Technical report CEDRIC RC391-2002 updated in 2010
http://cedric.cnam.fr

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SDP_S:
A TOOL TO FORMULATE AND SOLVE SEMIDEFINITE RELAXATIONS FOR BIVALENT QUADRATIC PROBLEMS

Version 1.1

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1. Introduction

SDP_S is a stand-alone program which formulates semidefinite programming (SDP) relaxations for any $0-1$ quadratic problem starting from a linear relaxation of the initial problem. Moreover, it can solve the SDP by using the Spectral Bundle Method of C. Helmberg [2], [3]. SDP_S is an implementation of the algorithm proposed by F. Roupin in [4]. One of the major advantages of SDP_S is that it requires no knowledge about semidefinite programming. Indeed, the input problem has just to be stated as a natural $0-1$ quadratic program:

$$(P\{0,1\}) \begin{cases} \text{Min or Max} & x^T A_0 x + b^T x + val & \\ \text{s.t.} & x^T A_i x + c_i^T x = (\text{or} \leq)\, d_i & i = 1, ..., m_1 \\ & c_i^T x = d_i & i = m_1, ..., m_2 \\ & d_i \leq c_i^T x \leq d'_i & i = m_2, ..., m \\ & x \in \{0,1\}^n & \end{cases}$$

where $1 \leq m_1 \leq m_2 \leq m$ and val is a real number. Note that some A_i $i \in \{0, ..., m\}$ can be equal to 0, and if it is true for all i in $\{0, ..., m\}$ then $(P\{0,1\})$ is a $0-1$ linear program. Moreover, some d_i or d'_i can be missing. More precisely, the semidefinite relaxation is obtained by following the rules (see [4] for details) in Table 1 starting from a linear relaxation P_L of $(P\{0,1\})$ (obviously some of the constraints in the first and second columns can be missing). Hence, the problem is seen as a linear relaxation of $(P\{0,1\})$.

2. How to use SDP_S

2.1. Installing and compiling SDP_S.

Install all the files by using the command "tar xvfz SDP_S.1.1.tar.gz". A directory SDP_S will be created. It contains: the source code directories "matrix", tools", "spectral" and "SDP_SOLVER", the documentation directory "docs", and a directory "examples" which contains several ready-to-be-used examples for SDP_S.

Second, to compile SDP_S, simply type "make" in the SDP_S directory. This will produce the stand-alone executable "SDP_S" located in SDP_S/, and a directory "Object" which contains all the object files. "Make clean" will erase all the object and executable files. Caution: the Makefile and SDP_S have been only tested on several Linux distributions. For other unix systems, you may have to modify the Makefile and/or the code. Please, read carefully all the documention files located in "SDP_S/docs".

2.2. Running SDP_S.

To obtain and solve a semidefinite relaxation (obtained by the recipe presented in [4]) of any bivalent quadratic problem, simply run SDP_S like this:

```
SDP_S <filename> [options]
```

where "filename" is the problem file (see Section 3). Options are:

- -fSB <filename> : The file "filename" will contain the problem in the SB format (for a latter use with the SB solver alone for instance).
- -fopt <filename> : Options contained in "filename" will be given to the SB solver. (See the manual "sbmanual.ps" for more details).
- -fres <filename> : The file "filename" will contain an output of the solving.
- -nosolve : the semidefinite program will not be solved, but a file written in the SB format will be created. This option is useful to test the syntax of the input files, or to use the SDP solver SB alone.
- -resume <filename> : resumes a old process using the resume file "filename".

P_L	$(SDP\{0,1\})$
$0 \leq x_i \leq 1 \, \forall i \in \{1,...,n\}$	$\begin{bmatrix} 1 & x^T \\ x & X \end{bmatrix} \succeq 0$ $\mathbf{d}(X) = x$
$A \bullet X + c^T x = (\leq) d$	$A \bullet X + c^T x = (\leq) d$
$\forall i < j < k < n \in \{1,...,n\}$ $0 \leq X_{ij}$ $X_{ij} \leq x_i$ $X_{ij} \leq x_j$ $x_i + x_j - 1 \leq X_{ij}$ $X_{ij} + X_{ik} + X_{jk} + 1 \geq X_{ij} + X_{ik} + X_{jk} + 1$	$\forall i < j < k < n \in \{1,...,n\}$ $0 \leq X_{ij}$ $X_{ij} \leq x_i$ $X_{ij} \leq x_j$ $x_i + x_j - 1 \leq X_{ij}$ $X_{ij} + X_{ik} + X_{jk} + 1 \geq X_{ij} + X_{ik} + X_{jk} + 1$
$c^T x = d$	$cc^T \bullet X - 2dc^T x + d^2 = 0$ $\Leftrightarrow \left\vert \begin{array}{l} cc^T \bullet X = d^2 \\ c^T x = d \end{array} \right.$ or $c^T x = d$ $\sum_{j=1}^n c_j X_{ij} = d x_i \, i = 1,...,n$
$d' \leq c^T x \leq d$	$cc^T \bullet X - (d + d') \, c^T x + dd' \leq 0$ or $\sum_{j=1}^n c_j X_{ij} \leq d x_i \quad i = 1,...,n$ $d' x_i \leq \sum_{j=1}^n c_j X_{ij} \quad i = 1,...,n$ $\sum_{j=1}^n c_j \, (x_j - X_{ij}) \leq d \, (1 - x_i) \quad i = 1,...,n$ $d' \, (1 - x_i) \leq \sum_{j=1}^n c_j \, (x_j - X_{ij}) \quad i = 1,...,n$
$c^T x \leq d$	$cc^T \bullet X - \left(\sum_{i \, s.t. \, c_i < 0} c_i + d\right) c^T x + d \sum_{i \, s.t. \, c_i < 0} c_i \leq 0$

TABLE 1. *Rules of the scheme*

If the -fopt option is not used, or if there is no "filename.opt" file in the current directory, then a default option file is created with the options "-si 1 -sh 1". This file has the extension ".opt". You can modify it to add options for SB without using the -fopt option, since SDP_S tests if an option file exists in the current directory.

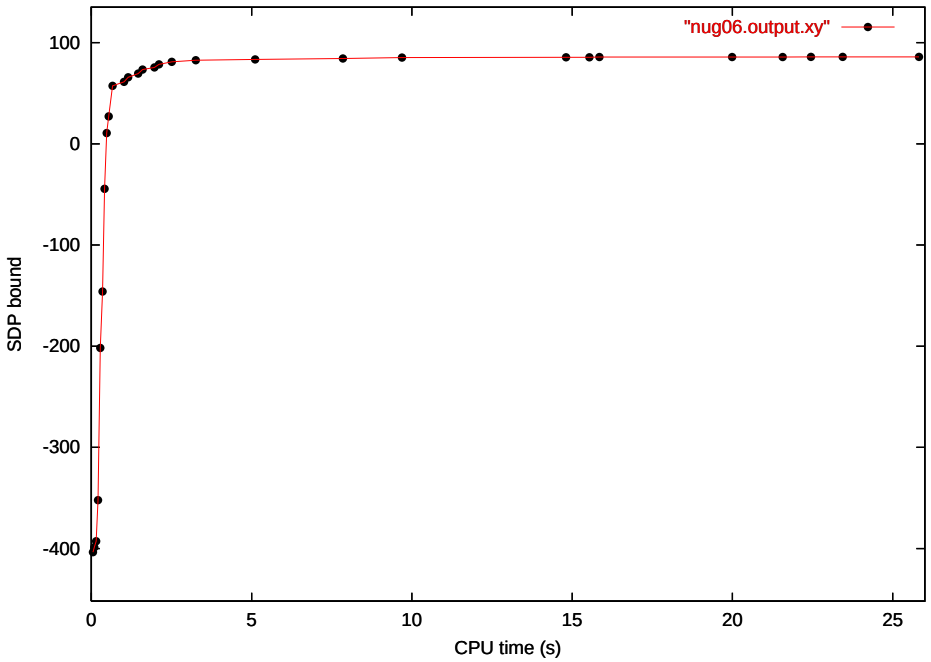
2.3. **Stopping SDP_S before normal termination.** The kill signals of SB can be used with SDP_S. These signals are :

(1) SIGTERM (15) : Save resume info, output summary, exit.
(2) SIGUSR2 (12) : Output summary and continue.
(3) SIGUSR1 (10) : Output summary, exit.
(4) SIGKILL (9) : Kill without any further output.

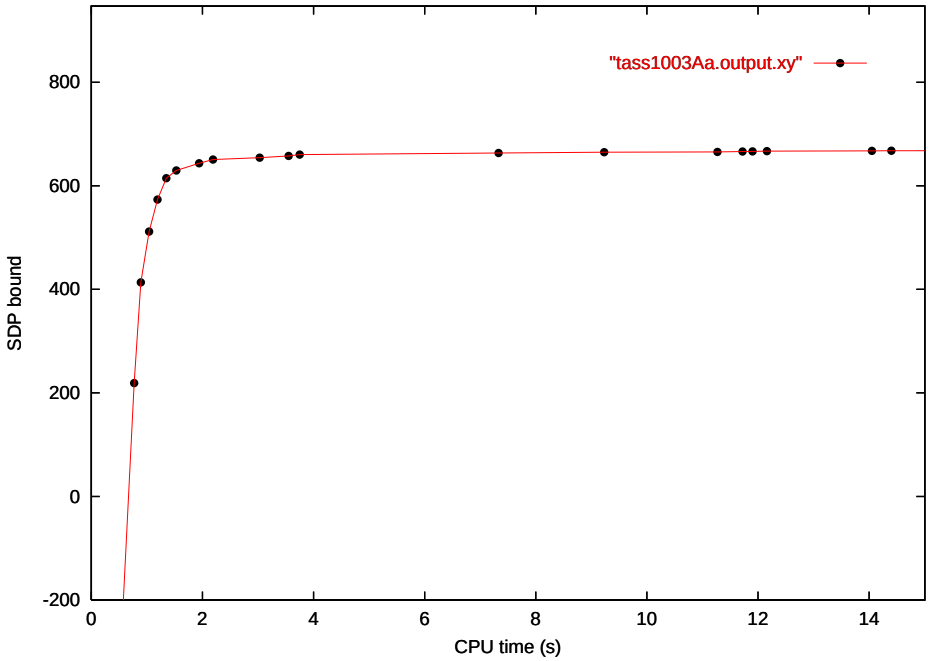
When the "SIGTERM (15)" is used, a resume file "SDPS_resume.dat" is created in the current directory (caution: it can be very large). This file can be used latter with the "-resume" option of SDP_S to continue the solving process. We do not recommend to use a "SIGKILL (9)" signal with SDP_S, especially if one has set the "-fres" option. Indeed, the output file may be incomplete. In all cases, the "SIGUSR1 (10)" and the "SIGTERM (15)" signals are the best ways to quit the program before normal termination.

2.4. **The res2plot utility.** Gnuplot and ghostview must be installed on your system to use the "res2plot" utility. The source code "res2plot.c" is located in SDP_S/tools. This program produces a postscript file from an output file produced by SDP_S (option "-fres"). The executable "res2Plot" is created with SDP_S in the "SDP_S/" directory when the "Make" command is used. res2Plot allows to visualize the convergence of the SDP solver. Several files are created in the current directory in order to output the graphical result. To obtain a better output it may be useful to change the ".plot" file to modify the ranges (x and y). Then gnuplot must be run again to create a new postscript file by using the command: "gnuplot filename.plot".

Example 2.1. This is the output obtained when running "res2plot nug06.output" in the SDP_S/examples/QAP directory:



Example 2.2. This is the output obtained when running "res2plot tass1003Aa.output" in the SDP_S/examples/QAP directory, then modifying the "tass1003Aa.output.plot" file by setting "set xrange [0:15]" "set yrange[-200:947]" and "set key 13,837" and running again gnuplot:



3. Input format

In this section we describe the syntax of the input files. The problem or "master" input file contains four main area:

(1) A commentary area.
(2) A quadratic or a linear function to maximize (or minimize).
(3) A general description of the problem (dimension, trace of the matrice if available, values of the bivalent variables, filenames describing the objective function).
(4) The constraints.

3.1. The commentary area.

It is possible to put some commentaries at the begin of the file. The commentaries must be written between two tokens "//". An example of commentary will be more explicit:

```
// This is a valid commentary //
```

Commentary can also be written on several lines:

```
// This is a //
// commentary //
// on several lines //
```

Or like that:

```
// This is
 a commentary
 on several lines //
```

But make sure that there is a space between the commentary and the tokens "//".

3.2. The objective function.

The objective function is the function to maximize or minimize. This area contains the following data:

(1) Maximization or Minimization.
(2) Does exist a quadratic part ?
(3) Does exist a linear part ?
(4) The constant value (which is not mandatory).

Caution: even if it is not mandatory to have a linear part *and* a quadratic part, there must be at least one of them.

Format

The general format of the objective function is:

$$< Min|Max >< (CX\ [+lx]\ [+'val']) \,|\, lx\ [+'val'] >$$

First, choose if you want to maximize or minimize. Keywords are Min or Max. Second, you must precise (in that order) if there are a quadratic part, a linear part and a constant. The quadratic part is precised by the presence of the character C followed by any character. The linear part is precised by the presence of the character l followed by any character. The constant is given by its value. Quadratic part, linear part and constant must be separated by the token "+".

Note that it must be the first data written after the commentary area. If the file do not begin with "Min or Max", an error will occur.

Example 3.1. One can write one of these objective functions :

```
* Max CX + 3
* Min lx + -5
* Max CX + lx and I can write what I want which does not begin with "+".
```

But not:

* `Max 3 :` *There is neither a quadratic or a linear part.*
* `Maximize CX :` *Maximize is not the same string as Max.*
* `Max CX - 5 :` *The character "+" must be written before the constant value, even if it is negative.*
* `Max 1x + CX :` *The quadratic part must be written before the linear part.*
* `# Max CX :` *The first word must be Min or Max.*

3.3. **Description of the problem.** In this part, the following informations are given:

- Dimension n of the problem (size of $x \in \{0,1\}^n$).
- The trace of the matrix $X = xx^T$ if it is a constant for the considered problem (it represents $\sum_{i=1}^n x_i = \sum_{i=1}^n X_{ii}$ because $\mathbf{d}(X) = x$). Since we work with $\{0,1\}$ variables, the trace of X is always bounded by n, but give a value to Trace(X) speeds up the convergence of the SDP solver SB [2]. This part is NOT mandatory.
- Values of the bivalent variables : $\{0,1\}$ or $\{-1,1\}$.
- Filenames for 'C' or/and 'l' (the quadratic and the linear parts of the objective function).

It is possible to put some commentaries between these informations. Caution : even if it is possible to use $\{-1,1\}$ variables in SDP_S, we *strongly* recommend to use $\{0,1\}$ variables instead. Indeed, several options are not available with the $\{-1,1\}$ model (for instance add triangle inequalities with the keyword "AddMoreConstr"). This is not restrictive since semidefinite relaxations obtained from the two bivalent models are equivalent (see for instance [4] for details).

Format

Each of the 6 labels that are possible has its own arguments:

(1) C : file < file type > : < filename >
(2) l : file < file type > : < filename >
(3) Dim : < Dimension of the problem >
(4) Trace : <Trace of the matrix X>
(5) Type : <$\{0,1\}$ | $\{-1,1\}$>
(6) AddMoreConstr : < integer in $\{0,...,31\}$>

Two different file types are possible to describe C and l : "Sparse" and "Dense":

- In the "Dense" format, "filename" must begin with the dimension of C followed by all the values of the matrix.
- In the "Sparse" format, "filename" begins by the dimension followed by the number of non-zero elements. Then, for each non-zero element, one has to write the 2 indices (row and column) followed by the value corresponding to these indices (indices start at one).

The "AddMoreConstr" option can be only used with the $\{0,1\}$ problems. Five kinds of constraints can be added:

$$\begin{cases} X_{ij} \geq 0 & 0 \leq i < j < n & (0) \\ X_{ij} \leq X_{ii} & 0 \leq i < j < n & (1) \\ X_{ij} \leq X_{jj} & 0 \leq i < j < n & \\ X_{ii} + X_{jj} \leq 1 + X_{ij} & 0 \leq i < j < n & (2) \\ X_{ik} + X_{jk} \leq X_{kk} + X_{ij} & 0 \leq i < j < n \text{ and } 0 \leq k < n & (3) \\ X_{ij} + X_{ik} + X_{jk} + 1 \geq X_{ii} + X_{jj} + X_{kk} & 0 \leq i < j < k < n & (4) \end{cases}$$

There is not yet such triangle inequalities implemented for the $\{-1,1\}$ problems. To choose the constraints, the "bit" of the corresponding constraints must be set to one. Then, the resulting integer number (belonging to $\{0,...,31\}$) must be written after "AddMoreConstr". For example, to add the constraints (0) and (2), the number must be equal to: $2^0 + 2^2 = 5$.

Example 3.2. Assuming that "FileC.dat" and "FileL.dat" exist, the following problem file is valid:

```
Max CX + lx + -4
C : File Sparse : FileC.dat
l : File Dense : FileL.dat
Dim : 23
Trace : 10
Type : {0,1}
AddMoreConstr : 5
```

3.4. **The constraints.** For this part, several labels can be used. In order to test quickly several different relaxations, one can disable constraints by putting a "_" at the beginning of the corresponding lines. The beginning of the constraints area is indicated by the label "Constraints:". For now, five labels of constraints are available:

- "File :"
- "Single :"
- "Sum :"
- "ProductConstrVect"
- "ProductConstrBarVect"
- "Simple"

3.4.1. *The "File" label.* This label indicates that the constraints are described in an external file. For instance : "File Sparse : constraints1.dat" or "File Dense : constraints1.dat" means that the constraints are described in the file "constraints1.dat".

Constraint File format:

```
< "Q" | "l" > < "=" | "<" | ">" | "<>" >
< dimension >
< number of constraints in this file >
< constant value of the first equality | inequality >
< vector/matrix of the first equality | inequality >
< constant value of the second equality | inequality >
< vector/matrix of the second equality | inequality >
...
< constant value of the last equality | inequality >
< vector/matrix of the last equality | inequality >
```

Description

First, one must precise if the constraints described in the file are linear or quadratic: the first character of the file will be a 'Q' for quadratic constraints or a 'l' for linear ones. Second, one must precise if the constraints are equality constraints or inequality constraints. In this last case only an upper bound can be written ('<' or '>'), or a lower and an upper bounds ('<>'). Hence, if m is the number of constraints, one must use:

(1) 'Q=' if the constraints that can be stated as "$B_i \bullet X + c_i^T x = d_i$" for $i \in \{1, ..., m\}$.
(2) 'l=' if the file contains constraints that can be stated as "$c_i^T x = d_i$" for $i \in \{1, ..., m\}$.
(3) 'Q<' if the constraints can be stated as "$B_i \bullet X + c_i^T x \leq d_i$" for $i \in \{1, ..., m\}$.
(4) 'l>' if the constraints can be stated as: "$c^T x \geq d_i$" for $i \in \{1, ..., m\}$.
(5) "l<>" if the constraints can be stated as: "$d_i' \leq c_i^T x \leq d_i$" for $i \in \{1, ..., m\}$.

Second, the dimension of x and the number m of constraints described in the file must be indicated. Third, for each constraint, the value of d and/or d' must be written:

(1) In the case '=', write d.
(2) In the cases '<' and '>', write d' or d.
(3) In the case "<>", write d' and d.

Finally, if the constraint is linear, then one has to write the vector (using the "Dense" or "Sparse" format according to the keyword written after the "File" label), else the constraint is quadratic, and the vector and the matrix have to be written (using the dense or spare format according to the keyword written after the "File" label). See the Section 4 for an example.

3.4.2. *The "Single :" label.* This label allows to write some constraints that will be treated like singletons. Here, no external file is needed.
Format:

```
Single : < i > < j > <operator> < value > [ range i ][ range j ]
```

Description:
Without using range variables, this label allows to write a constraint involving only one variable: "Single : 1 2 = 3.0" is to write $X_{12} = 3$. But if one wants to write many of constraints of this kind, range variables 'i' and 'j' can be used. The format of the range is: $\{a..b\}$ where 'a' and 'b' are integer or equal to 'n' (the size of $x \in \{0,1\}^n$). If the two indices have the same name, only one range have to be given. Caution: spaces are always forbidden between braces.

Example 3.3. The following examples are valid:

* `Single : 1 1 = 0 :` $X_{11} = x_1 = 0$.
* `Single : 1 j > 0` $\{1..2\}$: $X_{11} \geq 0$ *and* $X_{12} \geq 0$.
* `Single : i j > 0` $\{1..n\}$ $\{1..n\}$ *: All variables are non-negative.*
* `Single : i i = 1` $\{1..n\}$ *: All the diagonal elements are set to 1.*

But these examples are not valid:

* `Single : 1 1 < N :` *the value must be a number.*
* `Single : i j = 0` $\{1..n\}$ *: each variable must have a range.*
* `Single : 0 0 = 0 :` *indices start at one and not at zero.*
* `Single : i j = 0` $\{1..n\}$ $\{n..\ \ n\}$ *: spaces are forbidden between braces.*
* `Single : 1 2 ? 3 :` *The operator must be '=', '<' or '>'.*

3.4.3. *The "Sum :" label.* This label allows to write that the sum of some variables is equal to a constant number. To precise which kind of sum must be written, six keywords are allowed: "DiagSum", "VarSum", "RowSum", "SquareSum" ,"TriangSum" and "StrictTriangSum".

Format

```
DiagSum <operator> <value of the sum of the diagonal elements>
VarSum <operator> <value> {i..j}
RowSum <number of the line> <operator> <value of the row sum>
SquareSum {i1..i2} {j1..j2}<operator> <value of the sum>
TriangSum {i..j} <operator> <value of the sum >
StrictTriangSum {i..j} <operator> <value of the sum>
```

Examples

* `DiagSum = 2.0 :` $\sum_{i=1}^{n} X_{ii} = 2$
* `VarSum = 10` $\{1..n\}$ `:` $\sum_{i=1}^{n} x_i = 10 \Leftrightarrow \sum_{i=1}^{n} X_{n+1,i} = 10$
* `RowSum 2 < 3.0 :` $\sum_{j=1}^{n} X_{2j} \leq 3$
* `SquareSum` $\{1..2\}$ $\{3..n\}$ `> 5 :` $\sum_{i=1}^{2} \sum_{j=3}^{n} X_{ij} \geq 5$
* `TriangSum` $\{1..3\}$ `= 5 :` $\sum_{i=1}^{3} \sum_{j=i}^{3} X_{ij} = 5$
* `StrictTriangSum` $\{1..3\}$ `= 5 :` $\sum_{i=1}^{3} \sum_{j=i+1}^{3} X_{ij} = 5$

3.4.4. *The "ProductConstrVect" and "ProductConstrBarVect" labels.* These two labels allow to make the product of linear constraints with the components x_i or $1 - x_i$ ($i \in \{1, ..., n\}$) of the vector x in order to build new "quadratic" constraints. The keywords are: "ProductConstrVect" to multiply some linear constraints by x_i, and ProductConstrBarVect to multiply by $1 - x_i$ ($i \in \{1, ..., n\}$) . These constraints have been introduced by Adams and Sherali [1].

Format of ProductConstrVect

```
ProductConstrVect[Right | Left]< filename >
```

Format of ProductConstrBarVect

```
ProductConstrBarVect[Right | Left]< filename >
```

Here, the file "filename" is read (it must contains only linear constraints !), and for each constraint in this file, the quadratic constraints are equal to the product of the constraint by each component x_i $i \in \{1, ..., n\}$ (or $1 - x_i$ for the *ProductConstrBarVect* function) of the vector x (and will be added to the semidefinite relaxation). The filename indicated between the parentheses must be declared before in the list of all the constraints, in order to know if the file is written in the "Sparse" or "Dense" format. But one may comment it by putting a "_" at the beginning of the line where it is declared (if one wants to use only the product constraints). The option 'Right' or 'left' is useful only for linear constraints that are of the "<>" type. If 'Right' is precised, only the right inequality will be quadratized. If 'Left' is precised, only the left inequality will be quadratized. The default value is 'Right'. Hence, to obtain "quadratized" constraints by multiplying all the inequalities of a "l<>" file "constraints.lin" by x_i $i \in \{1, ..., n\}$, one must write:

```
ProductConstrVectRight(constraints.lin)
ProductConstrVectLeft(constraints.lin)
```

Example 3.4. In this example, the SDP relaxation will contains the linear constraints *and* the constraints that are quadratized (by multiplying by x_i $i \in \{1, ..., n\}$) from the file "constraints_lin.dat":

```
File Sparse : constraints_lin.dat
ProductConstrVectRight(constraints_lin.dat)
```

Example 3.5. Here, only the constraints that are quadratized from the file "constraints_lin.dat" will be considered in the SDP relaxation (here we mupltiply them by both x_i and $1 - x_i$ $i \in \{1, ..., n\}$):

```
_File Sparse : constraints_lin.dat
ProductConstrVectRight(constraints_lin.dat)
ProductConstrBarVectRight(constraints_lin.dat)
```

3.4.5. *The Simple label.* In some cases, one may simply wants to copy the linear constraints of $(P\{0, 1\})$, i.e. without replacing it by on of the two sets of constraints described in Table 1. It is possible by using the **simple** label as follows.

```
_File Sparse : constraints_lin.dat
Simple(constraints_lin.dat)
```

4. A small example

All the files corresponding to this example can be found in the directory SDP_S/examples/PROBLEM. Assume that we want to obtain a lower bound by semidefinite programming (by using SDP_S)

for the following problem :

$$\begin{cases} \text{Maximize} & 2x_1x_2 + x_2x_3 + x_1 + x_2 + x_3 + 2 \\ \text{Subject to} & x_1x_2 + x_3 \leq 1 \\ & x_1 + x_2 + x_3 \leq 2 \\ & 2x_1 + 3x_2 + 5x_3 \leq 6 \\ & x \in \{0,1\}^n \end{cases}$$

Five files have to be written: the problem file, the "C" file, the "l" file, the linear constraints file, and the quadratic constraints file. Here, in addition we decide to use the function "ProdConstrVect" with all the linear constraints, and we add the constraints $X_{ij} \geq 0$ for all $i < j \in \{1, 2, 3\}$.

4.1. **The problem or "master" file: "problem.pb".** This file contains the general description of the problem.

```
Max CX + lx + 2
C : file Sparse : problem.C
l : file Sparse : problem.l
Dim : 3
Type : {0,1}
AddMoreConstr : 1
Constraints :
File Sparse : problem.lin
File Sparse : problem.quad
ProductConstrVect(problem.lin)
```

4.2. **The "C" file: "problem.C".** This file contains the values of all the quadratic part of the objective function:

```
3  //dimension
2  //number of quadratic terms
1 2 1.0
2 3 0.5
```

(since the matrix described is symmetric).

4.3. **The "l" file: "problem.l".** This file contains the values of all the linear part of the objective function:

```
3  //dimension
3  //number of linear terms
1 1.0
2 1.0
3 1.0
```

4.4. **The linear constraints file: "problem.lin".** This file contains the two linear constraints:

```
l<
3  //dimension
2  //number of the constraints
2  //upper bound
3  //number of the non-zero terms of the vector
1 1.0
2 1.0
3 1.0
6
3
1 2.0
2 3.0
3 5.0
```

4.5. **The quadratic constraints file : "problem.quad".** This file contains the quadratic constraint:

```
Q<
3  //dimension
1  //number of constraints
1  //upper bound
1  //linear part
3 1.0
1  //quadratic part
1 2 0.5
```

4.6. **The option file : "problem.opt".** This file is created by SDP_S if it is missing in the current directory. But here, we have chosen to write our own:

```
-si 1 -sh 1 -te 1e-3
```

"-te 1e-e3" is an option of the SB solver which specifies the relative precision for termination. The default value (equal to 1e-5) is too small for our small combinatorial problem.

4.7. **Output.** Running this small example, you should obtain this output (the file can be found in the directory SDP_S/examples/PROBLEM):

```
SDP_S version 1.0, Copyright (C) 2002 G.DELAPORTE, S.JOUTEAU, F.ROUPIN
SDP_S comes with ABSOLUTELY NO WARRANTY
Maximizing
Dim : 3
Filename C : problem.C
Filename l : problem.l
Reading Linear constraints in file "problem.lin".
2 linear constraints read.
Reading Quadratic constraints in file "problem.quad".
1 quadratic constraints read.
Adding 6 product constraints made from file problem.lin
Adding constraints : Xij >= 0 ; 1<=i<j<=n
SBmethod version 1.1.1, Copyright (C) 2000 Christoph Helmberg
SBmethod comes with ABSOLUTELY NO WARRANTY
elapsed time: 00:00:00.00 ---- Fri Sep 20 15:39:46 2002
Dimension = 5 Number of constraints = 16
00:00:00.00 Primal : 0 Bound : (7.7308752)
00:00:00.00 Primal : 6.9162863 Bound : (7.1941383)
00:00:00.00 Primal : 7.0532889 Bound : (7.1012386)
00:00:00.00 Primal : 6.9580565 Bound : (7.0363676)
00:00:00.01 Primal : 6.9093106 Bound : (6.9968169)
00:00:00.02 Primal : 6.8362214 Bound : (6.9593700)
00:00:00.02 Primal : 6.7742534 Bound : (6.8696101)
00:00:00.02 Primal : 6.6361816 Bound : (6.7666645)
00:00:00.02 Primal : 6.4939071 Bound : (6.5743581)
00:00:00.02 Primal : 6.2397256 Bound : (6.2868662)
00:00:00.02 Primal : 5.8958654 Bound : (6.1128962)
00:00:00.03 Primal : 5.9091241 Bound : (6.0623258)
00:00:00.04 Primal : 5.9129459 Bound : (6.0332024)
00:00:00.04 Primal : 5.9007371 Bound : (6.0176165)
00:00:00.05 Primal : 5.9322197 Bound : (6.0086182)
00:00:00.12 Primal : 5.9663304 Bound : (6.0039070)
00:00:00.25 Primal : 5.9839651 Bound : (6.0018570)
00:00:00.58 Primal : 5.9925850 Bound : (6.0009266)
00:00:00.69 Primal : 5.9939318 Bound : (6.0009266)
>>>elapsed time: 00:00:00.69 ---- Fri Sep 20 15:39:46 2002
-------- SDP bound : 6.000927
Vector x:
0.999920 0.999521 0.000156
```

We obtain here an integral solution $(1, 1, 0)$ and the optimal value 6. Look the other examples in SDP_S/examples. In particular, there are the source codes to generate the input files for

SDP_S for the Quadratic Assignment and the Memory-Constrained Allocation problems. Details are given in the corresponding "README" files in /examples/QAP and /examples/MCAP.

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[4] F. Roupin. *From Linear to Semidefinite Programming : an Algorithm to obtain Semidefinite relaxations for bivalent Quadratic problems*. Journal Of Combinatorial Optimization 8(4):469-493, 2004.

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