M2 PLS

2024-2025

Complex systems
Part 1: Model checking

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Objectives of the module

- introduce formal models for critical systems specification
  1. in an untimed setting
  2. in a timed setting
  3. in a parametric timed setting
- use model checking to verify their properties
  - properties expressed in extensions of the LTL and CTL logics

Objectives of this part of the module

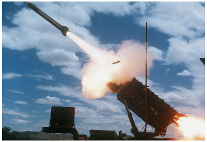
- introduce formal models for critical systems specification
  - finite-state automata
  - their extensions
- use model checking to verify their properties
  - reachability
  - properties expressed in LTL and CTL logics
- mention symbolic representations

Context: Verifying complex timed systems

- Critical systems: Failures may result in dramatic consequences
- Need for early bug detection
  - Bugs discovered when final testing: expensive

  ⇝ Need for a thorough specification and verification phase

Northeast blackout
(USA, 2003)



MIM-104 Patriot Missile Failure
(Iraq, 1991)

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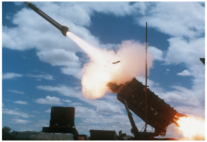

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  - Abstract interpretation
  - Theorem proving
  - Model checking

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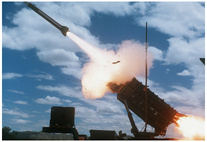

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The Therac-25 radiation therapy machine (1/2)

- Radiation therapy machine used in the 1980s
- Involved in accidents between 1985 and 1987, in which patients were given massive overdoses of radiation
  - Approximately 100 times the intended dose!
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"The failure only occurred when a particular nonstandard sequence of keystrokes was entered on the VT-100 terminal which controlled the PDP-11 computer: an `X` *to (erroneously) select 25MV photon mode followed by* ↑, `E` *to (correctly) select 25 MeV Electron mode, then* `Enter`*, all within eight seconds."*

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Limits of testing

This case illustrates the difficulty of bug detection without formal methods.

Bugs can be difficult to find

…and can have dramatic consequences for critical systems:

- health-related devices
- aeronautics and aerospace transportation
- smart homes and smart cities
- military devices
- etc.

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Hence, high need for formal verification

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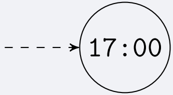

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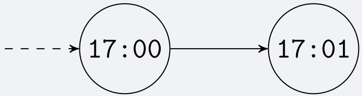

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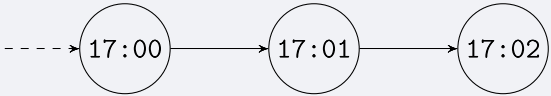

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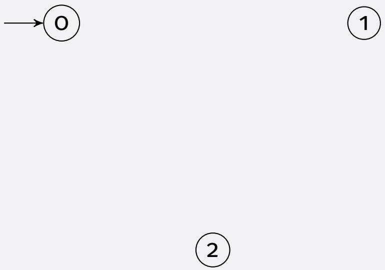

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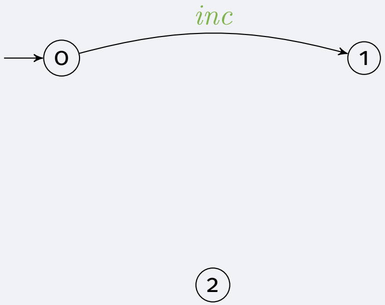

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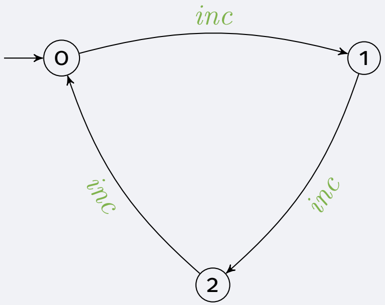

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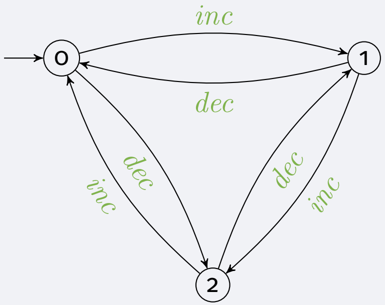

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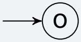

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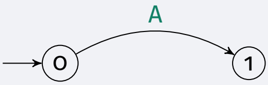

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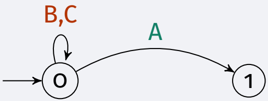

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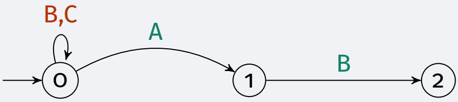

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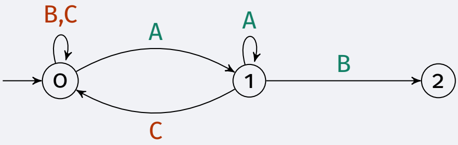

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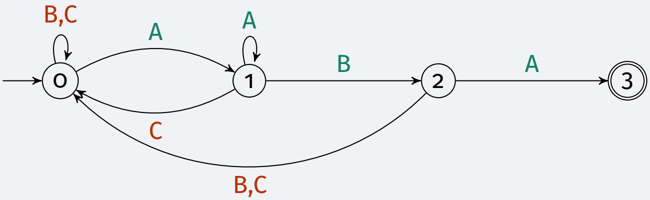

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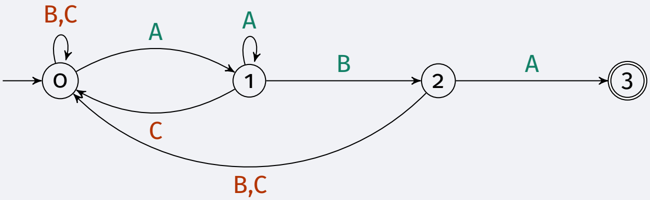
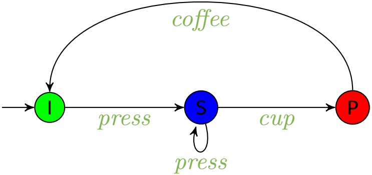


Remark

The digits in the states represent the number of consecutive correct keys that have been pressed.

Example: The coffee machine

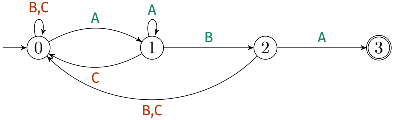


- I Idling
- S Adding sugar
- P Preparing coffee

Executions of a model (1/2)

Definition (Execution)

An execution is a sequence of states describing a possible evolution of the system.



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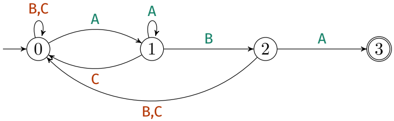

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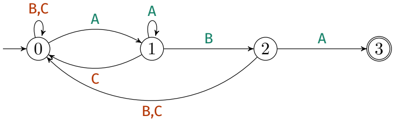

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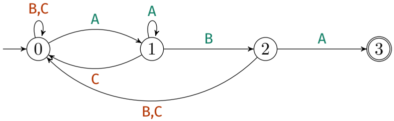

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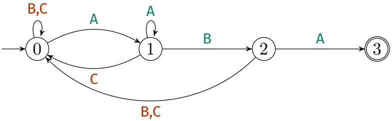

Examples of interesting questions on the numerical code example:

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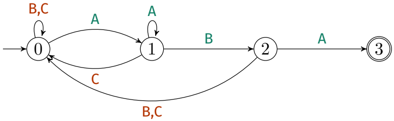

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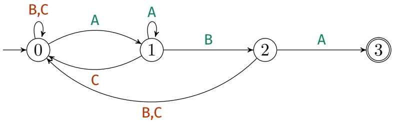

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- Is there a possible infinite execution?

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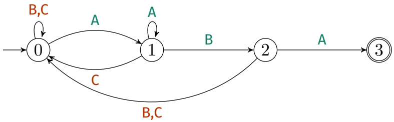

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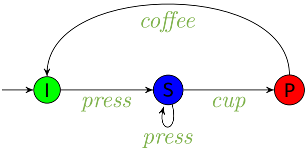

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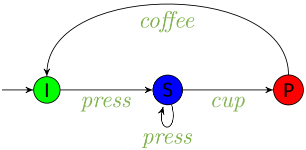

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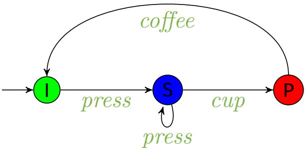

- Example of executions
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  - And so on

Execution tree

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A tree to represent all possible executions

- root: initial state of the automaton
- children of a node: its immediate successors (states accessible from the node in one step)

May be infinite!

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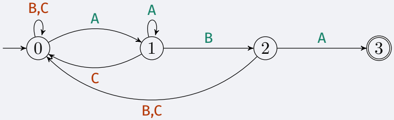

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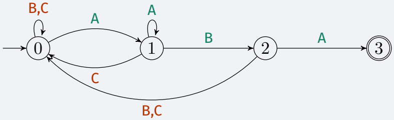

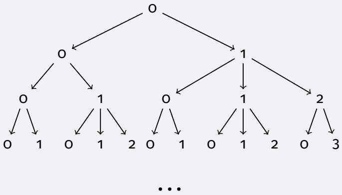

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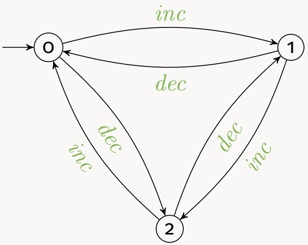
Execution tree for the modulo 3 counter



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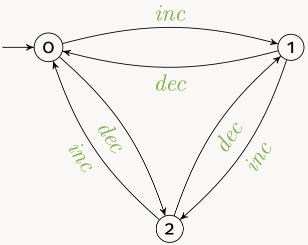

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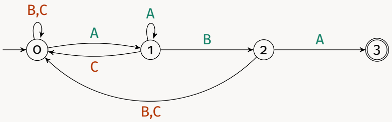

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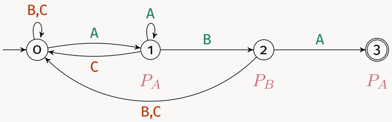

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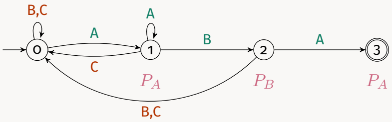

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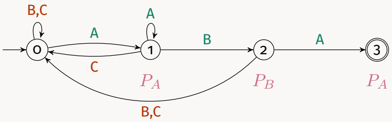

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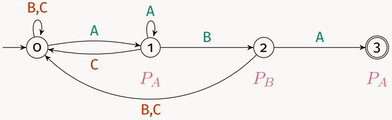

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Formal definition of automata

Definition (Automaton)

Let AP be a set of atomic propositions. An automaton is a tuple $\mathcal{A} = \langle Q, \Sigma, T, q_0, lab, F \rangle$ such that:

- Q is a finite set of states
- Σ is a finite set of transition labels
- $T \subseteq Q \times \Sigma \times Q$ is a set of transitions
- $q_0 \in Q$ is the (unique) initial state
- $lab : Q \to 2^{AP}$ associates with each state a finite set of atomic propositions
- $F \subseteq Q$ is a set of final states

Example

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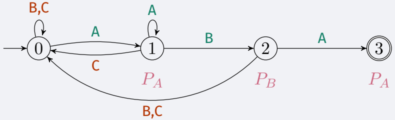

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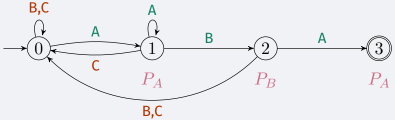

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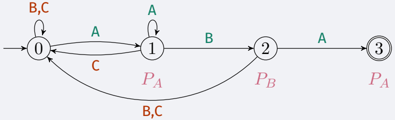

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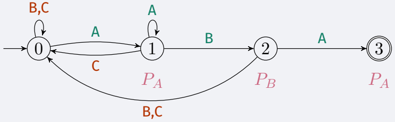

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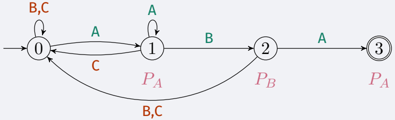

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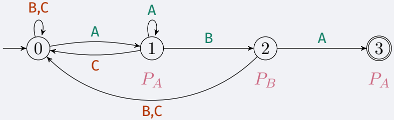

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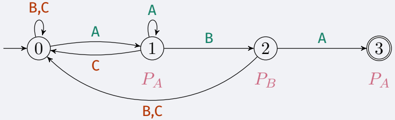

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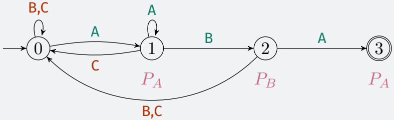

Exercise: draw the automaton

$\mathcal{A} = \langle Q, \Sigma, T, q_0, lab, F \rangle$, with

- $Q = \{q_1, q_2, q_3\}$
- $\Sigma = \{a, b, c, d\}$
- $q_0 = q_1$
- $F = \{q_2\}$
- $\forall q \in Q : lab(q) = \emptyset$
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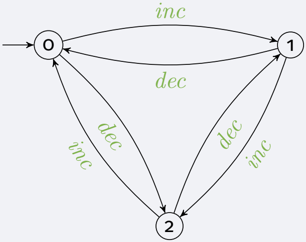
Exercise: formalize the automaton

Exercise (Formal representation of the modulo 3 counter)



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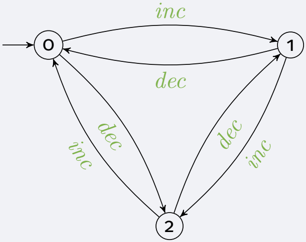

Behavior: runs

Definition (Run (or path))

- A run (or path) of an automaton $\mathcal{A}$ is a sequence ρ of successive transitions (q_i, a_i, q'_i) of $\mathcal{A}$, i. e., such that $\forall i, q_{i+1} = q'_i$.
  $\rho = q_1 \xrightarrow{a_1} q_2 \xrightarrow{a_2} q_3 \xrightarrow{a_3} q_4 \ldots$
- The length of a run ρ is its number of transitions $|\rho| \in \mathbb{N} \cup \{+\infty\}$
- The ith state of ρ is the state q_{i+1} reached after i transitions.

Behavior: executions

Definition (Execution)

- A partial execution of $\mathcal{A}$ is a run starting from the initial state q_0.
- A complete execution of $\mathcal{A}$ is an execution that is maximal. It is either infinite or ends in a state where no transition is possible. This state might be final (in F), or a deadlock.
- A state is reachable if there exists an execution in which it appears.
- The complete executions define the behavior of the automaton.

Exercise: mutual exclusion

Exercise (Mutual exclusion between two processes)

Specification

- two processes execute and need access to the same resource
- each process can request access to a critical section of its code
- they must not execute this part at the same time
- when they have finished they signal they exit their critical section and loop back to their initial state

Questions

1. Model this problem with an automaton
2. Associate atomic properties with each state
3. Is the mutual exclusion requirement satisfied?
4. Is the system fair?
5. What would happen if you wanted to add a third process?

Exercise: mutual exclusion (solution)

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Exercise: mutual exclusion (solution)

Outline

Extension with variables

Why and how to use variables?

- More compact models, improving readability (but not necessarily more expressive than pure automata!)
- Guards and updates on transitions

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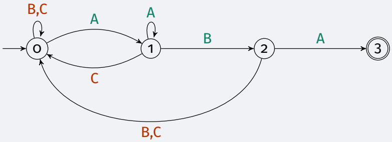

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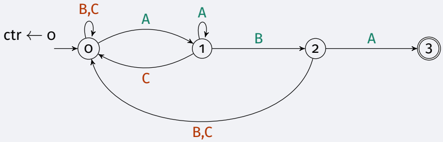

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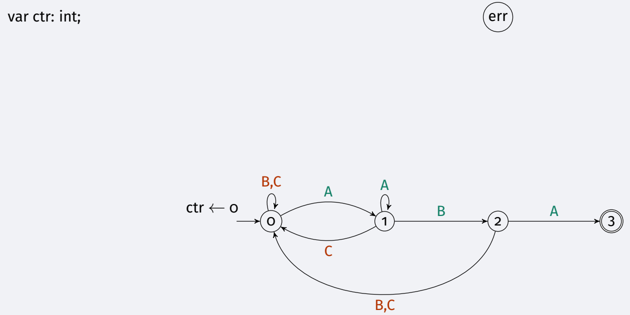

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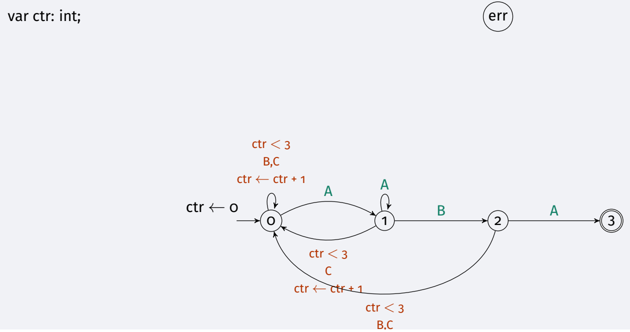

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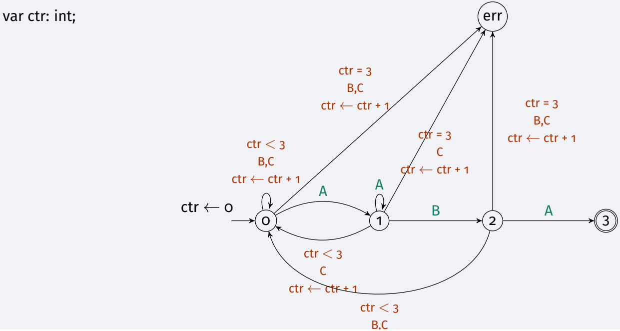

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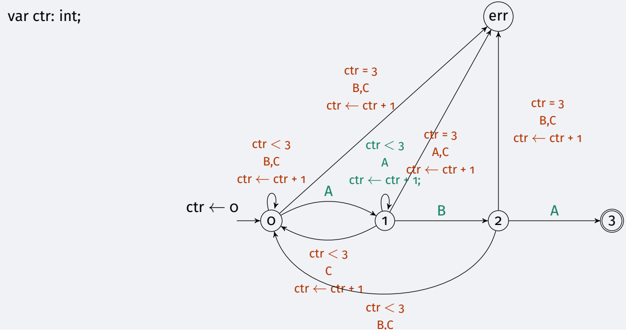

Extension with variables: without?

Exercise (The numerical code with 3 errors without variables)

Extension with variables: without?

Exercise (The numerical code with 3 errors without variables)

Synchronized product

Why?

- each component of the system is designed as an automaton
- composition of automata

Synchronized product

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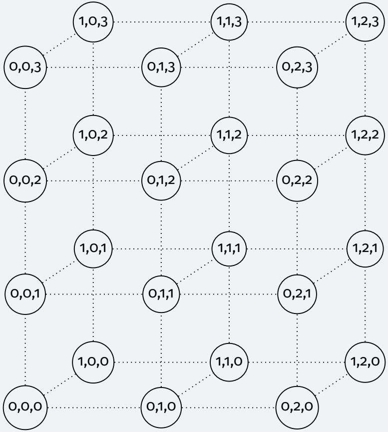
- each component of the system is designed as an automaton
- composition of automata

How?

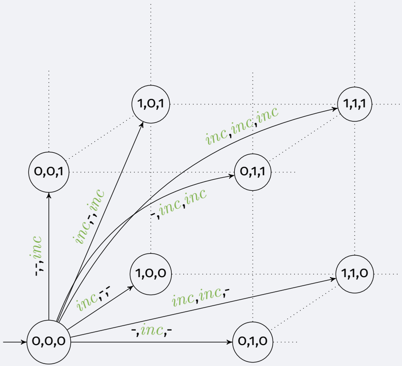
- independent actions lead to a Cartesian product of states
- synchronized actions occur simultaneously

Synchronized product: examples

Example (3 counters, modulo 2, 3, 4: states)

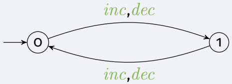


Example (3 counters: some transitions)

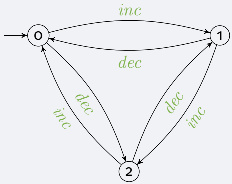


Example: Synchronized counters

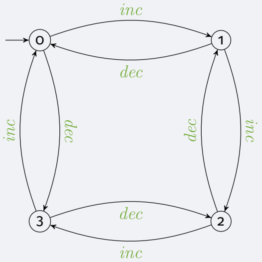
Modulo 2 counter



Modulo 3 counter



Modulo 4 counter



Example: Synchronized counters

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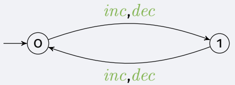

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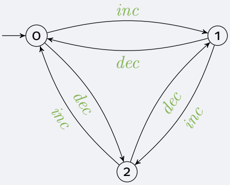

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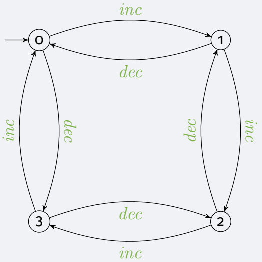

Synchronized actions: all counters increment or decrement simultaneously

Example: Synchronized counters

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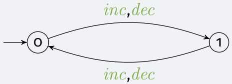

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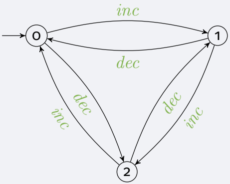

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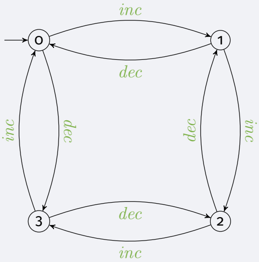

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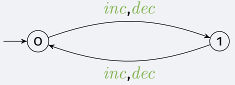

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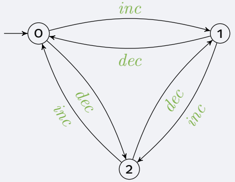

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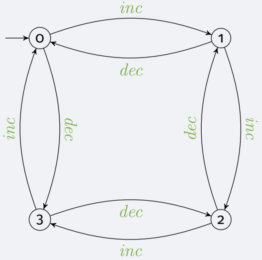

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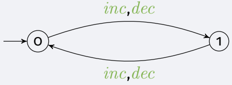

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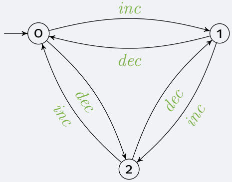

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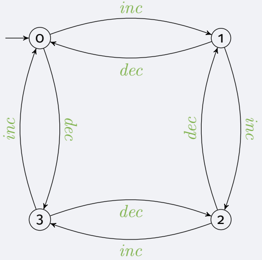

Synchronized actions: all counters increment or decrement simultaneously

Formal definition of the Cartesian product

Let $(\mathcal{A}_i)_{1 \leq i \leq n}$ be a family of automata $\mathcal{A}_i = \langle Q_i, \Sigma_i, T_i, q_{0i}, lab_i, F_i \rangle$.

Definition (Cartesian product of automata)

The Cartesian product $\mathcal{A}_1 \times \cdots \times \mathcal{A}_n$ of the automata in the family is the automaton $\mathcal{A} = \langle Q, \Sigma, T, q_0, lab, F \rangle$ such that :

- $Q = Q_1 \times \cdots \times Q_n$
- $\Sigma = \prod_{1 \leq i \leq n} \big(\Sigma_i \cup \{\epsilon\} \big)$ (where ϵ represents a silent action)
- $$T = \Big\{ \big((q_1, \dots, q_n), (a_1, \dots, a_n), (q'_1, \dots, q'_n) \big) \mid \forall 1 \leq i \leq n, (a_i = \epsilon \wedge q'_i = q_i) \vee \big(a_i \neq \epsilon \wedge (q_i, a_i, q'_i) \in T_i \big) \Big\}$$
- $q_0 = (q_{01}, \dots, q_{0n})$
- $\forall (q_1, \dots, q_n) \in Q : lab\big((q_1, \dots, q_n)\big) = \bigcup_{1 \leq i \leq n} lab_i(q_i)$
- $F = \big\{ (q_1, \dots, q_n) \in Q \mid \exists 1 \leq i \leq n, q_i \in F_i \big\}$

Formal definition of the synchronized product

Let $(\mathcal{A}_i)_{1 \leq i \leq n}$ be a family of automata $\mathcal{A}_i = \langle Q_i, \Sigma_i, T_i, q_{0i}, lab_i, F_i \rangle$.

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The synchronization set, denoted by $Sync$, describes all permitted simultaneous actions:

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Definition (Synchronized product of automata)

The synchronized product of $(\mathcal{A}_i)_{1 \leq i \leq n}$ over a set $Sync$ is the Cartesian product restricted to $\Sigma = Sync$.

Synchronization by message passing

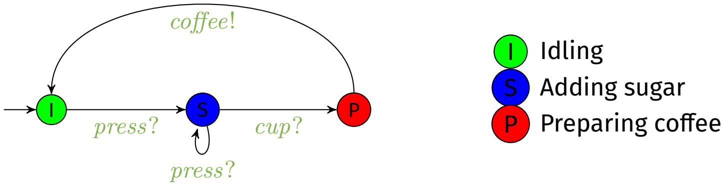
Message passing: a special case of synchronized product

$m!$ send a message m

$m?$ receive a message m

- reception and sending occur simultaneously
- they concern the same message

Synchronization by message passing: coffee machine



Exercise

Is this coffee machine environment-unfriendly (providing disposable cups) or environment-friendly (requesting the user to bring their own cup)?

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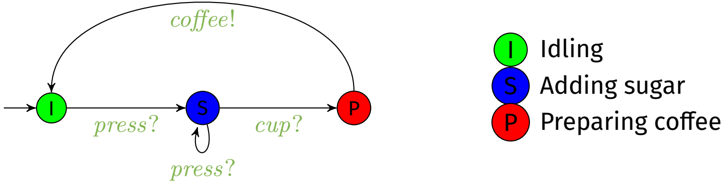

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A coffee drinker (sugarless)

- Specify a coffee drinker automaton $\mathcal{A}_{D1}$ that performs forever the following actions:
  1. press the button once
  2. place the cup
  3. wait for the coffee
  4. drink the coffee
  5. put the cup to the washing machine

  and so on

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- Specify a coffee drinker automaton $\mathcal{A}_{D2}$ that works just as $\mathcal{A}_{D1}$ except that they can nondeterministically ask for 0, 1 or 2 doses of sugar.

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A washing machine

- Specify a washing machine automaton $\mathcal{A}_W$ that accepts cups to wash, and once 5 cups are placed into the washing machine, then the machine washes all cups.

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Synchronization by message passing: lift example

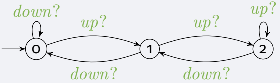
Example (A small lift)

Model of a lift in a 3-level building, made of:

- **the cabin** which goes up and down according to the current level and the lift controller commands
- **3 doors** (one per level) which open and close according to the controller's commands
- **a controller** which operates the lift

Synchronization by message passing: lift example (exercise)

Cabin



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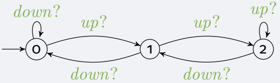

i^{th} door

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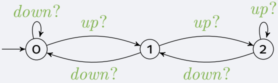

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Controller

Synchronization by message passing: lift example (exercise)

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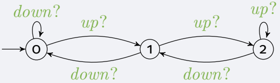

i^{th} door

Controller

Examples of properties

Exercise: Mutual exclusion problem

Exercise (Mutual exclusion problem)

1. Model the mutual exclusion problem with message passing:
   - one automaton per participating process (2 processes)
   - a controller
2. How do you add a new process? Give the model for 3 processes, and explain how to generalize it to n processes

Exercise: Mutual exclusion problem (solution)

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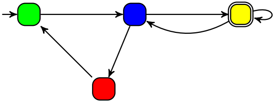
Outline

Model checking timed concurrent systems

- Principle of model checking [BK08]



A model of the system

 is unreachable

A property to be verified

- [BK08] Christel Baier and Joost-Pieter Katoen. *Principles of Model Checking.* MIT Press, 2008. ISBN: 978-0-262-02649-9

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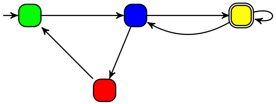

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$\overset{?}{\models}$

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- Question: does the model of the system satisfy the property?

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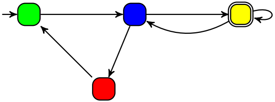

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$\stackrel{?}{\models}$

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A property to be verified

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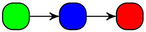
Yes

No



Counterexample

Turing award (2007) to Edmund M. Clarke, Allen Emerson and Joseph Sifakis

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Outline

Introduction to temporal logics

- Express dynamic behavior of the system
- Use formal syntax and semantics to avoid any ambiguity
- Capture statements and reasoning that involve the notion of order in time

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The logic CTL*

- Atomic propositions
- Logical (Boolean) operators:
  - `true`, `false`
  - $\neg$ (negation)
  - $\wedge$ (and), $\vee$ (or)
  - $\implies$ (logical implication), $\iff$ (if and only if)
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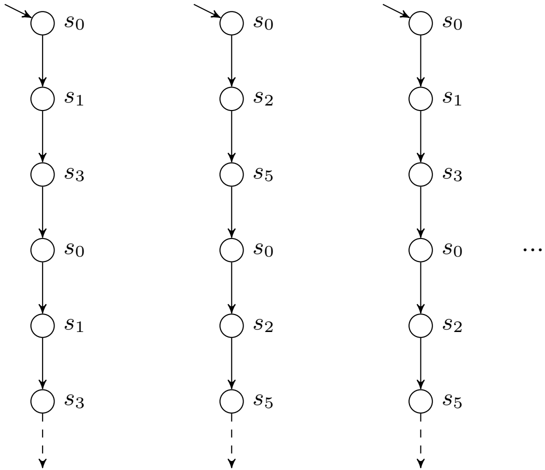
Two main subsets of CTL*

LTL Linear-time Temporal Logic: events are totally ordered

CTL Computation Tree Logic: events are partially ordered

Outline

LTL: Linear-time Temporal Logic



Syntax of LTL

LTL expresses formulas on the order between the future atomic propositions for one given path, over a set of atomic propositions AP

Definition (Minimal syntax of LTL)

$$LTL \ni \varphi ::= p \mid \neg\varphi \mid \varphi \vee \varphi \mid \mathsf{X}\varphi \mid \varphi \mathsf{U} \psi$$

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Informal illustration of the LTL semantics

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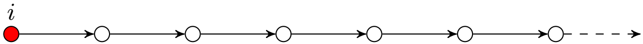

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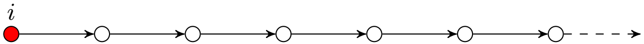

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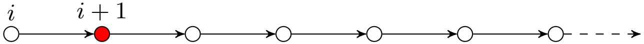

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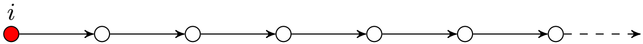

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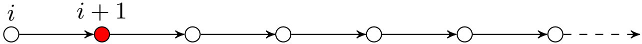

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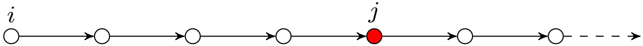

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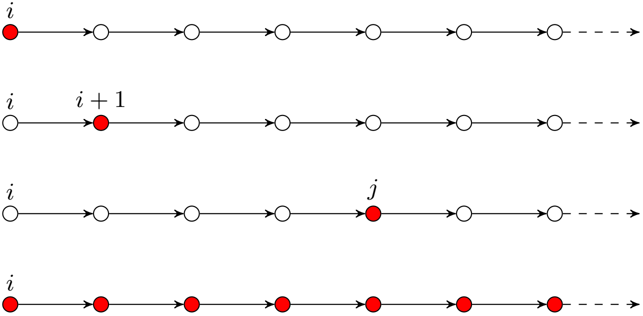

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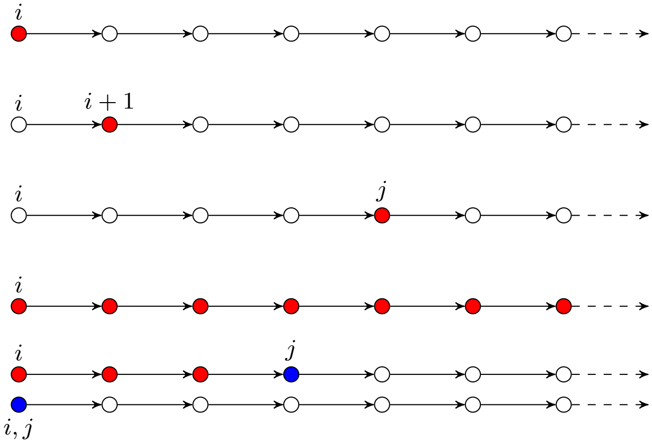

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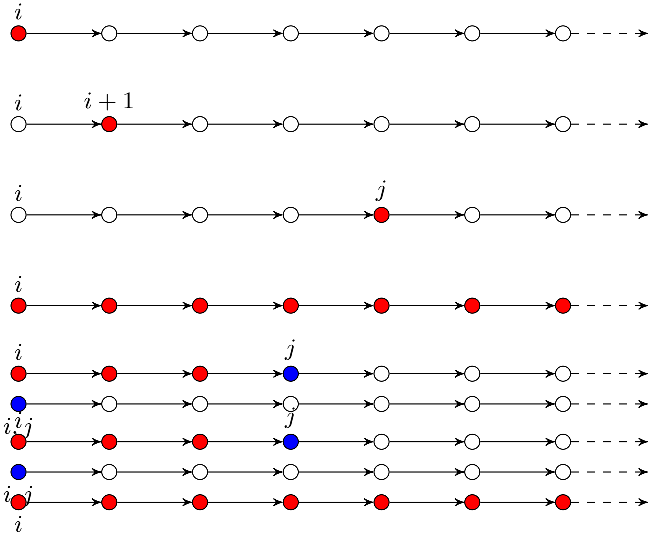

Semantics of LTL

Let ρ be a finite run and $p \in AP$ an atomic proposition.
"$\rho, i \models \varphi$" denotes that, at position i of its execution, ρ satisfies formula φ.

Definition (Semantics of LTL)

$\rho, i \models p$	if	$p \in lab\big(\rho(i)\big)$
$\rho, i \models \neg\varphi$	if	$\rho, i \not\models \varphi$
$\rho, i \models \varphi \wedge \psi$	if	$\rho, i \models \varphi$ and $\rho, i \models \psi$
$\rho, i \models \mathsf{X}\varphi$	if	$i < \lvert\rho\rvert$ and $\rho, i+1 \models \varphi$
$\rho, i \models \mathsf{F}\varphi$	if	$\exists j$ s.t. $i \leq j \leq \lvert\rho\rvert : \rho, j \models \varphi$
$\rho, i \models \mathsf{G}\varphi$	if	$\forall j$ s.t. $i \leq j \leq \lvert\rho\rvert : \rho, j \models \varphi$
$\rho, i \models \varphi \mathsf{U} \psi$	if	$\exists j$ s.t. $i \leq j \leq \lvert\rho\rvert : \rho, j \models \psi$ and $\forall k$ s.t. $i \leq k < j : \rho, k \models \varphi$

Exercise: Additional Boolean operators

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Express $\vee$, $\implies$, $\iff$ by using $\neg$ and $\wedge$

$$\varphi \vee \psi \quad \equiv$$

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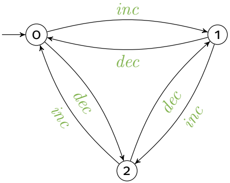
Examples of LTL formulae (counter)



- What do the following formulae mean?
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Example (Modulo 3 counter)

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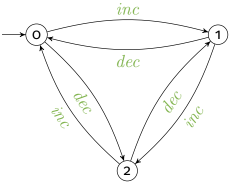

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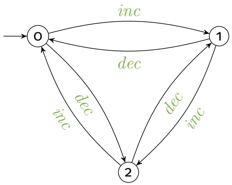

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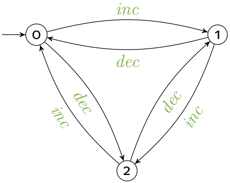

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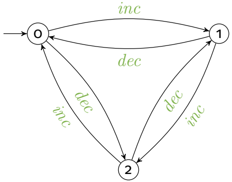

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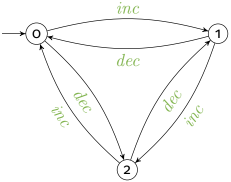

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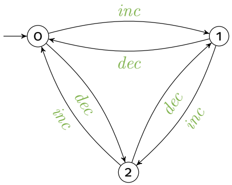

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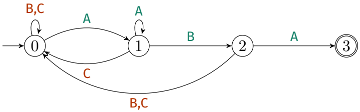

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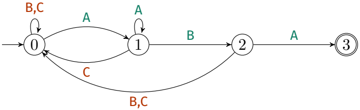

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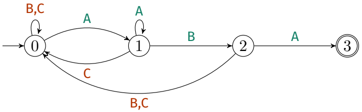

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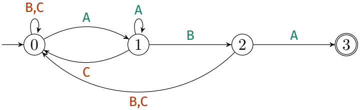

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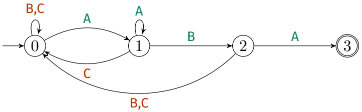

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Exercises: Writing LTL formulae (1/3)

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Express in LTL the following properties:

- "The plane will never crash"

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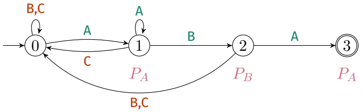

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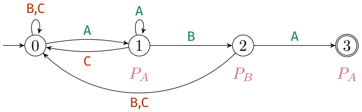

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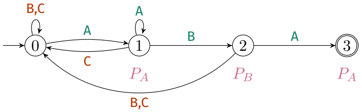

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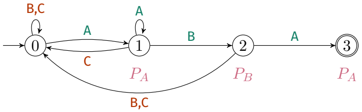

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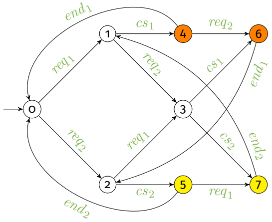

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Write an LTL formula satisfied by all runs where:

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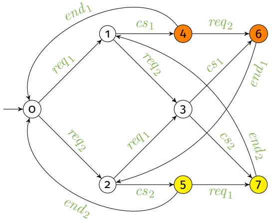

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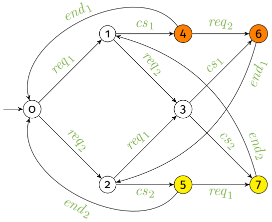

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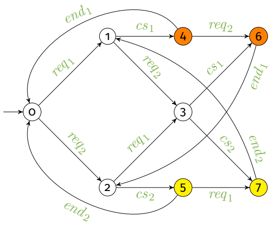

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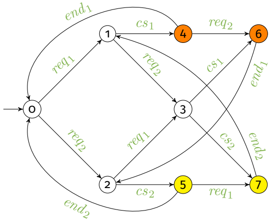

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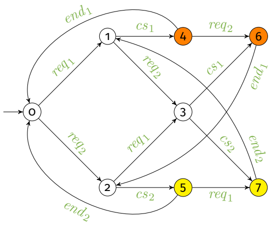

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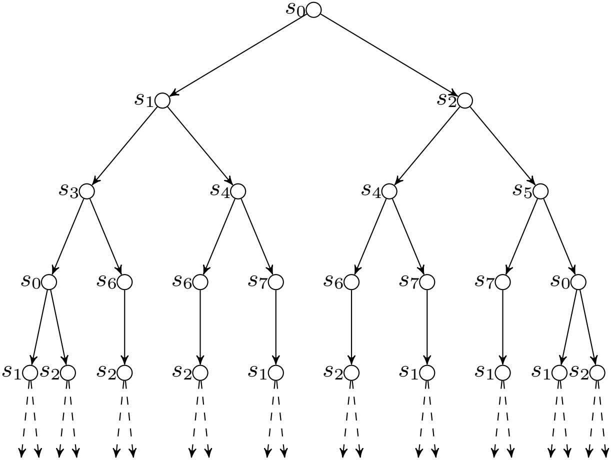
Outline

CTL: branching time



CTL (Computation tree logic)

CTL expresses formulas on the order between the future atomic propositions for some or for all paths, over a set of atomic propositions AP

Definition (Minimal syntax of CTL)

$$CTL \ni \varphi ::= p \mid \neg\varphi \mid \varphi \vee \varphi \mid \mathsf{EX}\varphi \mid \mathsf{E}\varphi\mathsf{U}\psi \mid \mathsf{A}\varphi\mathsf{U}\psi$$

Additional operators: F, G, R, W

Semantics of CTL

Same as LTL, plus:

$$\rho, i \models \mathsf{E}\varphi \quad \text{if} \quad \exists \rho' : \rho(0) \dots \rho(i) = \rho'(0) \dots \rho'(i) \text{ and } \rho', i \models \varphi$$
$$\rho, i \models \mathsf{A}\varphi \quad \text{if} \quad \forall \rho' : \rho(0) \dots \rho(i) = \rho'(0) \dots \rho'(i) \text{ we have } \rho', i \models \varphi$$

In CTL, each use of a temporal operator (X, F, G, U) must be in the immediate scope of a quantifier (E, A)

(This restriction does not apply in CTL*)

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A path quantifier must always be followed by a temporal operator.

Some useful combinations:

"there exists a path for which the next state is. . . "
"for all possible paths, the next state is. . . "
"it is possible that eventually. . . "
"in any case, eventually. . . "
"in any case, for all states. . . "

Illustration of the CTL semantics (1/8)

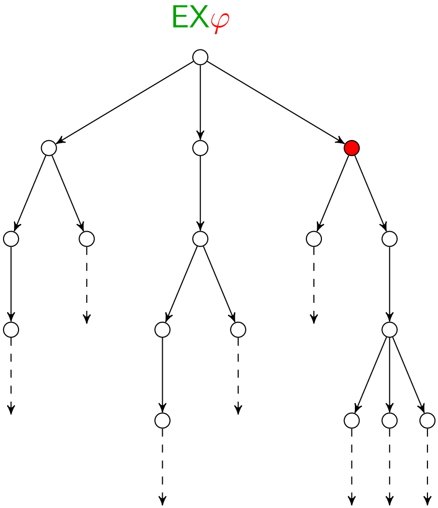


Illustration of the CTL semantics (2/8)

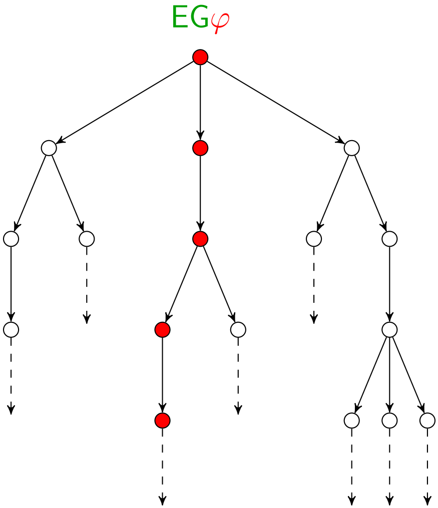


Illustration of the CTL semantics (3/8)

$\mathsf{E}\varphi\mathsf{U}\psi$

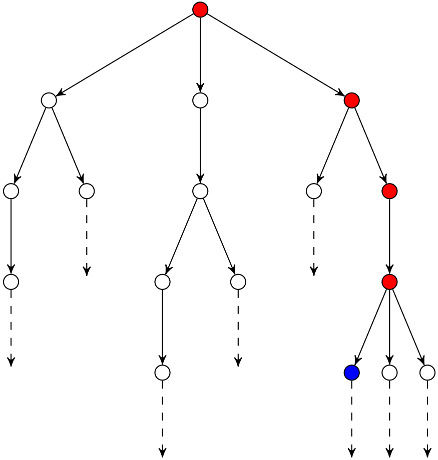
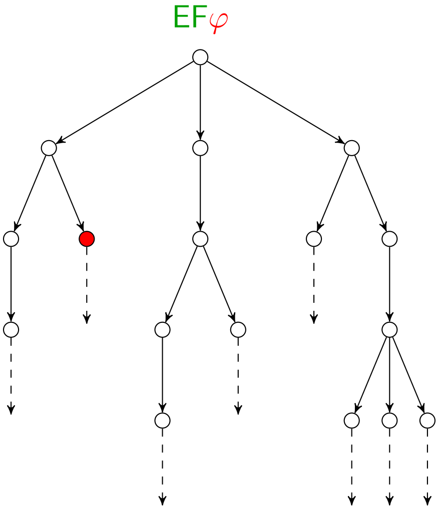


Illustration of the CTL semantics (4/8)



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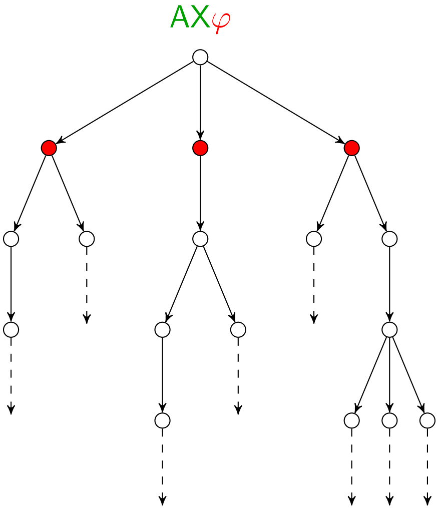


Illustration of the CTL semantics (6/8)

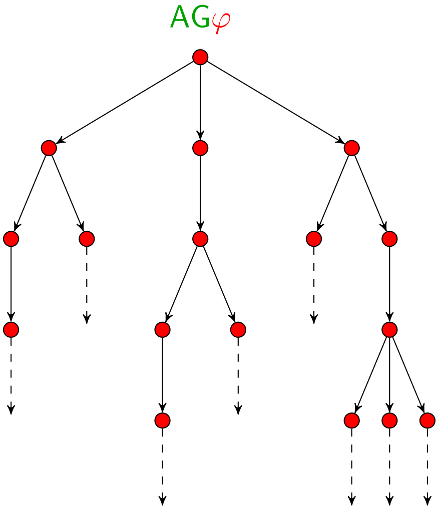


Illustration of the CTL semantics (7/8)

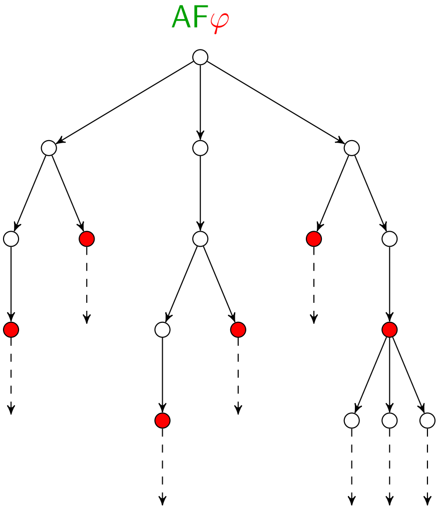


Illustration of the CTL semantics (8/8)

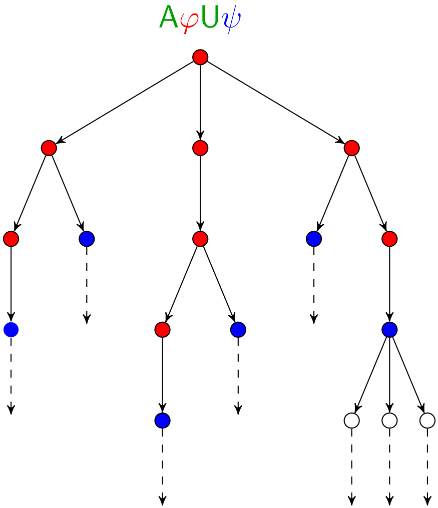
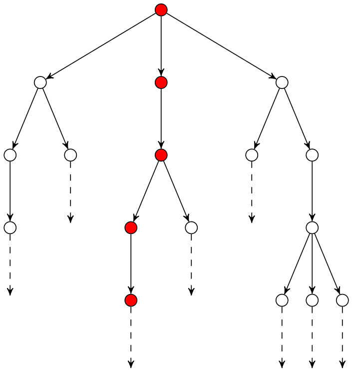


Illustration of the CTL semantics: Exercise

On which states are the following formulae valid?

1. $\mathsf{EX}\varphi$
2. $\mathsf{EF}\varphi$
3. $\mathsf{EG}\varphi$
4. $\mathsf{AX}\varphi$
5. $\mathsf{AG}\varphi$



Formally defining additional operators

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Examples of CTL formulae: Mutual exclusion

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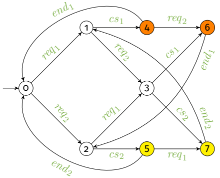

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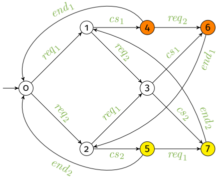

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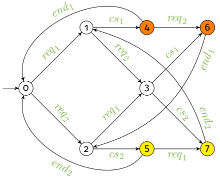

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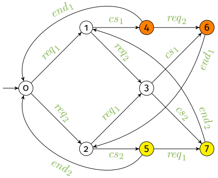

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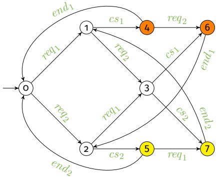

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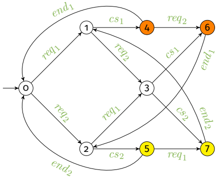

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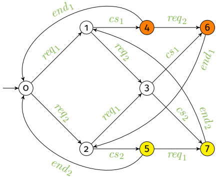

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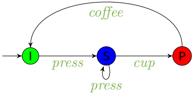
Express in CTL the following properties, and decide whether they are satisfied for the coffee machine

(We reason here with actions instead of state predicates)



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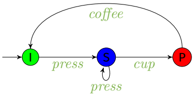

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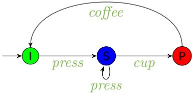

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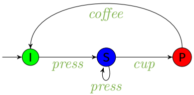

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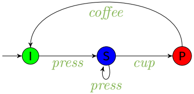

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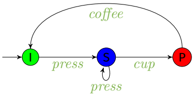

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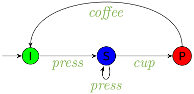

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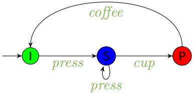

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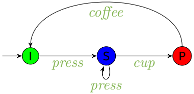

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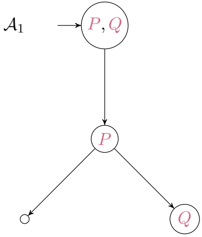
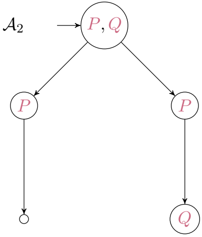
Outline

LTL and CTL do not recognize the same behaviors





LTL

Runs for both automata:

- $\{P, Q\}\ \{P\}\ \{-\}$
- $\{P, Q\}\ \{P\}\ \{Q\}$

$\forall \varphi : \mathcal{A}_1 \models \varphi \iff \mathcal{A}_2 \models \varphi$

CTL

$\mathcal{A}_1 \models \mathsf{AX}(\mathsf{EX}Q \land \mathsf{EX}\neg Q)$

$\mathcal{A}_2 \not\models \mathsf{AX}(\mathsf{EX}Q \land \mathsf{EX}\neg Q)$

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LTL model checking

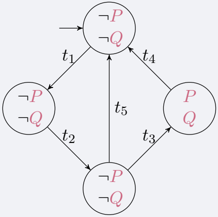
Algorithm working on path formulae

Principle for checking whether $\mathcal{A} \models \varphi$

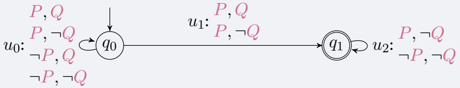
1. Construct the automaton $\mathcal{B}_{\neg\varphi}$ recognizing all executions not satisfying φ
2. Construct the synchronized product $\mathcal{A} \times \mathcal{B}_{\neg\varphi}$
3. If its recognized language is empty, then $\mathcal{A} \models \varphi$

Example

$\mathcal{A}$



$\mathcal{B}_{\neg\varphi}$ for $\varphi = \mathsf{G}(P \implies \mathsf{XF}Q)$



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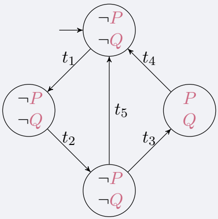

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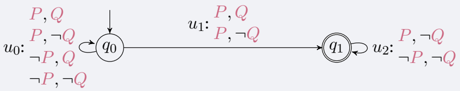

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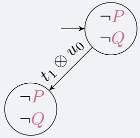

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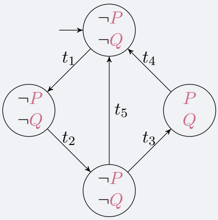
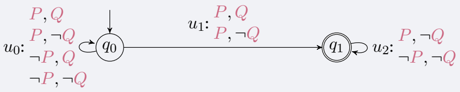
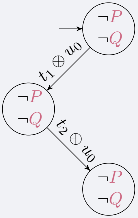


$\mathcal{B}_{\neg\varphi}$ for $\varphi = \mathsf{G}(P \implies \mathsf{XF}Q)$

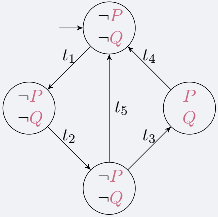


$\mathcal{A} \times \mathcal{B}_{\neg\varphi}$



Example

$\mathcal{A}$



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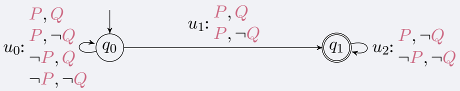

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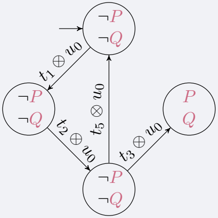
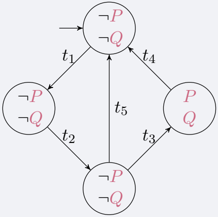


Example

$\mathcal{A}$



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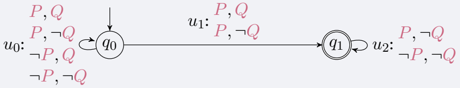

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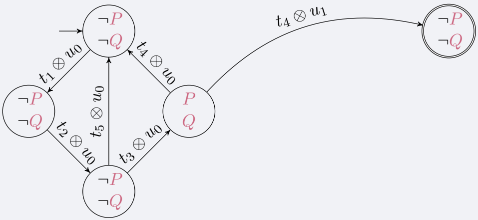
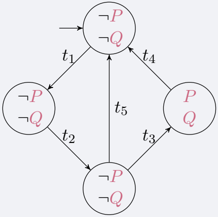


Example

$\mathcal{A}$



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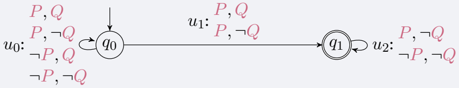

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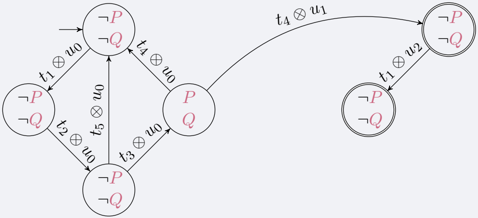
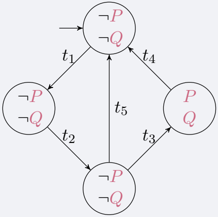


Example

$\mathcal{A}$



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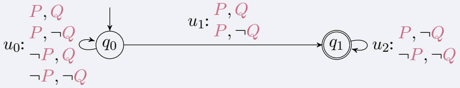

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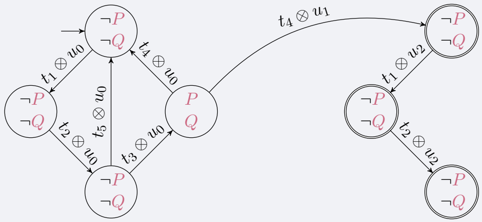
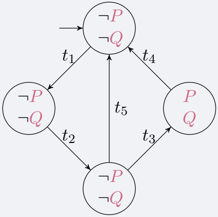


Example

$\mathcal{A}$



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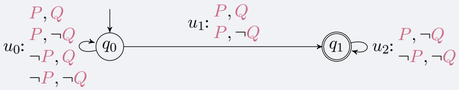

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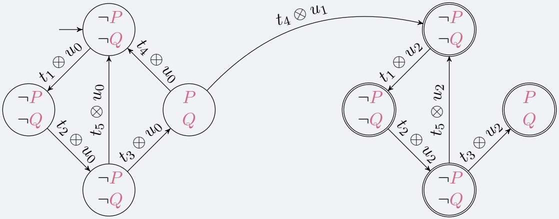
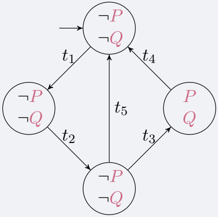


Example

$\mathcal{A}$



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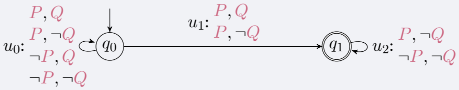
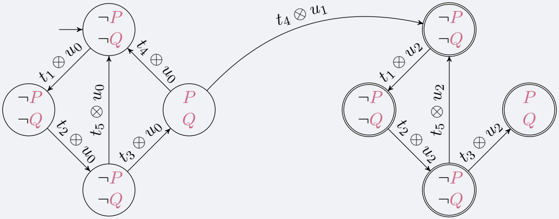


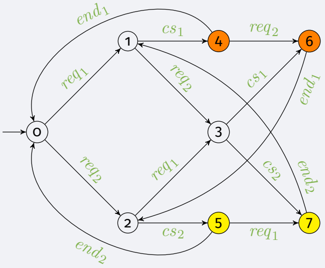
$\mathcal{A} \times \mathcal{B}_{\neg\varphi}$

$\mathcal{A} \not\models \varphi$



Exercise

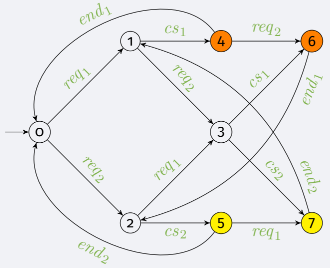
$\mathcal{A}$



$\mathcal{B}_{\neg\varphi}$ for $\varphi = \mathsf{G}\neg(cs_1 \wedge cs_2)$

Exercise

$\mathcal{A}$



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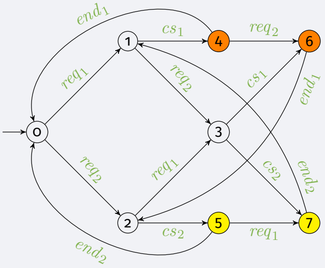
$\mathcal{A}$



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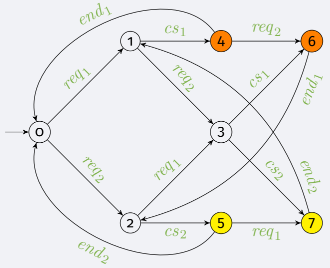
Exercise

$\mathcal{A}$

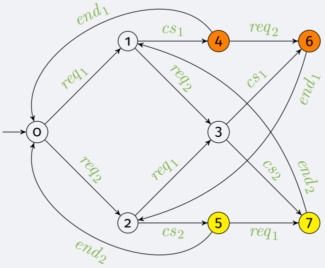


$\mathcal{B}_{\neg\varphi}$ for $\varphi = \mathsf{G}\neg(cs_1 \wedge cs_2)$

$\mathcal{A} \times \mathcal{B}_{\neg\varphi}$

Exercise

$\mathcal{A}$



$\mathcal{B}_{\neg\varphi}$ for $\varphi = \mathsf{G}\neg(cs_1 \wedge cs_2)$

$\mathcal{A} \times \mathcal{B}_{\neg\varphi}$

Outline

CTL model checking algorithm

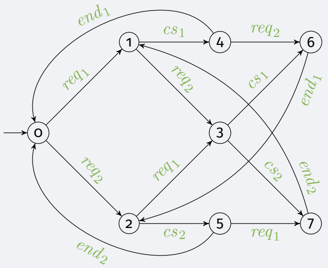
- Algorithm $markPred$ decides where a formula is satisfied
- Memorizes the already computed results
- Reuses the computed results of sub-formulae to compute new formulae

CTL model checking algorithm

Case 1: $\varphi = p$ (base case)

case $\varphi = p$ **do**
  forall $q \in Q$ **do**
    if $p \in lab(q)$ **then**
      $q.\varphi \leftarrow$ `true`
    else
      $q.\varphi \leftarrow$ `false`

$\varphi = R_1$



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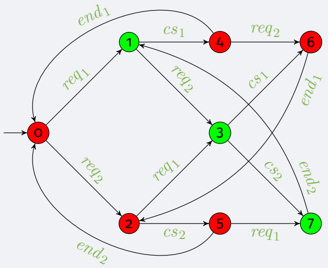

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Case 1: $\varphi = p$ (base case)

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- **forall** $q \in Q$ **do**
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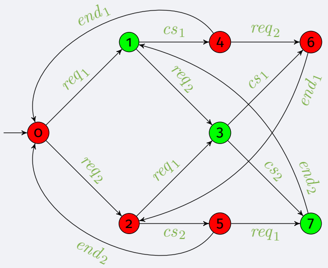
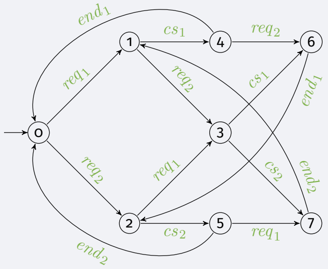


The formula is false on the initial state

Case 2: $\varphi = \neg\psi$

$markPred(\psi)$
forall $q \in Q$ **do**
$\quad q.\varphi \leftarrow \neg q.\psi$

$\varphi = \neg R_1$



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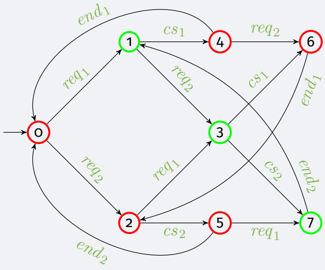

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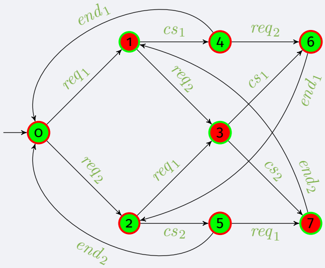

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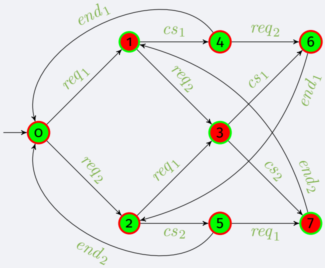
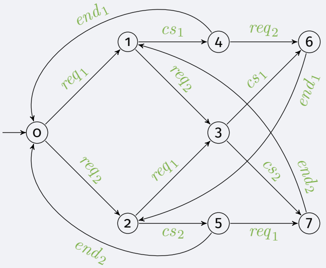


The formula is true on the initial state

Case 3: $\varphi = \psi_1 \wedge \psi_2$

$markPred(\psi_1)$
$markPred(\psi_2)$
forall $q \in Q$ **do**
  $q.\varphi \leftarrow q.\psi_1 \wedge q.\psi_2$

$\varphi = R_1 \wedge R_2$



Case 3: $\varphi = \psi_1 \land \psi_2$

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forall $q \in Q$ **do**
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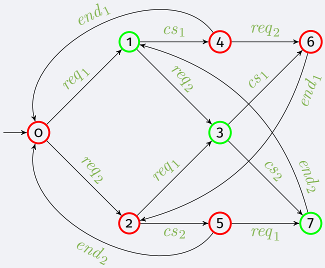
$\varphi = R_1 \land R_2$



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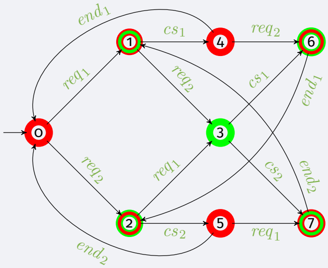

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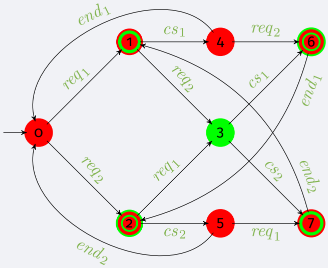

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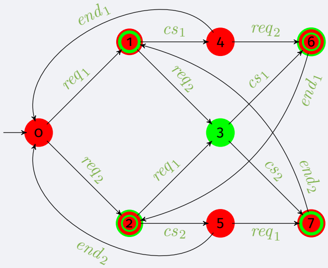


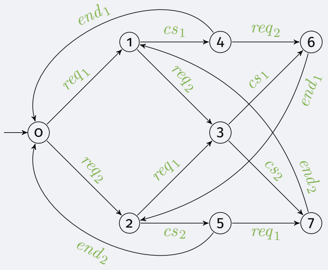
The formula is false on the initial state

Case 4: $\varphi = \mathsf{EX}\psi$

```
/* Init */
markPred(ψ)
forall q ∈ Q do
  q.φ ← false

/* Main loop */
forall (q, _, q') ∈ T do
  if q'.ψ = true then
    q.φ ← true
```

$\varphi = \mathsf{EX} R_1$



Case 4: $\varphi = \mathsf{EX}\psi$

```
/* Init */
```
$markPred(\psi)$

forall $q \in Q$ **do**

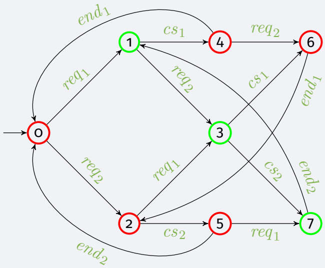
  $q.\varphi \leftarrow$ `false`

```
/* Main loop */
```
forall $(q, \_, q') \in T$ **do**

  if $q'.\psi =$ `true` **then**

    $q.\varphi \leftarrow$ `true`

$\varphi = \mathsf{EX} R_1$



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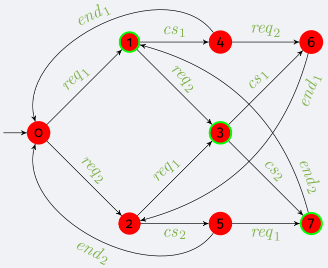

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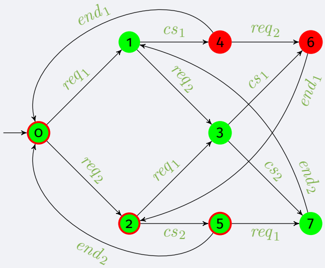

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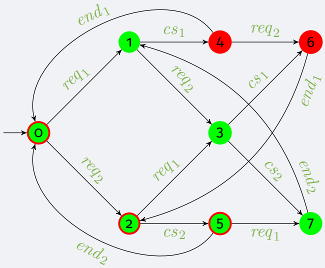


The formula is true on the initial state

Case 5: $\varphi = \mathsf{E}\psi_1\mathsf{U}\psi_2$

/* Init */

$markPred(\psi_1)$

$markPred(\psi_2)$

forall $q \in Q$ **do**

  $q.\varphi \leftarrow$ `false`

  $q.seenbefore \leftarrow$ `false`

$L \leftarrow \emptyset$

forall $q \in Q$ **do**

  if $q.\psi_2 =$ `true` **then**

    $L \leftarrow L \cup \{q\}$

/* Main loop */

while $L \neq \emptyset$ **do**

  pick q from L; $L \leftarrow L \setminus \{q\}$

  $q.\varphi \leftarrow$ `true`

  forall $(q', \_, q) \in T$ **do**

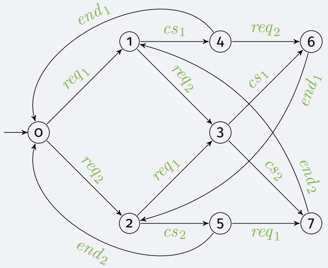
    if $q'.seenbefore =$ `false` **then**

      $q'.seenbefore \leftarrow$ `true`

      if $q'.\psi_1 =$ `true` **then**

        $L \leftarrow L \cup \{q'\}$

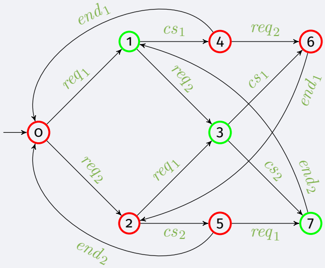
$\varphi = \mathsf{E}R_1\mathsf{U}CS_1$



Case 5: $\varphi = \mathsf{E}\psi_1\mathsf{U}\psi_2$

```
/* Init */
markPred(ψ1)
markPred(ψ2)
forall q ∈ Q do
    q.φ ← false
    q.seenbefore ← false
L ← ∅
forall q ∈ Q do
    if q.ψ2 = true then
        L ← L ∪ {q}
/* Main loop */
while L ≠ ∅ do
    pick q from L; L ← L \ {q}
    q.φ ← true
    forall (q', _, q) ∈ T do
        if q'.seenbefore = false
          then
            q'.seenbefore ← true
            if q'.ψ1 = true then
                L ← L ∪ {q'}
```

$\varphi = \mathsf{E}R_1\mathsf{U}CS_1$



Case 5: $\varphi = \mathsf{E}\psi_1 \mathsf{U} \psi_2$

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$L \leftarrow \emptyset$

forall $q \in Q$ **do**

  if $q.\psi_2 =$ true **then**

    $L \leftarrow L \cup \{q\}$

/* Main loop */

while $L \neq \emptyset$ **do**

  pick q from L; $L \leftarrow L \setminus \{q\}$

  $q.\varphi \leftarrow$ true

  forall $(q', \_, q) \in T$ **do**

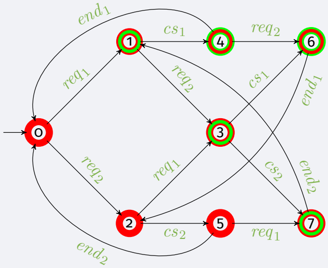
    if $q'.seenbefore =$ false **then**

      $q'.seenbefore \leftarrow$ true

      if $q'.\psi_1 =$ true **then**

        $L \leftarrow L \cup \{q'\}$

$\varphi = \mathsf{E} R_1 \mathsf{U} CS_1$



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/* Main loop */
while L ≠ ∅ do
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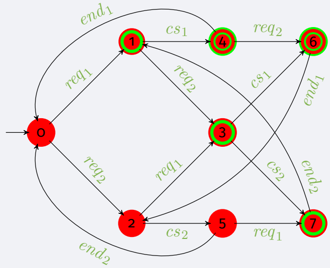
    q.φ ← true
    forall (q', _, q) ∈ T do
        if q'.seenbefore = false
          then
            q'.seenbefore ← true
            if q'.ψ1 = true then
                L ← L ∪ {q'}
```

$\varphi = \mathsf{E} R_1 \mathsf{U} CS_1$

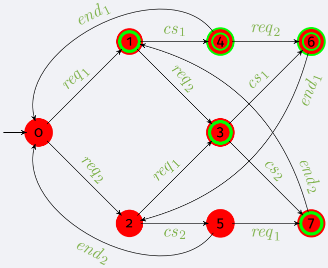


Case 5: $\varphi = \mathsf{E}\psi_1 \mathsf{U} \psi_2$

```
/* Init */
markPred(ψ1)
markPred(ψ2)
forall q ∈ Q do
    q.φ ← false
    q.seenbefore ← false
L ← ∅
forall q ∈ Q do
    if q.ψ2 = true then
        L ← L ∪ {q}

/* Main loop */
while L ≠ ∅ do
    pick q from L; L ← L \ {q}
    q.φ ← true
    forall (q′, _, q) ∈ T do
        if q′.seenbefore = false
          then
            q′.seenbefore ← true
            if q′.ψ1 = true then
                L ← L ∪ {q′}
```

$\varphi = \mathsf{E} R_1 \mathsf{U}\, CS_1$

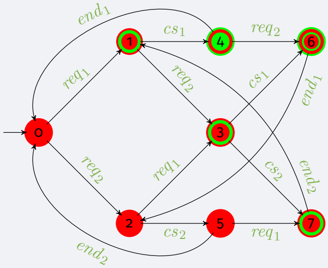


Case 5: $\varphi = \mathsf{E}\psi_1\mathsf{U}\psi_2$

/* Init */
$markPred(\psi_1)$
$markPred(\psi_2)$
forall $q \in Q$ **do**
  $q.\varphi \leftarrow$ false
  $q.seenbefore \leftarrow$ false
$L \leftarrow \emptyset$
forall $q \in Q$ **do**
  if $q.\psi_2 =$ true **then**
    $L \leftarrow L \cup \{q\}$

/* Main loop */
while $L \neq \emptyset$ **do**
  pick q from L; $L \leftarrow L \setminus \{q\}$
  $q.\varphi \leftarrow$ true
  forall $(q', \_, q) \in T$ **do**
    if $q'.seenbefore =$ false **then**
      $q'.seenbefore \leftarrow$ true
      if $q'.\psi_1 =$ true **then**
        $L \leftarrow L \cup \{q'\}$

$\varphi = \mathsf{E}R_1\mathsf{U}CS_1$



Case 5: $\varphi = \mathsf{E}\psi_1\mathsf{U}\psi_2$

```
/* Init */
```
$markPred(\psi_1)$
$markPred(\psi_2)$
forall $q \in Q$ **do**
  $q.\varphi \leftarrow$ `false`
  $q.seenbefore \leftarrow$ `false`
$L \leftarrow \emptyset$
forall $q \in Q$ **do**
  if $q.\psi_2 =$ `true` **then**
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```
/* Main loop */
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while $L \neq \emptyset$ **do**
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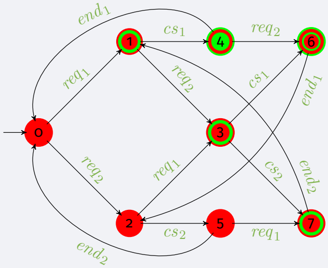
$\varphi = \mathsf{E}R_1\mathsf{U}CS_1$



Case 5: $\varphi = \mathsf{E}\psi_1 \mathsf{U} \psi_2$

/* Init */

$markPred(\psi_1)$

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forall $q \in Q$ **do**

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- $q.seenbefore \leftarrow$ `false`

$L \leftarrow \emptyset$

forall $q \in Q$ **do**

- **if** $q.\psi_2 =$ `true` **then**
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while $L \neq \emptyset$ **do**

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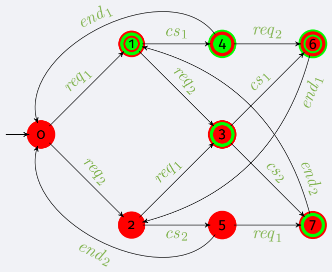
$\varphi = \mathsf{E} R_1 \mathsf{U} CS_1$



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forall $q \in Q$ **do**

- **if** $q.\psi_2 =$ `true` **then**
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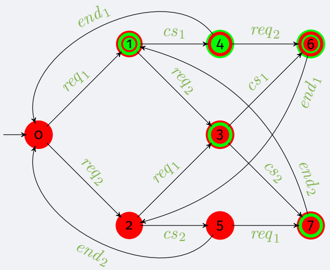

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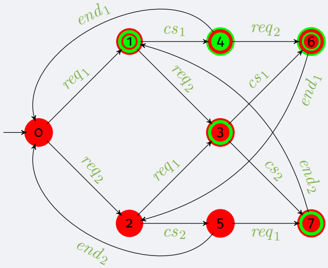

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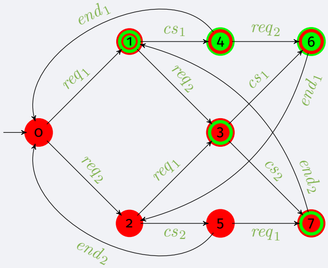
    q.seenbefore ← false
L ← ∅
forall q ∈ Q do
    if q.ψ2 = true then
        L ← L ∪ {q}
/* Main loop */
while L ≠ ∅ do
    pick q from L; L ← L \ {q}
    q.φ ← true
    forall (q', _, q) ∈ T do
        if q'.seenbefore = false
          then
            q'.seenbefore ← true
            if q'.ψ1 = true then
                L ← L ∪ {q'}
```

$\varphi = \mathsf{E}R_1\mathsf{U}CS_1$



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  pick q from L; $L \leftarrow L \setminus \{q\}$
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    if $q'.seenbefore =$ false **then**
      $q'.seenbefore \leftarrow$ true
      if $q'.\psi_1 =$ true **then**
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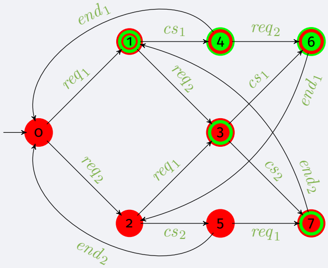
$\varphi = \mathsf{E} R_1 \mathsf{U} CS_1$



Case 5: $\varphi = \mathsf{E}\psi_1\mathsf{U}\psi_2$

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/* Main loop */
while L ≠ ∅ do
    pick q from L; L ← L \ {q}
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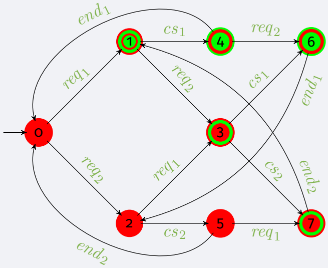
    forall (q', _, q) ∈ T do
        if q'.seenbefore = false
          then
            q'.seenbefore ← true
            if q'.ψ1 = true then
                L ← L ∪ {q'}
```

$\varphi = \mathsf{E}R_1\mathsf{U}CS_1$



Case 5: $\varphi = \mathsf{E}\psi_1 \mathsf{U} \psi_2$

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    forall (q', _, q) ∈ T do
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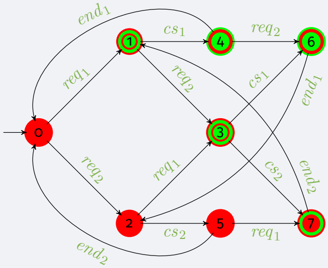
          then
            q'.seenbefore ← true
            if q'.ψ1 = true then
                L ← L ∪ {q'}
```

$\varphi = \mathsf{E} R_1 \mathsf{U} CS_1$



Case 5: $\varphi = \mathsf{E}\psi_1\mathsf{U}\psi_2$

/* Init */

$markPred(\psi_1)$

$markPred(\psi_2)$

forall $q \in Q$ **do**

  $q.\varphi \leftarrow$ `false`

  $q.seenbefore \leftarrow$ `false`

$L \leftarrow \emptyset$

forall $q \in Q$ **do**

  if $q.\psi_2 =$ `true` **then**

    $L \leftarrow L \cup \{q\}$

/* Main loop */

while $L \neq \emptyset$ **do**

  pick q from L; $L \leftarrow L \setminus \{q\}$

  $q.\varphi \leftarrow$ `true`

  forall $(q', \_, q) \in T$ **do**

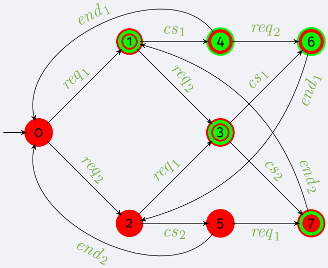
    if $q'.seenbefore =$ `false` **then**

      $q'.seenbefore \leftarrow$ `true`

      if $q'.\psi_1 =$ `true` **then**

        $L \leftarrow L \cup \{q'\}$

$\varphi = \mathsf{E}R_1\mathsf{U}CS_1$



Case 5: $\varphi = \mathsf{E}\psi_1\mathsf{U}\psi_2$

```
/* Init */
markPred(ψ1)
markPred(ψ2)
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    q.φ ← false
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L ← ∅
forall q ∈ Q do
    if q.ψ2 = true then
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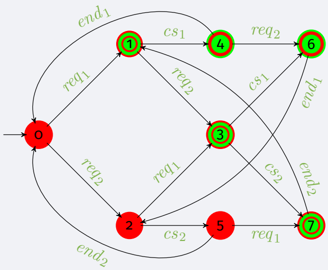
/* Main loop */
while L ≠ ∅ do
    pick q from L; L ← L \ {q}
    q.φ ← true
    forall (q', _, q) ∈ T do
        if q'.seenbefore = false
          then
            q'.seenbefore ← true
            if q'.ψ1 = true then
                L ← L ∪ {q'}
```

$\varphi = \mathsf{E}R_1\mathsf{U}CS_1$



Case 5: $\varphi = \mathsf{E}\psi_1\mathsf{U}\psi_2$

```
/* Init */
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forall q ∈ Q do
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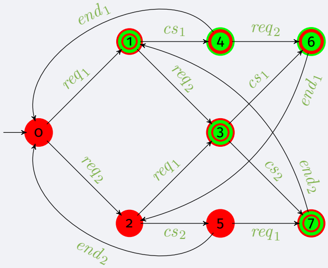
L ← ∅
forall q ∈ Q do
    if q.ψ2 = true then
        L ← L ∪ {q}

/* Main loop */
while L ≠ ∅ do
    pick q from L; L ← L \ {q}
    q.φ ← true
    forall (q′, _, q) ∈ T do
        if q′.seenbefore = false
          then
            q′.seenbefore ← true
            if q′.ψ1 = true then
                L ← L ∪ {q′}
```

$\varphi = \mathsf{E}R_1\mathsf{U}CS_1$

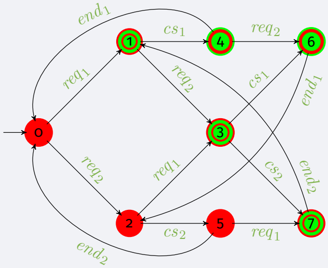


Case 5: $\varphi = \mathsf{E}\psi_1 \mathsf{U} \psi_2$

/* Init */
$markPred(\psi_1)$
$markPred(\psi_2)$
forall $q \in Q$ **do**
  $q.\varphi \leftarrow$ false
  $q.seenbefore \leftarrow$ false
$L \leftarrow \emptyset$
forall $q \in Q$ **do**
  if $q.\psi_2 =$ true **then**
    $L \leftarrow L \cup \{q\}$

/* Main loop */
while $L \neq \emptyset$ **do**
  pick q from L; $L \leftarrow L \setminus \{q\}$
  $q.\varphi \leftarrow$ true
  forall $(q', \_, q) \in T$ **do**
    if $q'.seenbefore =$ false **then**
      $q'.seenbefore \leftarrow$ true
      if $q'.\psi_1 =$ true **then**
        $L \leftarrow L \cup \{q'\}$

$\varphi = \mathsf{E} R_1 \mathsf{U} CS_1$

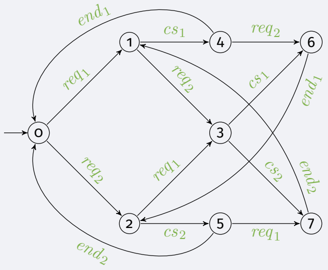


The formula is false on the initial state

Case 6: $\varphi = \mathsf{A}\psi_1\mathsf{U}\psi_2$

```
/* Init */
markPred(ψ1)
markPred(ψ2)
L ← ∅
forall q ∈ Q do
    q.nb ← degree(q)
    q.φ ← false
forall q ∈ Q do
    if q.ψ2 = true then
        L ← L ∪ {q}
/* Main loop */
while L ≠ ∅ do
    pick q from L; L ← L \ {q}
    q.φ ← true
    forall (q', _, q) ∈ T do
        q'.nb ← q'.nb − 1
        if q'.nb = 0 ∧ q'.ψ1 =
          true ∧ q'.φ = false then
            L ← L ∪ {q'}
```

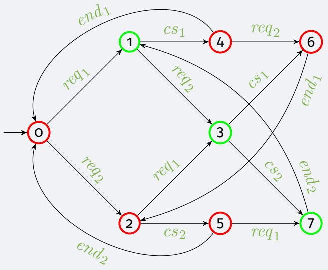
$\varphi = \mathsf{A}R_1\mathsf{U}CS_1$



Case 6: $\varphi = \mathsf{A}\psi_1 \mathsf{U} \psi_2$

/* Init */
$markPred(\psi_1)$
$markPred(\psi_2)$
$L \leftarrow \emptyset$
forall $q \in Q$ **do**
  $q.nb \leftarrow degree(q)$
  $q.\varphi \leftarrow$ false
forall $q \in Q$ **do**
  if $q.\psi_2 =$ true **then**
    $L \leftarrow L \cup \{q\}$
/* Main loop */
while $L \neq \emptyset$ **do**
  pick q from L; $L \leftarrow L \setminus \{q\}$
  $q.\varphi \leftarrow$ true
  forall $(q', \_, q) \in T$ **do**
    $q'.nb \leftarrow q'.nb - 1$
    if $q'.nb = 0 \wedge q'.\psi_1 =$ true $\wedge q'.\varphi =$ false **then**
      $L \leftarrow L \cup \{q'\}$

$\varphi = \mathsf{A} R_1 \mathsf{U} CS_1$



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$markPred(\psi_1)$
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forall $q \in Q$ **do**
  $q.nb \leftarrow degree(q)$
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forall $q \in Q$ **do**
  if $q.\psi_2 =$ true **then**
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while $L \neq \emptyset$ **do**
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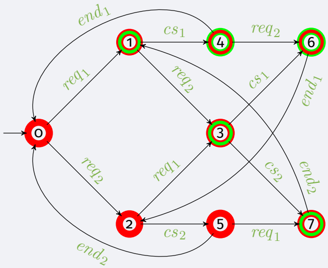
  forall $(q', \_, q) \in T$ **do**
    $q'.nb \leftarrow q'.nb - 1$
    if $q'.nb = 0 \land q'.\psi_1 =$ true $\land q'.\varphi =$ false **then**
      $L \leftarrow L \cup \{q'\}$

$\varphi = \mathsf{A} R_1 \mathsf{U}\, CS_1$



Case 6: $\varphi = \mathsf{A}\psi_1\mathsf{U}\psi_2$

```
/* Init */
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$markPred(\psi_1)$
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$L \leftarrow \emptyset$

forall $q \in Q$ **do**
- $q.nb \leftarrow degree(q)$
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forall $q \in Q$ **do**
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```
/* Main loop */
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while $L \neq \emptyset$ **do**
- pick q from L; $L \leftarrow L \setminus \{q\}$
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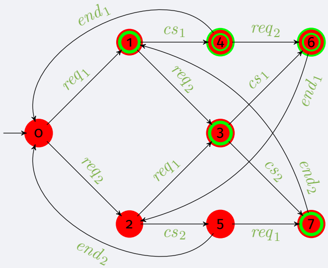
$\varphi = \mathsf{A}R_1\mathsf{U}CS_1$



Case 6: $\varphi = \mathsf{A}\psi_1\mathsf{U}\psi_2$

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/* Init */
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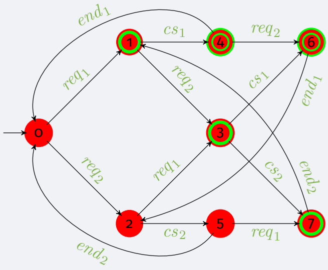
$\varphi = \mathsf{A} R_1 \mathsf{U}\, CS_1$



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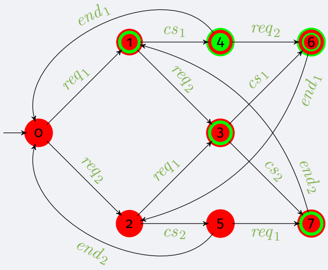

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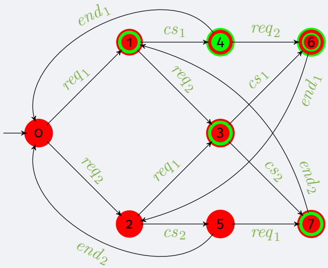
$\varphi = \mathsf{A} R_1 \mathsf{U}\, CS_1$



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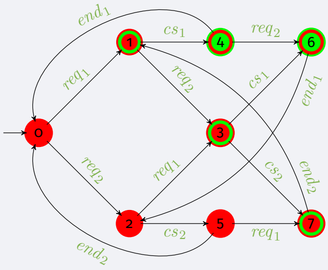
/* Main loop */
while $L \neq \emptyset$ **do**
    pick q from L; $L \leftarrow L \setminus \{q\}$
    $q.\varphi \leftarrow$ true
    forall $(q', \_, q) \in T$ **do**
        $q'.nb \leftarrow q'.nb - 1$
        if $q'.nb = 0 \wedge q'.\psi_1 =$ true $\wedge q'.\varphi =$ false **then**
            $L \leftarrow L \cup \{q'\}$

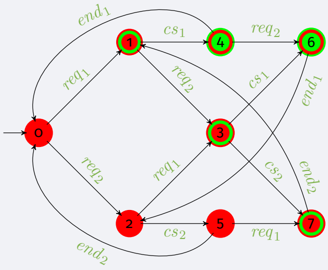
$\varphi = \mathsf{A}R_1\mathsf{U}CS_1$



Case 6: $\varphi = \mathsf{A}\psi_1\mathsf{U}\psi_2$

```
/* Init */
markPred(ψ1)
markPred(ψ2)
L ← ∅
forall q ∈ Q do
    q.nb ← degree(q)
    q.φ ← false
forall q ∈ Q do
    if q.ψ2 = true then
        L ← L ∪ {q}
/* Main loop */
while L ≠ ∅ do
    pick q from L; L ← L \ {q}
    q.φ ← true
    forall (q′, _, q) ∈ T do
        q′.nb ← q′.nb − 1
        if q′.nb = 0 ∧ q′.ψ1 =
          true ∧ q′.φ = false then
            L ← L ∪ {q′}
```

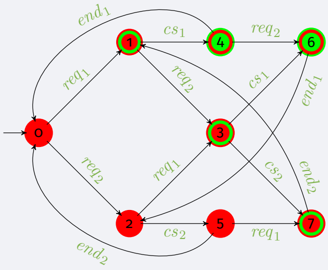
$\varphi = \mathsf{A} R_1 \mathsf{U}\, CS_1$



Case 6: $\varphi = \mathsf{A}\psi_1 \mathsf{U} \psi_2$

```
/* Init */
markPred(ψ1)
markPred(ψ2)
L ← ∅
forall q ∈ Q do
    q.nb ← degree(q)
    q.φ ← false
forall q ∈ Q do
    if q.ψ2 = true then
        L ← L ∪ {q}
/* Main loop */
while L ≠ ∅ do
    pick q from L; L ← L \ {q}
    q.φ ← true
    forall (q', _, q) ∈ T do
        q'.nb ← q'.nb − 1
        if q'.nb = 0 ∧ q'.ψ1 =
          true ∧ q'.φ = false then
            L ← L ∪ {q'}
```

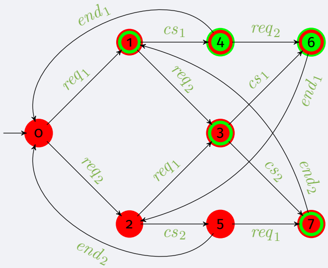
$\varphi = \mathsf{A} R_1 \mathsf{U}\, CS_1$



Case 6: $\varphi = \mathsf{A}\psi_1\mathsf{U}\psi_2$

/* Init */
$markPred(\psi_1)$
$markPred(\psi_2)$
$L \leftarrow \emptyset$
forall $q \in Q$ **do**
  $q.nb \leftarrow degree(q)$
  $q.\varphi \leftarrow$ false
forall $q \in Q$ **do**
  if $q.\psi_2 =$ true **then**
    $L \leftarrow L \cup \{q\}$
/* Main loop */
while $L \neq \emptyset$ **do**
  pick q from L; $L \leftarrow L \setminus \{q\}$
  $q.\varphi \leftarrow$ true
  forall $(q', \_, q) \in T$ **do**
    $q'.nb \leftarrow q'.nb - 1$
    if $q'.nb = 0 \wedge q'.\psi_1 =$ true $\wedge q'.\varphi =$ false **then**
      $L \leftarrow L \cup \{q'\}$

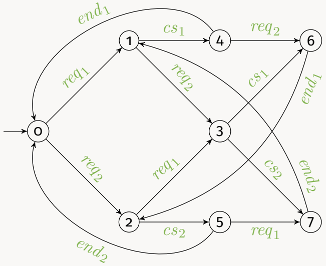
$\varphi = \mathsf{A}R_1\mathsf{U}CS_1$



The formula is false on the initial state

Exercise: Check a CTL formula (mutual exclusion)

Check $\mathsf{AG}\big(\mathsf{EF}(I_1 \wedge I_2)\big)$



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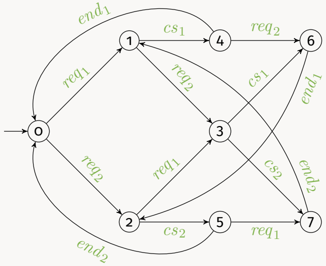
Check $\mathsf{AG}\big(\mathsf{EF}(I_1 \wedge I_2)\big)$



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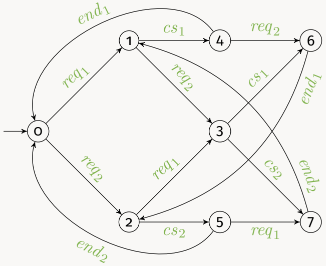
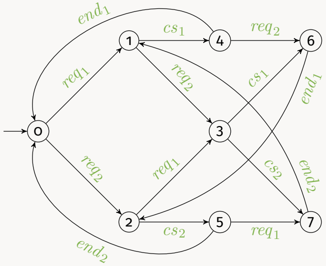
Check $\mathsf{AG}\big(\mathsf{EF}(I_1 \wedge I_2)\big)$



Exercise: Check a CTL formula (mutual exclusion)

Check $\mathsf{AG}\big(\mathsf{EF}(I_1 \land I_2)\big)$



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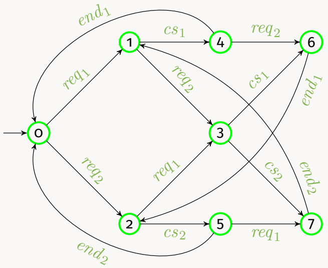
Check $\mathsf{AG}\big(\mathsf{EF}(I_1 \wedge I_2)\big)$



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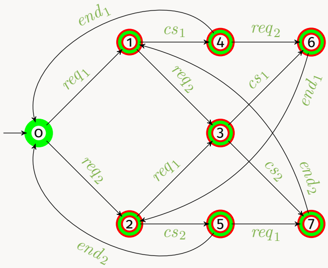
Check $\mathsf{AG}\big(\mathsf{EF}(I_1 \wedge I_2)\big)$



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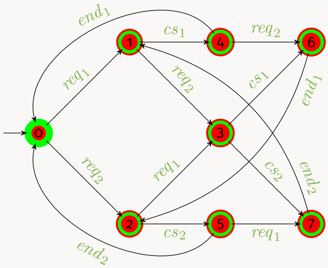
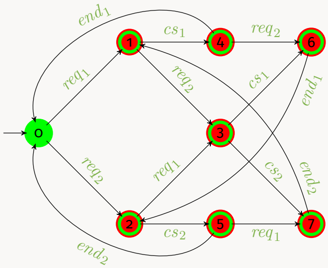
Check $\mathsf{AG}\big(\mathsf{EF}(I_1 \wedge I_2)\big)$



Exercise: Check a CTL formula (mutual exclusion)

Check $\mathsf{AG}\big(\mathsf{EF}(I_1 \wedge I_2)\big)$



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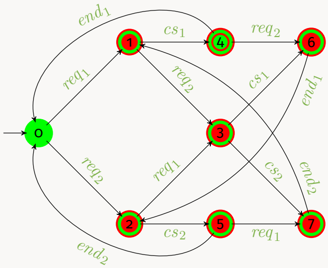
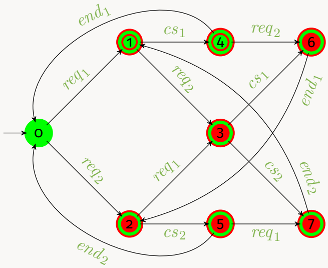
Check $\mathsf{AG}\big(\mathsf{EF}(I_1 \wedge I_2)\big)$



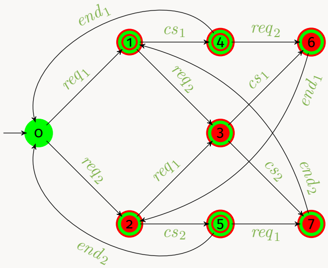
Exercise: Check a CTL formula (mutual exclusion)

Check $\mathsf{AG}\big(\mathsf{EF}(I_1 \wedge I_2)\big)$



Exercise: Check a CTL formula (mutual exclusion)

Check $\mathsf{AG}\big(\mathsf{EF}(I_1 \wedge I_2)\big)$



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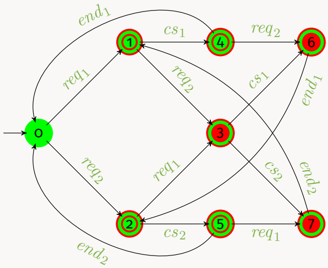
Check $\mathsf{AG}\big(\mathsf{EF}(I_1 \wedge I_2)\big)$



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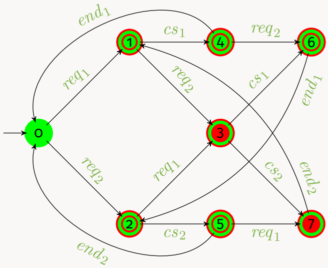
Check $\mathsf{AG}\big(\mathsf{EF}(I_1 \wedge I_2)\big)$



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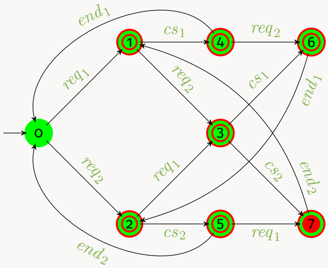
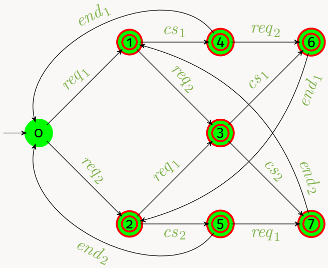


Exercise: Check a CTL formula (mutual exclusion)

Check $\mathsf{AG}\big(\mathsf{EF}(I_1 \wedge I_2)\big)$



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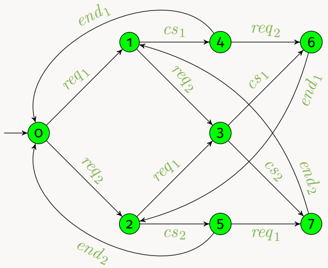
Check $\mathsf{AG}\big(\mathsf{EF}(I_1 \wedge I_2)\big)$



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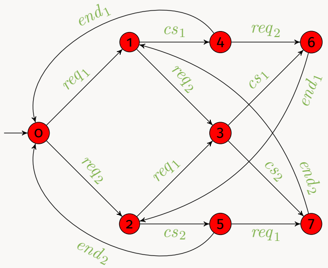
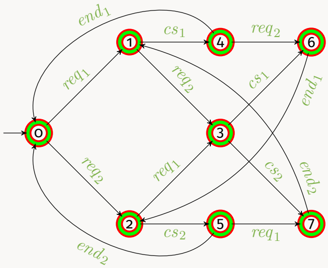
Check $\mathsf{AG}\big(\mathsf{EF}(I_1 \wedge I_2)\big)$



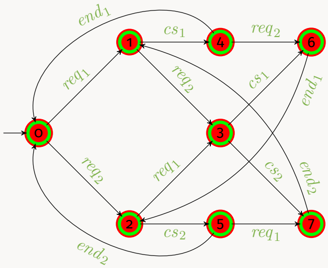
Exercise: Check a CTL formula (mutual exclusion)

Check $\mathsf{AG}\big(\mathsf{EF}(I_1 \wedge I_2)\big)$



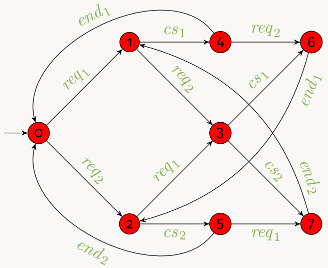
Exercise: Check a CTL formula (mutual exclusion)

Check $\mathsf{AG}\big(\mathsf{EF}(I_1 \land I_2)\big)$



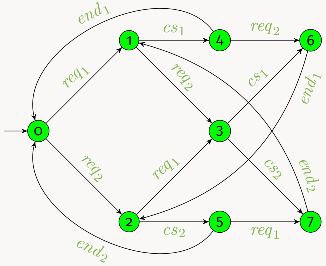
Exercise: Check a CTL formula (mutual exclusion)

Check $\mathsf{AG}\big(\mathsf{EF}(I_1 \wedge I_2)\big)$



Exercise: Check a CTL formula (mutual exclusion)

Check $\mathsf{AG}\big(\mathsf{EF}(I_1 \land I_2)\big)$

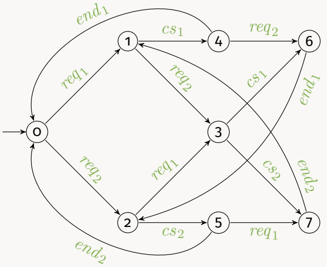


The formula is true

Exercise: Check another CTL formula (mutual exclusion)

/* Init */
$markPred(\psi_1)$
$markPred(\psi_2)$
$L \leftarrow \emptyset$
forall $q \in Q$ **do**
    $q.nb \leftarrow degree(q)$
    $q.\varphi \leftarrow$ false
forall $q \in Q$ **do**
    if $q.\psi_2 =$ true **then**
        $L \leftarrow L \cup \{q\}$
/* Main loop */
while $L \neq \emptyset$ **do**
    pick q from L; $L \leftarrow L \setminus \{q\}$
    $q.\varphi \leftarrow$ true
    forall $(q', \_, q) \in T$ **do**
        $q'.nb \leftarrow q'.nb - 1$
        if $q'.nb = 0 \wedge q'.\psi_1 =$ true $\wedge\, q'.\varphi =$ false **then**
            $L \leftarrow L \cup \{q'\}$

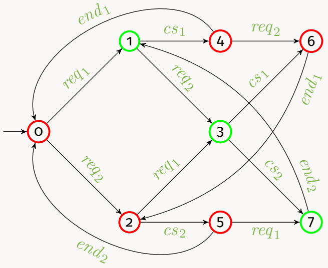
$\varphi = \mathsf{A} R_1 \mathsf{U} CS_2$



Exercise: Check another CTL formula (mutual exclusion)

/* Init */
$markPred(\psi_1)$
$markPred(\psi_2)$
$L \leftarrow \emptyset$
forall $q \in Q$ **do**
  $q.nb \leftarrow degree(q)$
  $q.\varphi \leftarrow$ false
forall $q \in Q$ **do**
  if $q.\psi_2 =$ true **then**
    $L \leftarrow L \cup \{q\}$
/* Main loop */
while $L \neq \emptyset$ **do**
  pick q from L; $L \leftarrow L \setminus \{q\}$
  $q.\varphi \leftarrow$ true
  forall $(q', \_, q) \in T$ **do**
    $q'.nb \leftarrow q'.nb - 1$
    if $q'.nb = 0 \land q'.\psi_1 =$ true $\land q'.\varphi =$ false **then**
      $L \leftarrow L \cup \{q'\}$

$\varphi = \mathsf{A} R_1 \mathsf{U} CS_2$



Exercise: Check another CTL formula (mutual exclusion)

/* Init */
$markPred(\psi_1)$
$markPred(\psi_2)$
$L \leftarrow \emptyset$
forall $q \in Q$ **do**
  $q.nb \leftarrow degree(q)$
  $q.\varphi \leftarrow$ false

forall $q \in Q$ **do**
  if $q.\psi_2 =$ true **then**
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/* Main loop */
while $L \neq \emptyset$ **do**
  pick q from L; $L \leftarrow L \setminus \{q\}$
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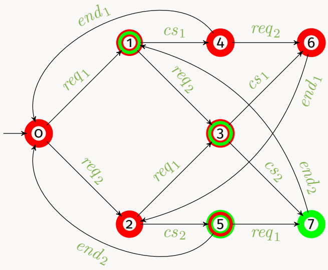
  forall $(q', \_, q) \in T$ **do**
    $q'.nb \leftarrow q'.nb - 1$
    if $q'.nb = 0 \wedge q'.\psi_1 =$ true $\wedge q'.\varphi =$ false **then**
      $L \leftarrow L \cup \{q'\}$

$\varphi = \mathsf{A} R_1 \mathsf{U} CS_2$



Exercise: Check another CTL formula (mutual exclusion)

/* Init */
$markPred(\psi_1)$
$markPred(\psi_2)$
$L \leftarrow \emptyset$
forall $q \in Q$ **do**
    $q.nb \leftarrow degree(q)$
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forall $q \in Q$ **do**
    if $q.\psi_2 =$ true **then**
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/* Main loop */
while $L \neq \emptyset$ **do**
    pick q from L; $L \leftarrow L \setminus \{q\}$
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    forall $(q', \_, q) \in T$ **do**
        $q'.nb \leftarrow q'.nb - 1$
        if $q'.nb = 0 \wedge q'.\psi_1 =$ true $\wedge q'.\varphi =$ false **then**
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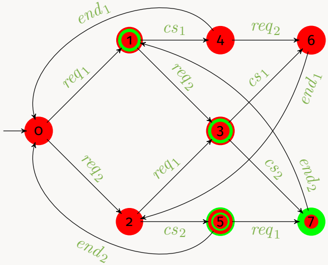
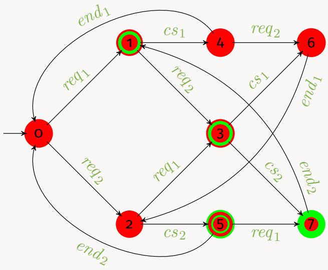
$\varphi = \mathsf{A} R_1 \mathsf{U} CS_2$



Exercise: Check another CTL formula (mutual exclusion)

```
/* Init */
markPred(ψ1)
markPred(ψ2)
L ← ∅
forall q ∈ Q do
    q.nb ← degree(q)
    q.φ ← false
forall q ∈ Q do
    if q.ψ2 = true then
        L ← L ∪ {q}
/* Main loop */
while L ≠ ∅ do
    pick q from L; L ← L \ {q}
    q.φ ← true
    forall (q′, _, q) ∈ T do
        q′.nb ← q′.nb − 1
        if q′.nb = 0 ∧ q′.ψ1 =
          true ∧ q′.φ = false then
            L ← L ∪ {q′}
```

$\varphi = \mathsf{A} R_1 \mathsf{U} CS_2$



Exercise: Check another CTL formula (mutual exclusion)

/* Init */
$markPred(\psi_1)$
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$L \leftarrow \emptyset$
forall $q \in Q$ **do**
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/* Main loop */
while $L \neq \emptyset$ **do**
    pick q from L; $L \leftarrow L \setminus \{q\}$
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    forall $(q', \_, q) \in T$ **do**
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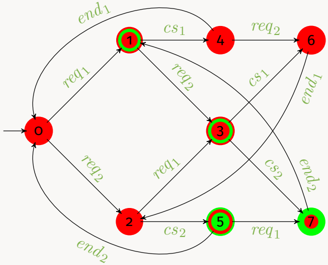

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```
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```
/* Main loop */
```
while $L \neq \emptyset$ **do**
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  $q.\varphi \leftarrow$ `true`
  forall $(q', \_, q) \in T$ **do**
    $q'.nb \leftarrow q'.nb - 1$
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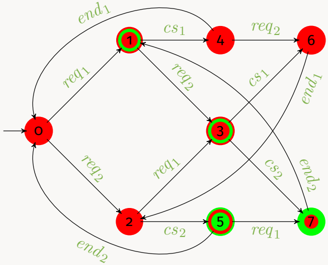

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forall $q \in Q$ **do**

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/* Main loop */

while $L \neq \emptyset$ **do**

  pick q from L; $L \leftarrow L \setminus \{q\}$

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  forall $(q', \_, q) \in T$ **do**

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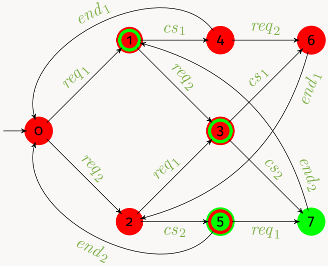

Exercise: Check another CTL formula (mutual exclusion)

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/* Main loop */
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    q.φ ← true
    forall (q', _, q) ∈ T do
        q'.nb ← q'.nb − 1
        if q'.nb = 0 ∧ q'.ψ1 =
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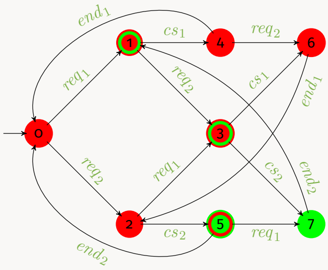
            L ← L ∪ {q'}
```

$\varphi = \mathsf{A} R_1 \mathsf{U} CS_2$



Exercise: Check another CTL formula (mutual exclusion)

/* Init */

$markPred(\psi_1)$

$markPred(\psi_2)$

$L \leftarrow \emptyset$

forall $q \in Q$ **do**

- $q.nb \leftarrow degree(q)$
- $q.\varphi \leftarrow$ `false`

forall $q \in Q$ **do**

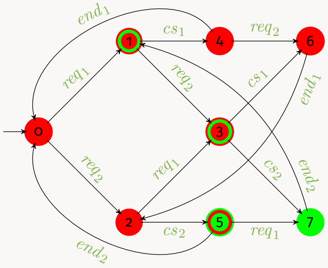
- **if** $q.\psi_2 =$ `true` **then**
  - $L \leftarrow L \cup \{q\}$

/* Main loop */

while $L \neq \emptyset$ **do**

- pick q from L; $L \leftarrow L \setminus \{q\}$
- $q.\varphi \leftarrow$ `true`
- **forall** $(q', \_, q) \in T$ **do**
  - $q'.nb \leftarrow q'.nb - 1$
  - **if** $q'.nb = 0 \wedge q'.\psi_1 =$ `true` $\wedge\, q'.\varphi =$ `false` **then**
    - $L \leftarrow L \cup \{q'\}$

$\varphi = \mathsf{A} R_1 \mathsf{U}\, CS_2$



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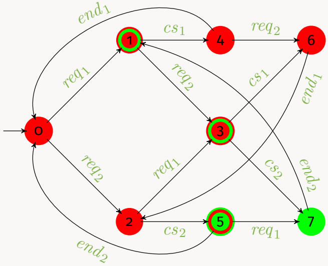
$markPred(\psi_1)$
$markPred(\psi_2)$
$L \leftarrow \emptyset$
forall $q \in Q$ **do**
  $q.nb \leftarrow degree(q)$
  $q.\varphi \leftarrow$ `false`
forall $q \in Q$ **do**
  if $q.\psi_2 =$ `true` **then**
    $L \leftarrow L \cup \{q\}$
/* Main loop */
while $L \neq \emptyset$ **do**
  pick q from L; $L \leftarrow L \setminus \{q\}$
  $q.\varphi \leftarrow$ `true`
  forall $(q', \_, q) \in T$ **do**
    $q'.nb \leftarrow q'.nb - 1$
    if $q'.nb = 0 \land q'.\psi_1 =$ `true` $\land q'.\varphi =$ `false` **then**
      $L \leftarrow L \cup \{q'\}$

$\varphi = \mathsf{A} R_1 \mathsf{U} CS_2$



Outline

Reachability properties

How to characterize reachability properties?

A reachability property states that some particular situation can be reached.

It may:

- be simple
- be conditional: restrict the form of paths reaching the state
- apply to any reachable state

Often, the negation of reachability (safety) is the interesting property.

Reachability properties

Examples

- we can obtain $n < 0$
- we can enter the critical section
- we cannot have $n < 0$
- we cannot reach the *crash* state
- we can enter the critical section without traversing $n = 0$
- we can return to the initial state
- we can always return to the initial state

Reachability properties

Examples

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Reachability properties

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- we can enter the critical section without traversing $n = 0$ (conditional)
- we can return to the initial state (simple)
- we can always return to the initial state (any reachable state)

Outline

Reachability in CTL

Form of formulae in CTL

- use the EF modal operator: EFφ
- φ is a propositional formula without temporal operator
- EU for conditional reachability
- Nesting AG and EF when considering any reachable state

Reachability in CTL: examples

Examples

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- we can enter the critical section:
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Outline

4 Reachability Properties

- Reachability in CTL
- Computing the state space
- Specifying properties using observers

Computation of the reachability graph

Forward chaining

- start from the initial state
- add its successors
- continue until saturation

Drawback: potential explosion of the set being constructed

Computation of the reachability graph

Backward chaining

Construct the set of states which can lead to some target states

- start from target states
- add their immediate predecessors
- continue until saturation
- test whether some initial state is in the computed set

Drawbacks:

- identify target states
- computing predecessors can be more difficult than computing successors (e. g., for automata with variables)
- target states may be unreachable

Computation of the reachability graph

On-the-fly exploration

- check the property during exploration
- only partially construct the state space

Outline

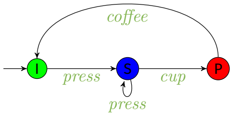
Verifying properties using observers

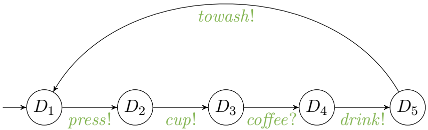
An observer is an automaton that observes the system behavior

- It synchronizes with other automata's actions
- It must be non-blocking (see example on the white board)
  - Note: a complete automaton is never blocking
- Its location(s) give an indication on the system property

Then verifying the property reduces to a reachability property on the observer (in parallel with the system)

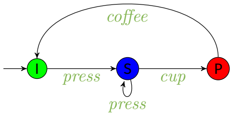
Observers for the coffee machine (1/3)

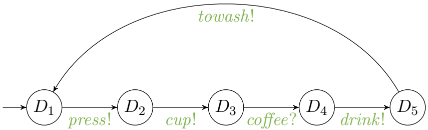




Design an observer for the coffee machine and the drinker verifying that it is possible to order a coffee with exactly one dose of sugar.
(…and check the validity of the property)

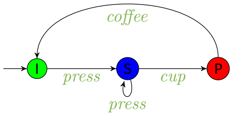
Observers for the coffee machine (2/3)

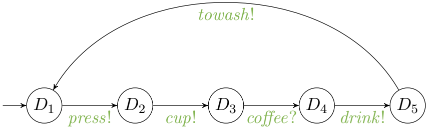




Design an observer for the coffee machine and the drinker verifying that it is possible to order a coffee with *at least* one dose of sugar.
(...and check the validity of the property)

Observers for the coffee machine (3/3)

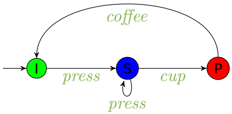


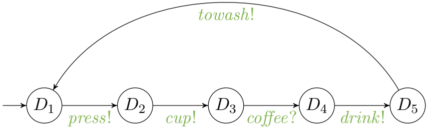


Consider the following property on the coffee machine and the coffee drinker: "whenever the (first) coffee comes, no cup was put to the washing machine before"

1. express this property using CTL
2. write an observer verifying this property

Observers for the coffee machine (3/3)





Consider the following property on the coffee machine and the coffee drinker: “whenever the (first) coffee comes, no cup was put to the washing machine before”

1. express this property using CTL $\mathsf{A}(\neg towash)\mathsf{U}\,coffee$
2. write an observer verifying this property

Outline

The state space explosion problem

Systems are often designed in a compositional manner. Example:

- a server
- 10 clients
- a network

The synchronized product becomes exponentially larger

- even worse in the presence of variables

The state space explosion problem

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The synchronized product becomes exponentially larger

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State space explosion problem

The state space explosion problem is the main obstacle to model checking algorithms, because of the necessity to construct the entire state space.

The state space explosion problem: examples

Example

For a system made of 10 components with 10 states each, the (maximum) number of different states in the global system is

The state space explosion problem: examples

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Example

For a system made of 100 components with 10 states each, the (maximum) number of different states in the global system is

The state space explosion problem: examples

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Alleviating the state space explosion problem

An active research field since the 1980s!

Some techniques (among many others):

- construct only the part of the state space needed
  - on-the-fly construction of the synchronized product
- parallel/distributed verification
  - more computational power
  - ...but not all algorithms can be parallelized!
- parameterized verification [AD16]
- symmetry reductions, partial order reductions...
  - remove "similar" parts of the state spaces
- symbolic model checking [Ake78] [Bur+92]

- [AD16] Parosh Aziz Abdulla and Giorgio Delzanno. « Parameterized verification ». In: *International Journal on Software Tools for Technology Transfer* 18.5 (2016), pp. 469–473
- [Ake78] Sheldon B. Akers Jr. « Binary Decision Diagrams ». In: *IEEE Transactions on Computers* 27.6 (1978), pp. 509–516
- [Bur+92] Jerry R. Burch, Edmund M. Clarke, Kenneth L. McMillan, David L. Dill, and L. J. Hwang. « Symbolic Model Checking: 10^{20} States and Beyond ». In: *Information and Computation* 98.2 (1992), pp. 142–170

Motivation for symbolic approaches

- Idea: represent symbolically states and transitions

- A symbolic state aims at representing concisely large sets of states

Outline

Symbolic computation of state sets

Let $\mathcal{A} = \langle Q, \Sigma, T, q_0, lab, F \rangle$ be an automaton, and $S \subseteq Q$ a set of its states.
Let φ be a CTL formula.

Notations

- $Pre(S) = \{q \in Q \mid (q, \_, q') \in T \wedge q' \in S\}$ is the set of immediate predecessors of states in S
- $Sat(\varphi) = \{q \in Q \mid q \models \varphi\}$ is the set of states of the automaton satisfying formula φ
- $Pre^*(S)$ is the set of predecessors of states in S, whatever the number of steps

Computing $Sat(\varphi)$

$$
\begin{aligned}
Sat(\neg\varphi) &= Q \setminus Sat(\varphi) \\
Sat(\psi_1 \wedge \psi_2) &= Sat(\psi_1) \cap Sat(\psi_2) \\
Sat(\mathsf{EX}\varphi) &= Pre\big(Sat(\varphi)\big) \\
Sat(\mathsf{AX}\varphi) &= Q \setminus Pre\big(Q \setminus Sat(\varphi)\big) \\
Sat(\mathsf{EF}\varphi) &= Pre^*\big(Sat(\varphi)\big)
\end{aligned}
$$

Symbolic features

- symbolic representations of the state sets
- functions to manipulate these symbolic representations

Example

- suppose the automaton has 2 integer variables $a, b \in \{0, \ldots, 255\}$
- each state is a triple (q, v_a, v_b) where v_a and v_b are values for a and b
- the set of reachable states can contain

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- a possible symbolic representation could be $(q_2, 3, \_)$ for all states in q_2 with $a = 3$ and any value for b
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Requirements for symbolic model checking

1. symbolic representation of $Sat(p)$ for each proposition $p \in AP$
2. algorithm to compute a symbolic representation of $Pre(S)$ from a symbolic representation of S
3. algorithms to compute the complement, union and intersection of symbolic representations of the sets
4. algorithm to compare symbolic representations of sets

Outline

Binary Decision Diagrams

- Data structure commonly used for the symbolic representation of state sets
- Efficiency: cheap basic operations, compact data structure
- Simplicity: data structure and associated algorithms simple to describe and implement
- Easy adaptation: appropriate for problems dealing with loosely correlated data
- Generality: not tied to a particular family of automata

BDD structure

n Boolean variables $x_1, \ldots, x_n$

- suppose $n = 4$.
  $\langle b_1, b_2, b_3, b_4 \rangle$ associates values with $x_1, \ldots, x_4$

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- Let us represent $S = \{\langle b_1, b_2, b_3, b_4 \rangle \mid (b_1 \vee b_3) \wedge (b_2 \implies b_4)\}$

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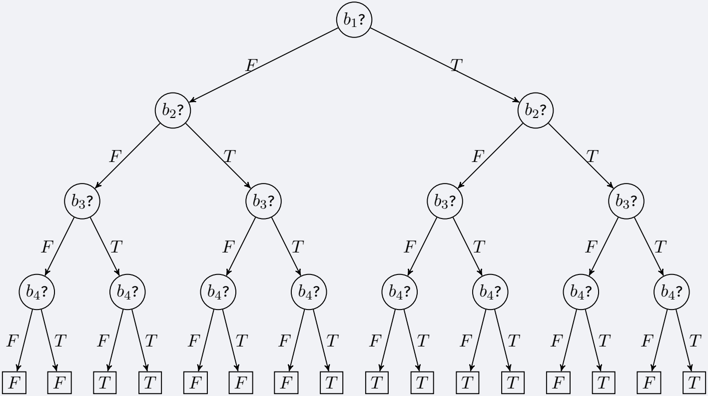
- Let us represent $S = \{\langle b_1, b_2, b_3, b_4 \rangle \mid (b_1 \vee b_3) \wedge (b_2 \implies b_4)\}$
- Possible representations:
  - $|S| = 9$: $$\begin{aligned} S = \{ \quad & \langle F, F, T, F \rangle, \langle F, F, T, T \rangle, \langle F, T, T, T \rangle, \\ & \langle T, F, F, F \rangle, \langle T, F, F, T \rangle, \langle T, F, T, F \rangle, \\ & \langle T, F, T, T \rangle, \langle T, T, F, T \rangle, \langle T, T, T, T \rangle \} \end{aligned}$$
  - the formula itself: $(b_1 \vee b_3) \wedge (b_2 \implies b_4)$
  - the formula in disjunctive normal form:
    $(b_1 \wedge \neg b_2) \vee (b_1 \wedge b_4) \vee (b_3 \wedge \neg b_2) \vee (b_3 \wedge b_4)$
  - a decision tree

Representation with a decision tree

$(b_1 \vee b_3) \wedge (b_2 \implies b_4)$



BDD: a reduced decision tree

- identical subtrees are shared $\rightsquigarrow$ directed acyclic graph (*dag*)
- internal superfluous nodes are deleted (where no choice is possible)

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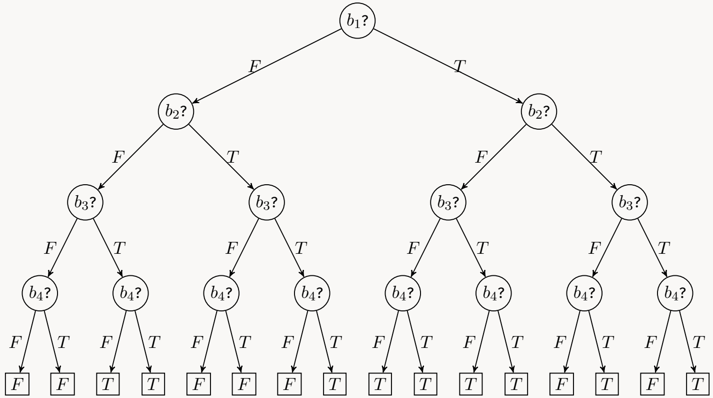

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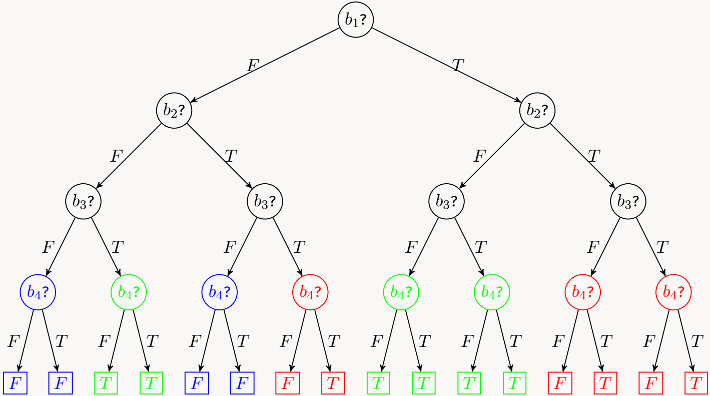

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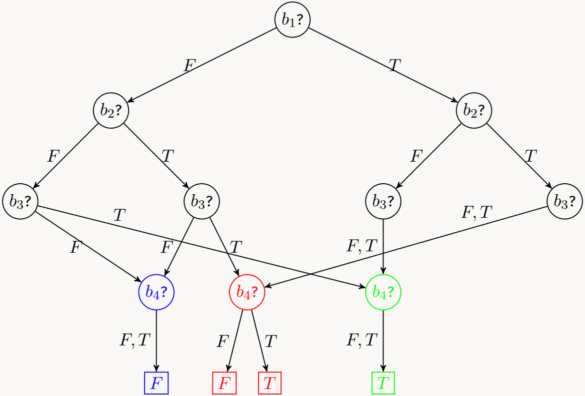

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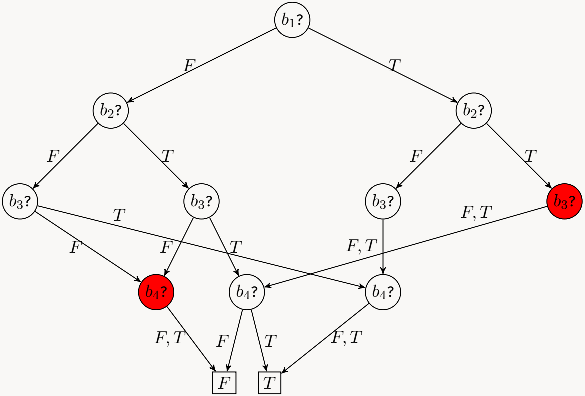

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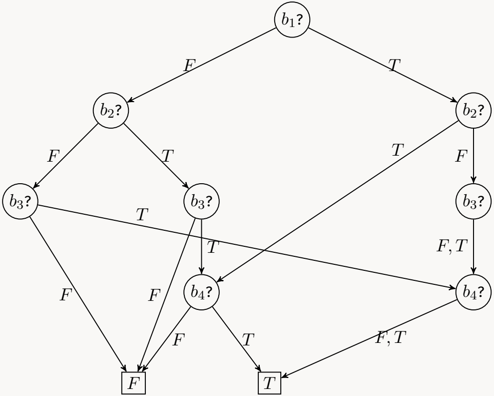

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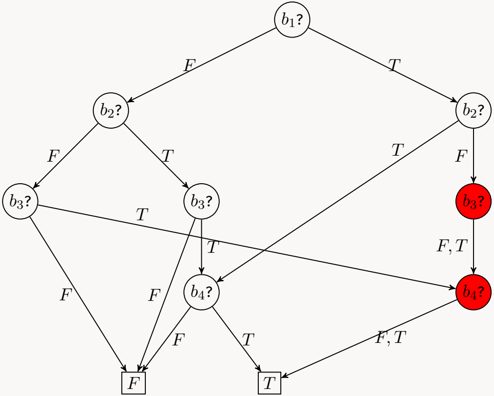

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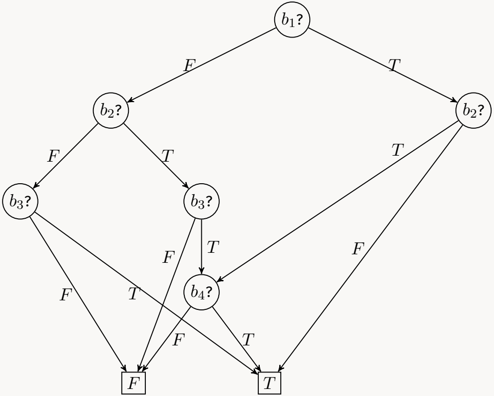
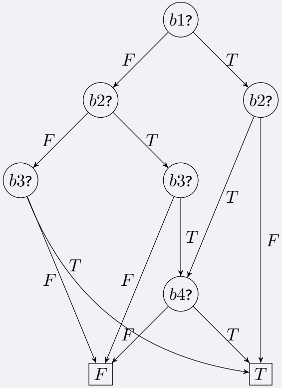


Testing whether a tuple belongs to the set

Are $\langle T, F, T, F\rangle$, $\langle F, F, T, F\rangle$ in S?



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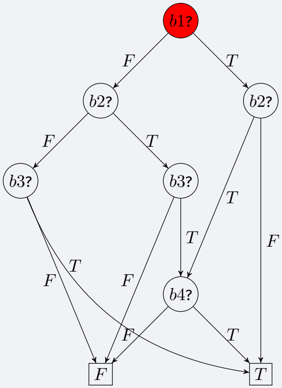

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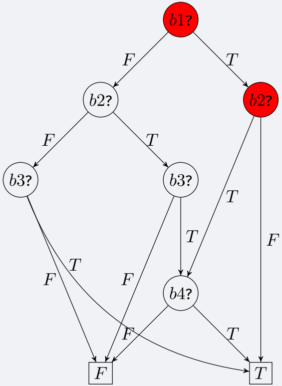

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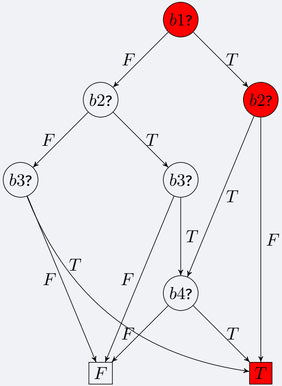

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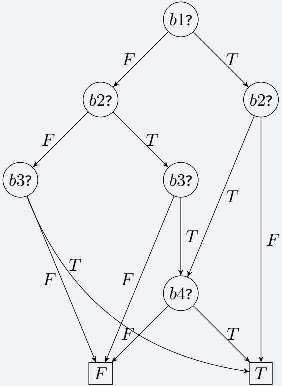

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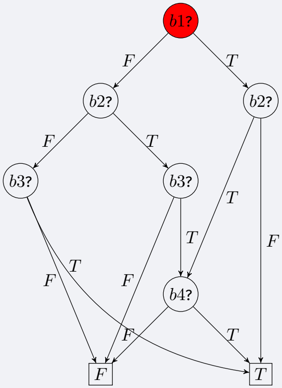

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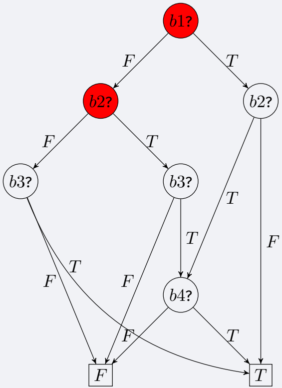

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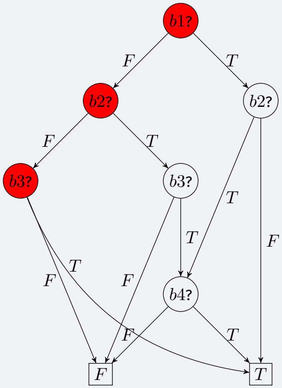

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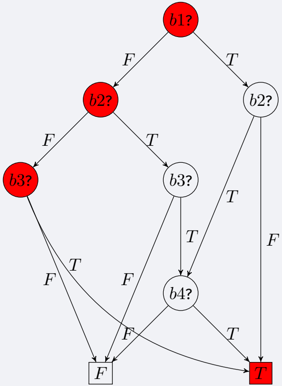

Exercise

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Give the BDD for $\neg\Big((b_1 \land (b_2 \lor b_4) \land b_5\Big) \lor \neg b_3) \lor \Big(b_4 \implies (b_3 \land b_5)\Big)$

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- existence of a canonical BDD structure:
  - unicity for a fixed order of the variables
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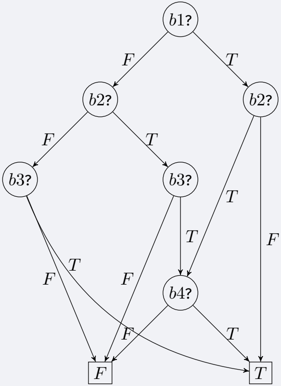
  - simple operations: complement, union, intersection, projection

Exercise: Complement

Exercise (Complement)



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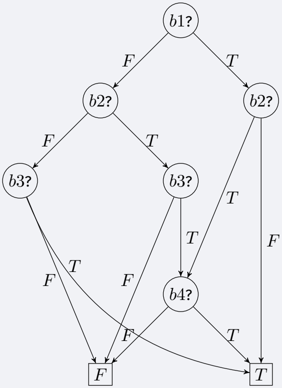

Exercise: Union

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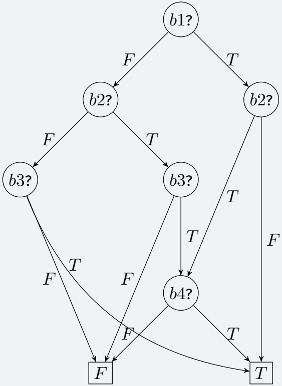

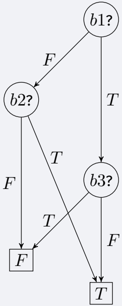

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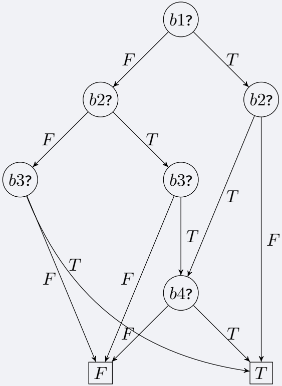

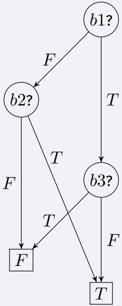

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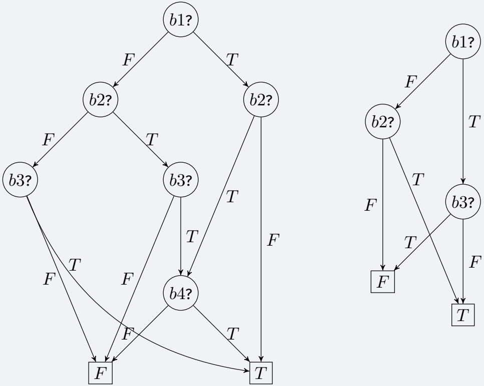

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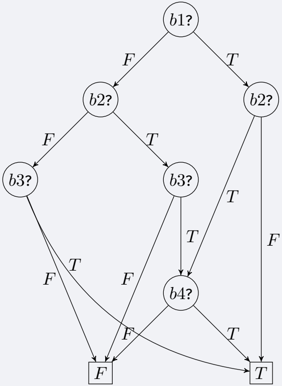

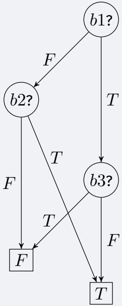

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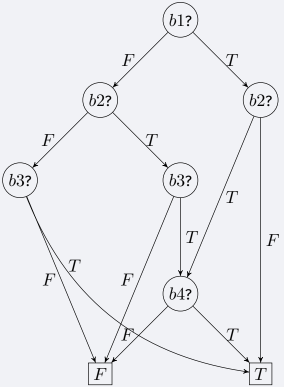
Exercise (Projection $S[b_3 \leftarrow T]$)



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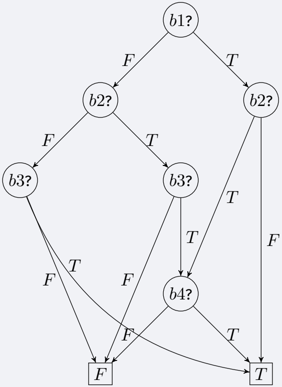

Outline

Representing automata by BDDs

Encoding of states

- Boolean encoding of states
- Boolean encoding of each individual variable

Representing automata by BDDs: example

Example

Let us consider an automaton with:

- $Q = \{q_0, \dots, q_6\}$
- an integer variable $digit \in \{0, \dots, 9\}$
- a Boolean variable $ready$

How many bits are necessary to encode a state $\langle q, d, r \rangle$?

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For example, $\langle q_3, 8, F \rangle$ is represented by:

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Representing a set of states

$$Sat\big(ready \implies (digit > 2)\big)$$

Representing a set of states

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Representing a transition

Transition seen as a pair of states

$\langle q_3, 8, F\rangle \longrightarrow \langle q_5, 0, F\rangle$ is represented by:

$$(\overbrace{\underset{b_1^1}{F}, \underset{b_1^2}{T}, \underset{b_1^3}{T}}^{q_3}, \overbrace{\underset{b_2^1}{T}, \underset{b_2^2}{F}, \underset{b_2^3}{F}, \underset{b_2^4}{F}}^{8}, \underset{b_3^1}{F}, \overbrace{\underset{b'^1_1}{T}, \underset{b'^2_1}{F}, \underset{b'^3_1}{T}}^{q_5}, \overbrace{\underset{b'^1_2}{F}, \underset{b'^2_2}{F}, \underset{b'^3_2}{F}, \underset{b'^4_2}{F}}^{0}, \underset{b'^1_3}{F})$$

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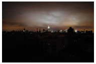
Additional information

Explanation for the 3 pictures in the beginning



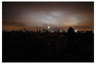
Allusion to the Northeast blackout (USA, 2003)
Computer bug
Consequences: 11 fatalities, huge cost
(Picture actually from the Sandy Hurricane, 2012)

Allusion to the MIM-104 Patriot Missile Failure (Iraq, 1991)
28 fatalities, hundreds of injured
Computer bug: software error (clock drift)
(Picture of an actual MIM-104 Patriot Missile, though not the one of 1991)

Allusion to the sinking of the Sleipner A offshore platform (Norway, 1991)
No fatalities
Computer bug: inaccurate finite element analysis modeling
(Picture actually from the Deepwater Horizon Offshore Drilling Platform)

Credits

Source of the graphics used I



Titre : Hurricane Sandy Blackout New York Skyline
Auteur : David Shankbone
Source : https://commons.wikimedia.org/wiki/File:Hurricane_Sandy_Blackout_New_York_Skyline.JP
Licence : BY

Titre : Deepwater Horizon Offshore Drilling Platform on Fire
Auteur : ideum
Source : https://secure.flickr.com/photos/ideum/4711481781/
Licence : BY SA

Titre : DA-SC-88-01663
Auteur : imcomkorea
Source : https://secure.flickr.com/photos/imcomkorea/3017886760
Licence : BY NC ND

Titre : Smiley green alien big eyes (aaah)
Auteur : LadyofHats
Source : https://commons.wikimedia.org/wiki/File:Smiley_green_alien_big_eyes.svg
Licence : PUBLIC DOMAIN

Titre : Smiley green alien big eyes (cry)
Auteur : LadyofHats

Source of the graphics used II

Source : https://commons.wikimedia.org/wiki/File:Smiley_green_alien_big_eyes.svg
Licence : PUBLIC DOMAIN

Titre : Coffee machine drawing
Auteur : Ysangkok
Source : https://commons.wikimedia.org/wiki/File:Coffee_machine.svg
Licence : PUBLIC DOMAIN

Titre : taking a coffee break
Auteur : chris
Source : https://commons.wikimedia.org/wiki/File:SMirC-coffeebreak.svg
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Authors: **Laure Petrucci** and **Étienne André**

(LaTeX source available to academic teachers upon request)

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