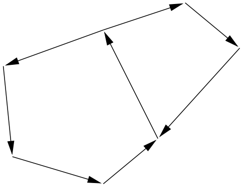

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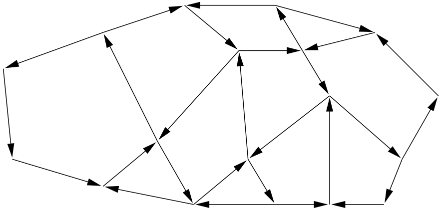

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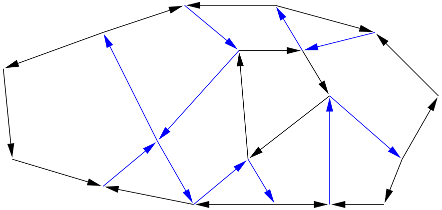

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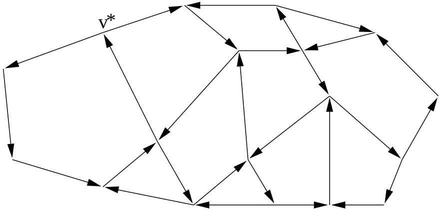
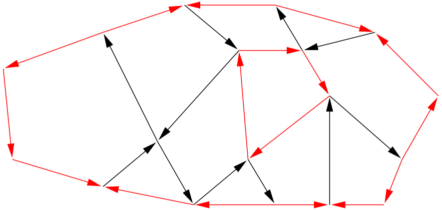


Graph of a planar polycell

The edges not in I form the set B of *boundary edges*.

Definition

The *graph* $G_{(\mathcal{C},I)}$ of a polycell $(\mathcal{C}, I)$ is formed by the graph $\cup_i C_i$ and, for each boundary edge $(a,b) \in B$, the reverse edge (b,a).



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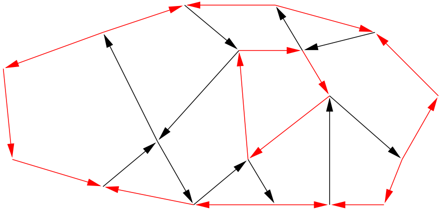

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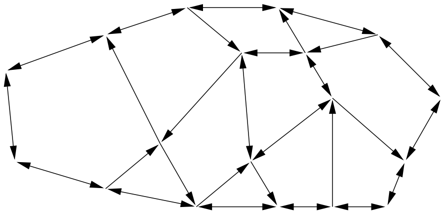
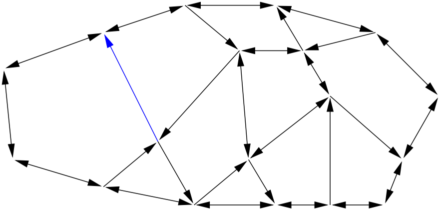


Tiling of a polycell (or of it's graph)

Let $(\mathcal{C}, I)$ be a polycell and $e \in I$ an inner edge. $Circ(e)$ denotes the subset of $\mathcal{C}$ formed by the cells which use the edge e.

Definition

A *tiling* of a polycell $(\mathcal{C}, I)$ is a subset T of I such that $\{Circ(e)\}_{e \in T}$ is a partition of $\mathcal{C}$.



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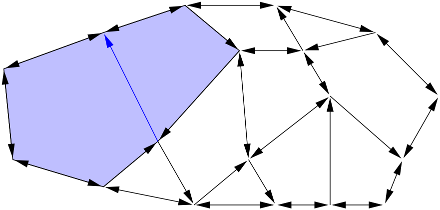

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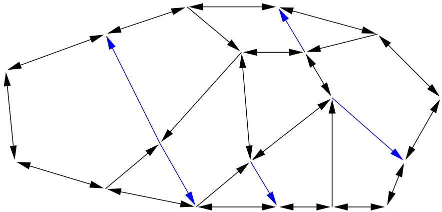


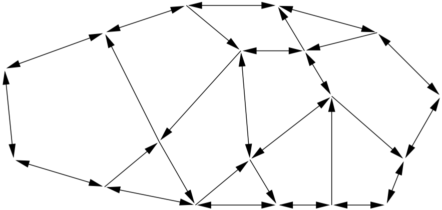
Outline

Counter

Definition

A *counter* ψ on a polycell $(\mathcal{C}, I)$ is a $\mathbb{R}$-valuation of the edges within $\mathcal{C}$ such that :

- the value of a boundary edge is always equal to zero ;
- the sum over the edges of each cell is equal to one.



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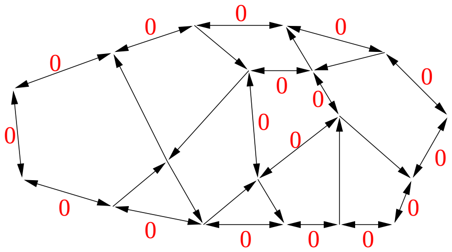

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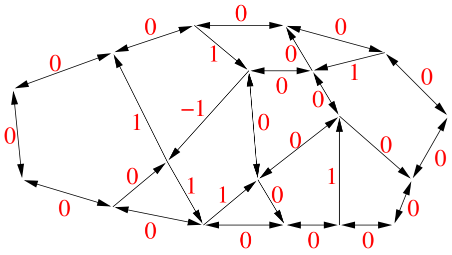
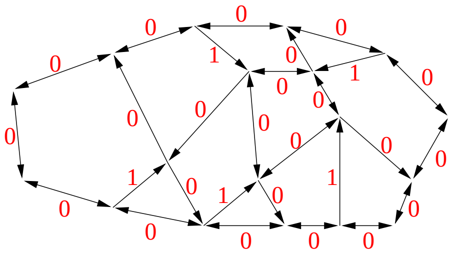


Binary counter

Definition

A *binary counter* is a counter wich takes only the values 0 or 1.

Each binary counter trivially corresponds to a tiling : the edges of the tiling are those with value 1.



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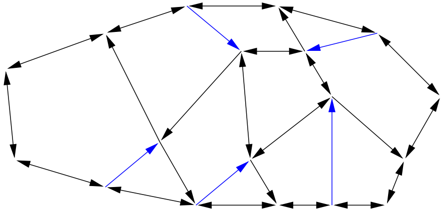
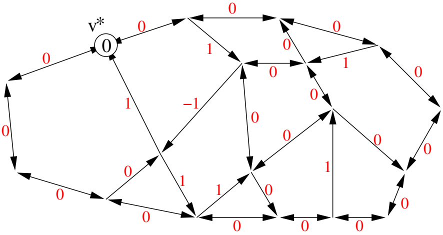


Height function of a counter

Definition

The *height function* h_ψ of a counter ψ on the polycell $(\mathcal{C}, I, v^*)$ maps each vertex v onto the ψ-weight of a shortest (directed) path from v^* to v.



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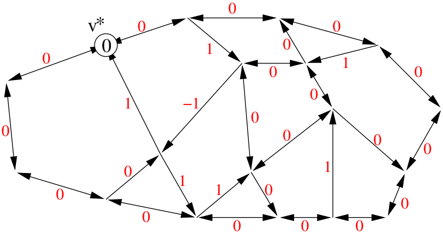

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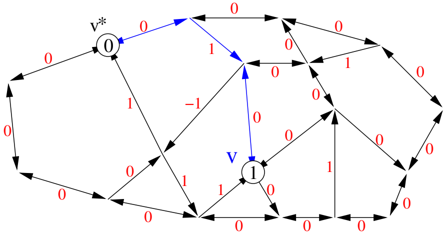

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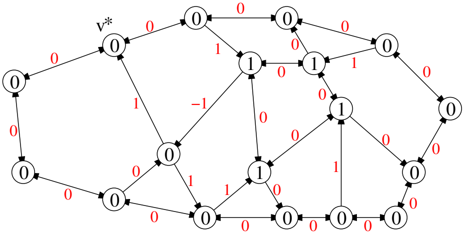

Height function of a counter

One proves :

Fact

- *The height is well-defined iff there exists (at least) a tiling of the polycell ;*
- *A counter ψ is uniquely determined by it's height function h_ψ.*

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Definition

A polycell $(\mathcal{C}, I, v^*)$ is *bipartite* if we can split $\mathcal{C}$ into two subset $\mathcal{C}_w$ and $\mathcal{C}_b$ such that two cells in the same subset does not have any common edge.

Theorem

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Let ψ be a counter on a planar polycell $(\mathcal{C}, I, v^*)$ which admits (at least) a tiling. Let δ be defined on each edge $e = (v, v')$ by :

$$\delta(e) = \psi(e) - \left(h_\psi(v') - h_\psi(v)\right).$$

Theorem

δ is a binary counter.

Moreover, $\forall v \quad h_\delta(v) = 0$: δ is called *the minimal counter.*

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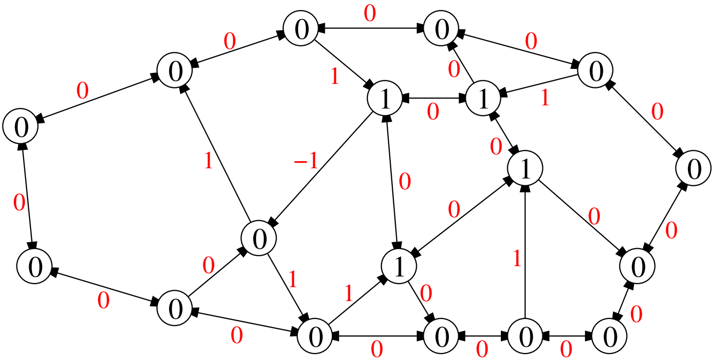
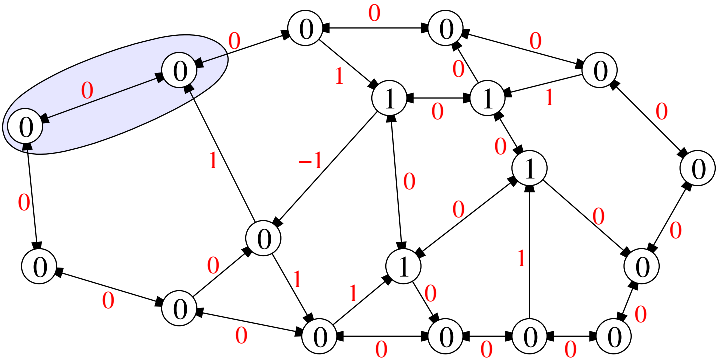


Let ψ be any one counter. We will construct the binary counter δ.



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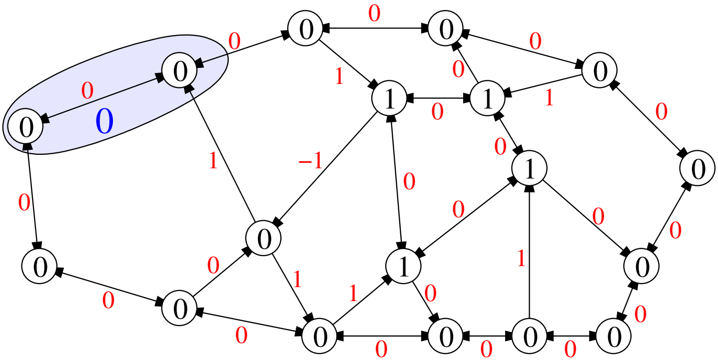

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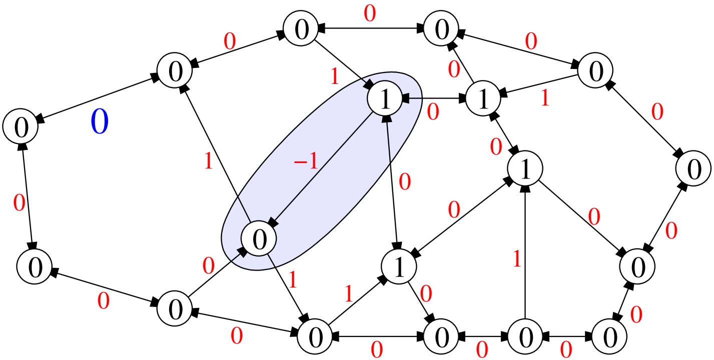
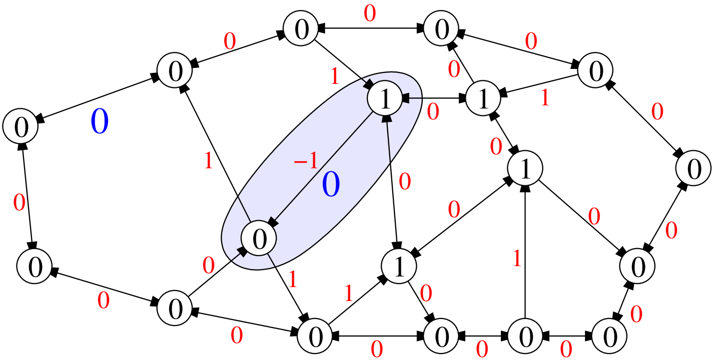


$\delta(e) = \psi(e) - (h_\psi(v') - h_\psi(v)) = -1 - (0 - 1) = 0$



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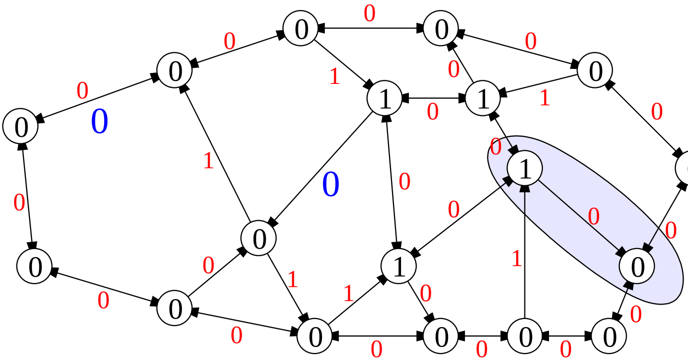

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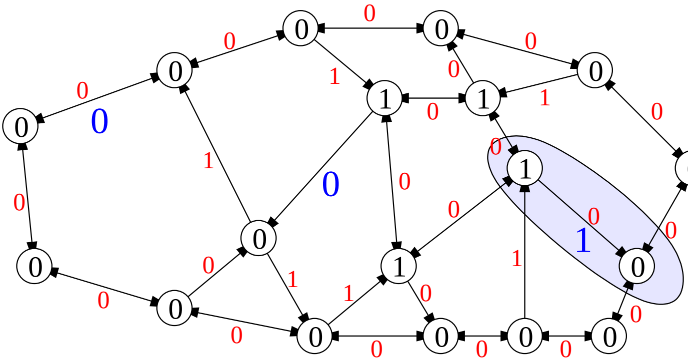

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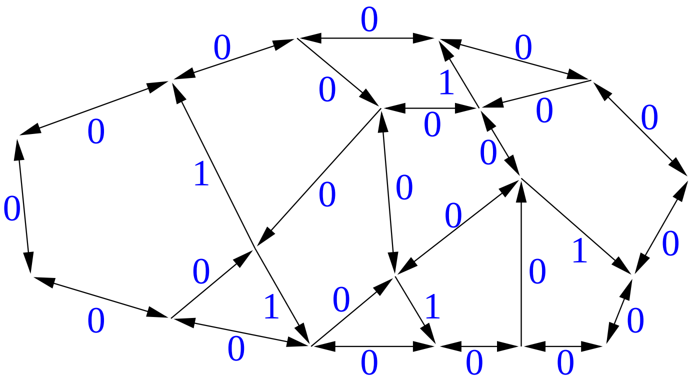
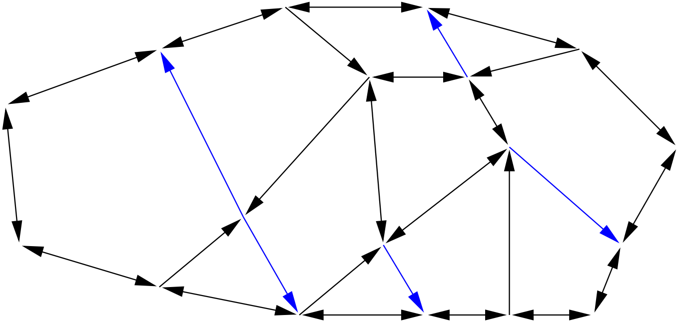


We thus obtain the binary counter δ,



and the correspondant tiling.

Hence, given a planar bipartite polycell, one can

- construct a counter ψ in time $\mathcal{O}(n)$;
- compute it's height function h_ψ (*Single Source Shortest Path* with negative weight edges) ;
- construct from ψ a binary counter δ in time $\mathcal{O}(n)$.

Since for planar graphs, SSSP can be solved in $\mathcal{O}(n \ln(n)^3)$, it proves :

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If a planar bipartite polycell has a tiling, one can construct a tiling in time $\mathcal{O}(n \ln(n)^3)$.

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Let $(\mathcal{C}, I, v^*)$ be a fixed planar polycell. Let $\mathcal{T}$ be the set of the tilings (or binary counters) of $(\mathcal{C}, I, v^*)$.

Lemma

If δ and δ' are binary counters with height functions h_δ and $h_{\delta'}$, then $min(h_\delta, h_{\delta'})$ is the height function of a binary counter, and $max(h_\delta, h_{\delta'})$ too.

We denote $\delta \wedge \delta'$ the binary counter with height function $min(h_\delta, h_{\delta'})$ and $\delta \vee \delta'$ the one with height function $max(h_\delta, h_{\delta'})$. $\wedge$ and $\vee$ are thus operations onto $\mathcal{T}$.

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$(\mathcal{T}, \wedge, \vee)$ is a (finite) distributive lattice.

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Hasse's diagram of $(\mathcal{T}, \wedge, \vee)$

We denote $\preceq$ the associated partial order :

$$\delta \preceq \delta' \Leftrightarrow (\forall v \;\; h_\delta(v) \leq h_{\delta'}(v)).$$

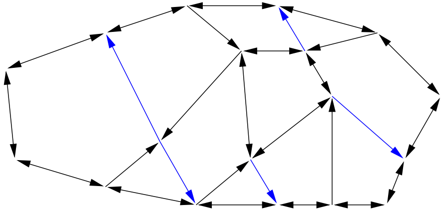
We say that δ' *covers* δ if $\delta \preceq \delta'$ and $(\delta \preceq \delta'' \preceq \delta') \Rightarrow \delta'' = \delta$

Outline

Nodule

Definition

Let $(\mathcal{C}, I, v^*)$ be a planar polycell and T a tiling. Let G_T be the graph obtained from the graph of the polycell removing the edges of T. The *nodules* of $(\mathcal{C}, I, v^*)$ are the strongly connected components of G_T.



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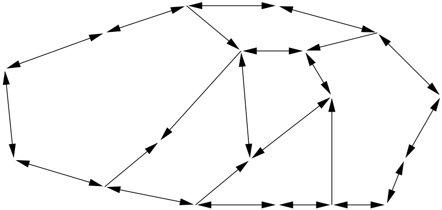

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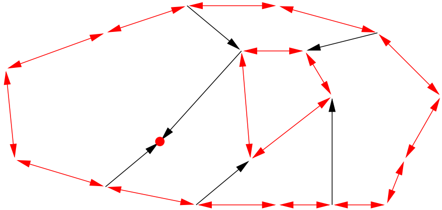

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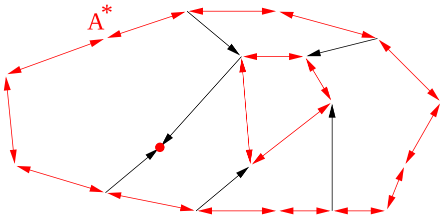

The nodule containing v^* is named A^*.

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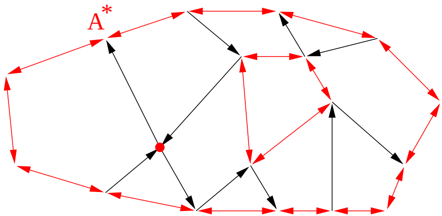


In fact, the nodules depend only on the polycell (not on T).

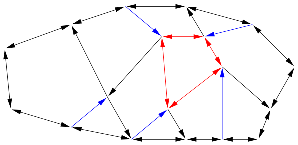
Flip

Definition

Let $T \in \mathcal{T}$ be a tiling and A a nodule other than A^*.

If all incoming edges on A are in T and none of the outcoming, the *decreasing flip* on A exchanges these edges in T.

The *increasing flip* is the reverse operation.



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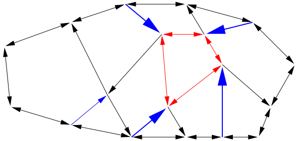

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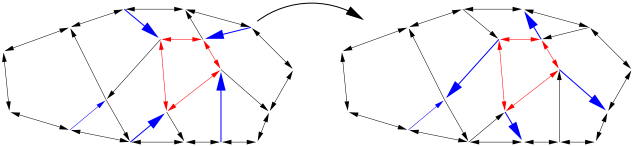

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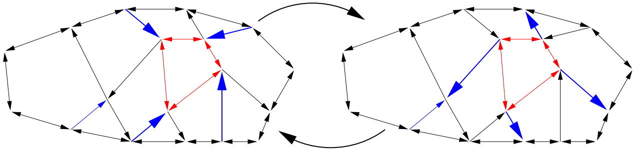

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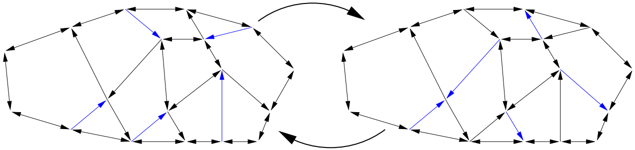

Flips are constructive operations on the set $\mathcal{T}$ of the tilings.

Flips and the lattice $(\mathcal{T}, \wedge, \vee)$

Lemma

δ' covers δ iff there exists a increasing flip that transforms δ into δ'.

Definition

The graph of flip-accessibility in $\mathcal{T}$ is the (undirected) graph whose vertices are the tilings of $\mathcal{T}$, linked iff co-accessible by a single flip.

Theorem

The Hasse's diagram of $(\mathcal{T}, \wedge, \vee)$ and the graph of flip-accessibility in $\mathcal{T}$ are isomorphic.

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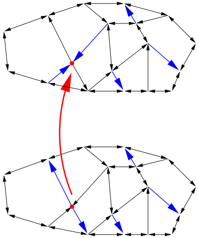

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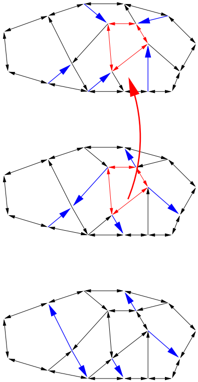

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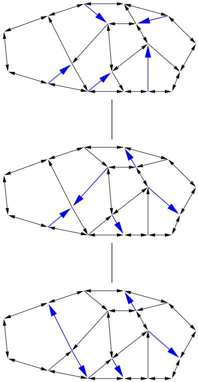

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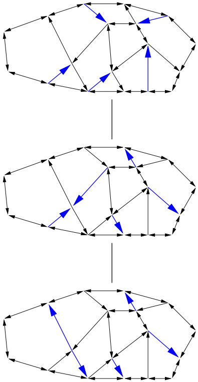

Flip-accessibility in $\mathcal{T}$

This isomorphism proves that any two tilings can be connected by a sequence of flips. More precisely :

Theorem

Let $(\mathcal{C}, I, v^*)$ be a planar polycell with tilings $\mathcal{T}$ and nodules $\mathcal{A}$. Let δ and δ' be any two binary counters (or tilings) with height functions h_δ and $h_{\delta'}$.
A shortest sequence of flips that transforms δ into δ' has length :

$$\sum_{A \in \mathcal{A}} |h_\delta(A) - h_{\delta'}(A)|$$

Moreover we can effectively compute such a sequence.

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$$\sum_{A \in \mathcal{A}} |h_\delta(A) - h_{\delta'}(A)| = \mathcal{O}(n^2)$$

Moreover we can effectively compute such a sequence.

Outline

Enumeration

A linear extension of the partial order $\preceq$ combined with the constructive operation of flip can be fruitful used for a planar polycell :

Theorem

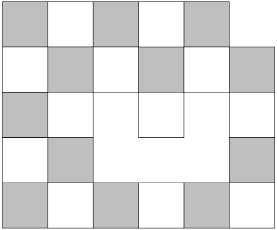
Given an initial tiling, one can enumerate the whole set of the tilings in linear time per tiling and with space $\mathcal{O}(n \ln n)$.

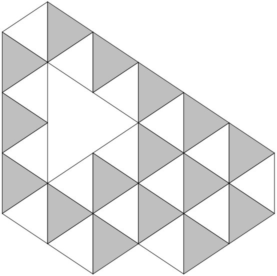
Random sampling

Theorem

Outline

Tilings with dominoes or lozenges





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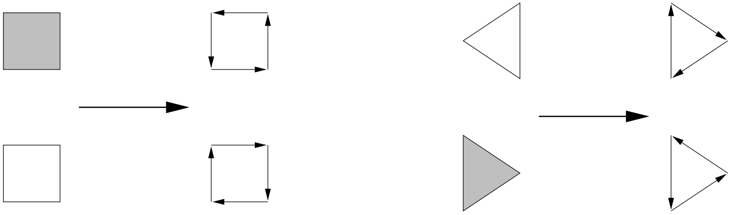

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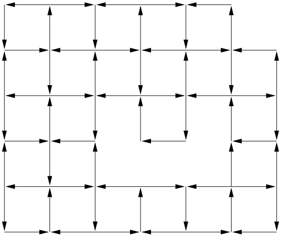

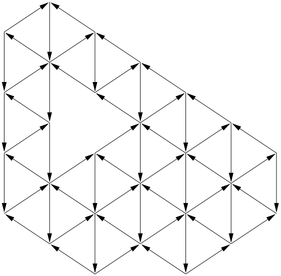

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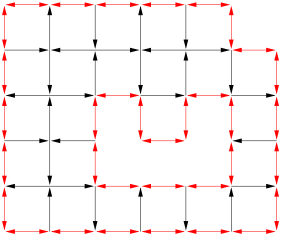

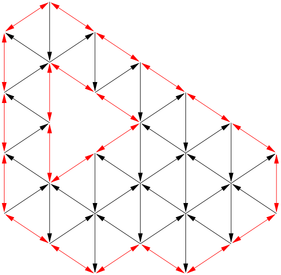

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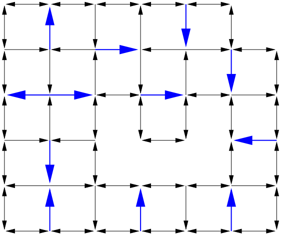

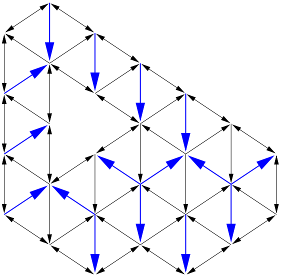

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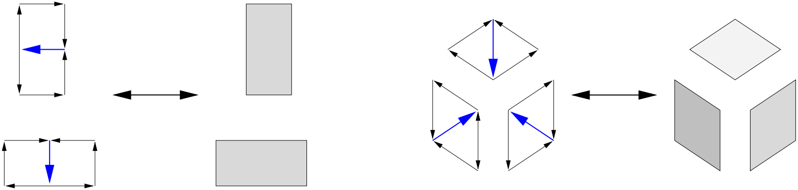

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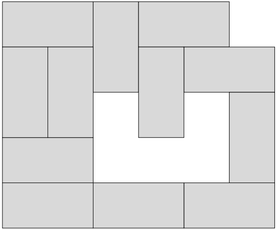

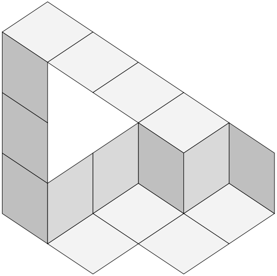

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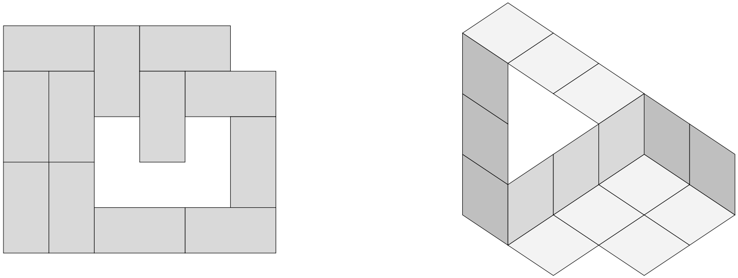

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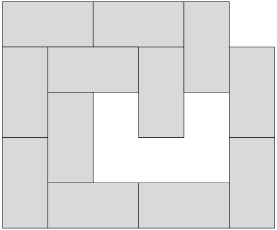

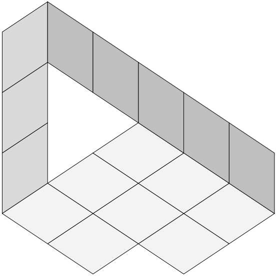

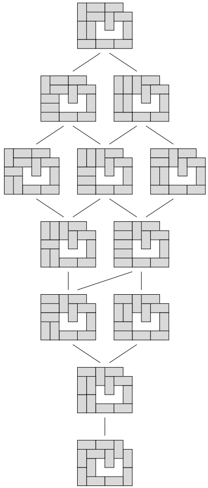

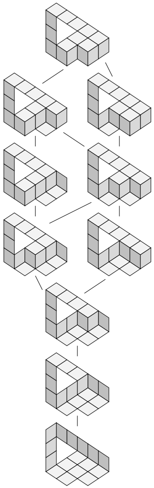

Perfect matchings of a planar bipartite graph