Cutting planar maps into slices

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Based on joint works with Emmanuel Guitter and Grégory Miermont

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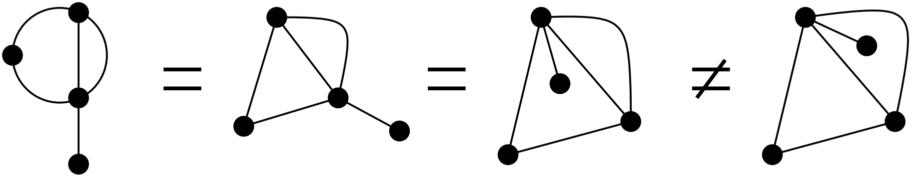
Outline

Planar maps: definitions

A planar map is a connected (multi)graph embedded into the sphere and considered up to homeomorphism. It is made of vertices, edges, faces and corners. The degree of a face or a vertex is its number of incident corners.



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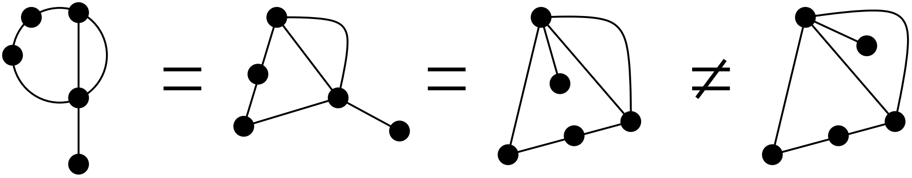

A planar map is bipartite if all its faces have even degree.

Context and motivations

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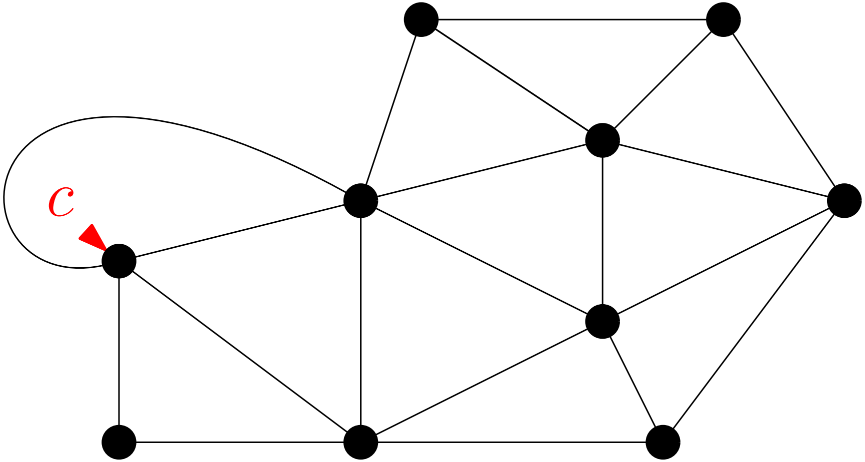
- but does it extend to maps of other topologies, similarly to the topological recursion discovered by Eynard and Orantin? With Guitter and Miermont we recently made partial progress in this direction.

Outline

Leftmost geodesic



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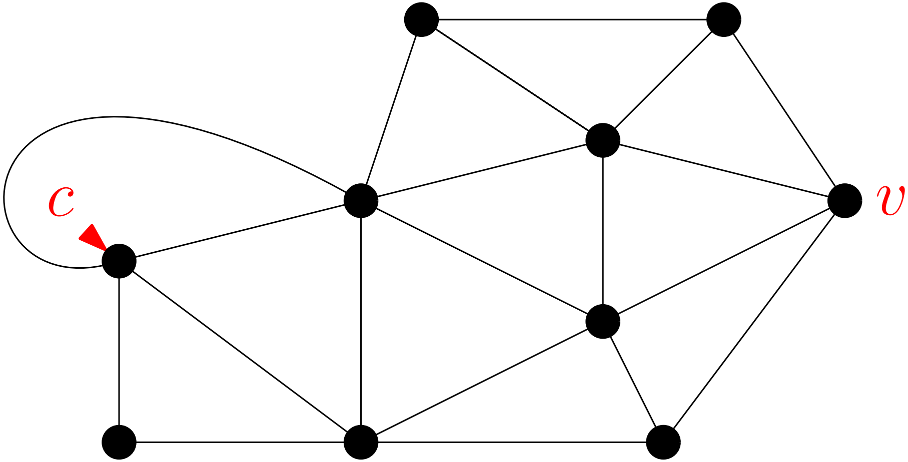

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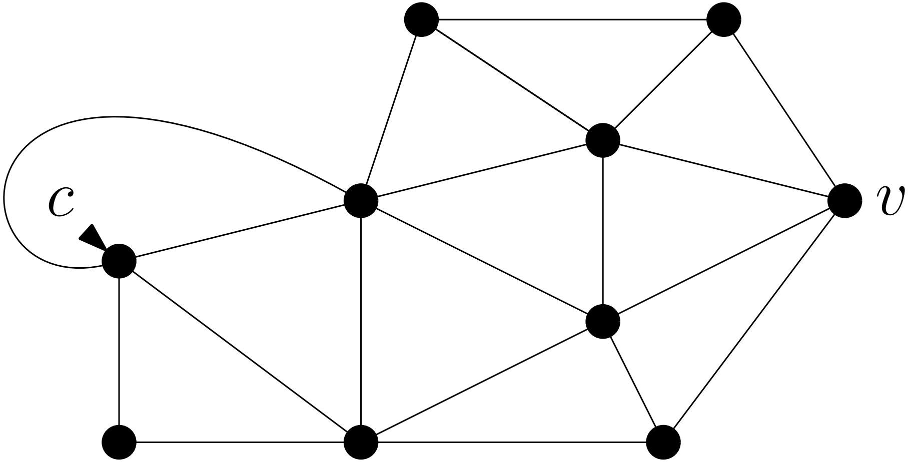

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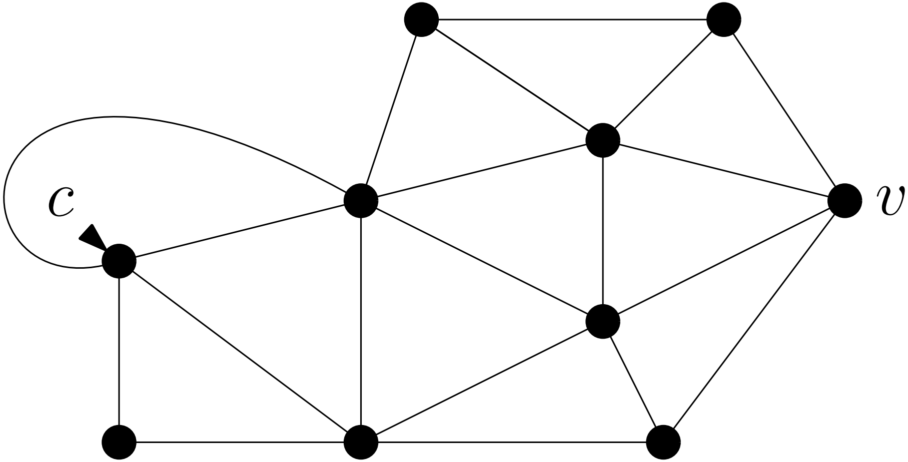

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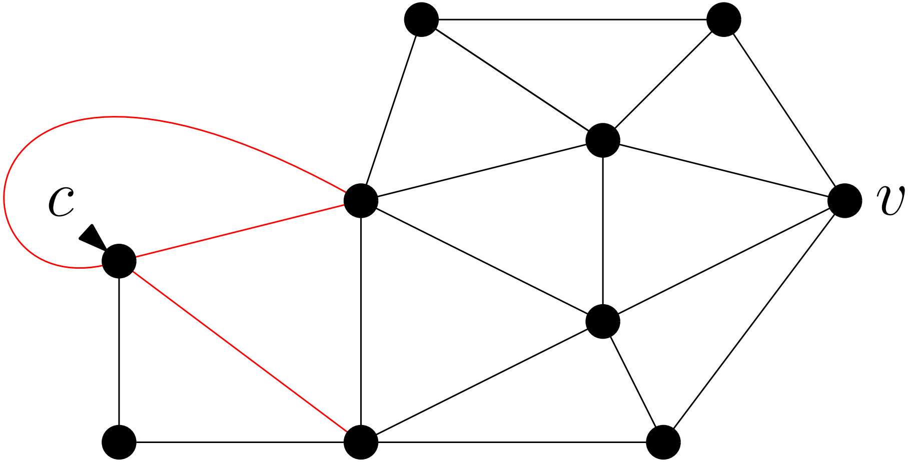

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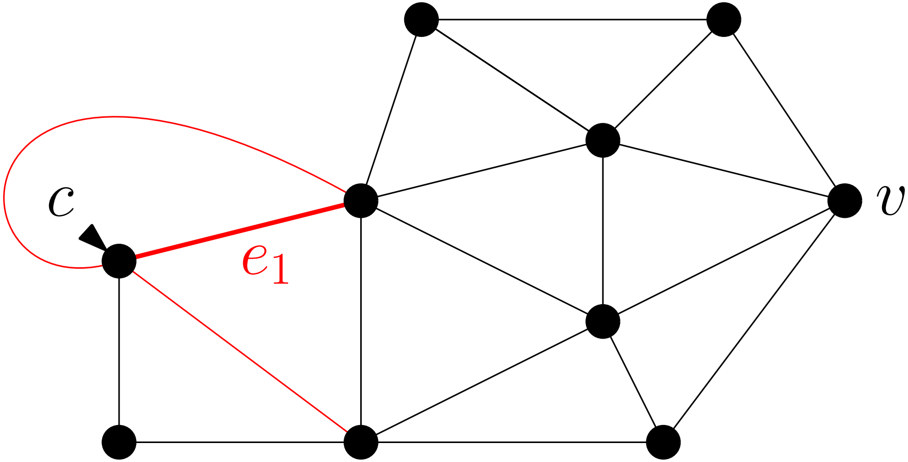

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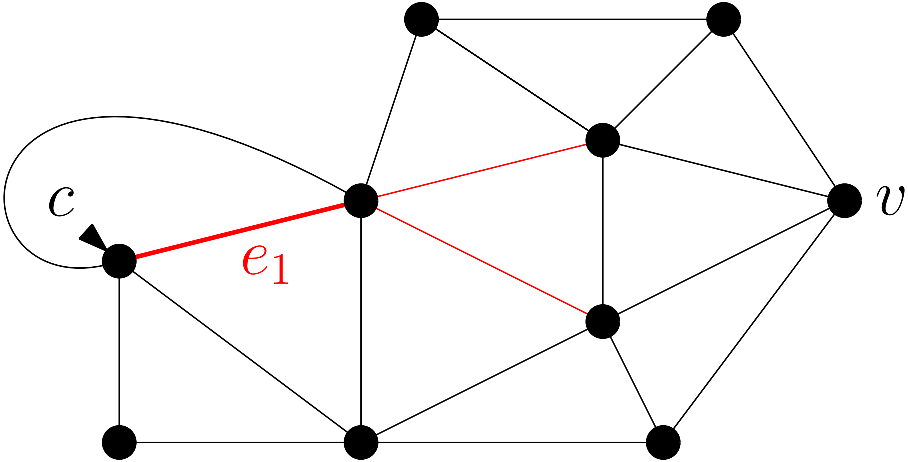

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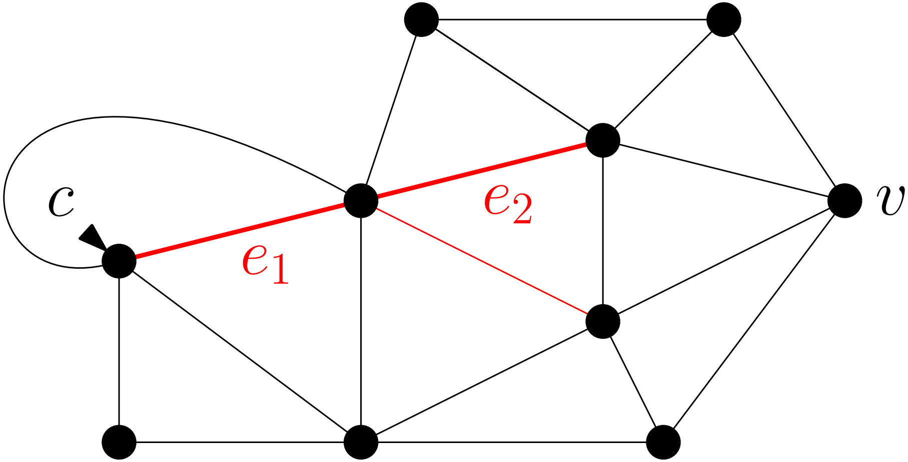

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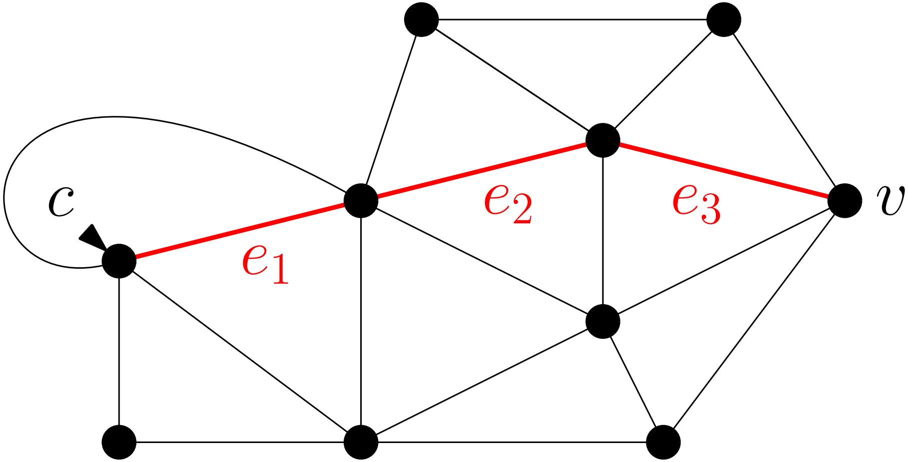

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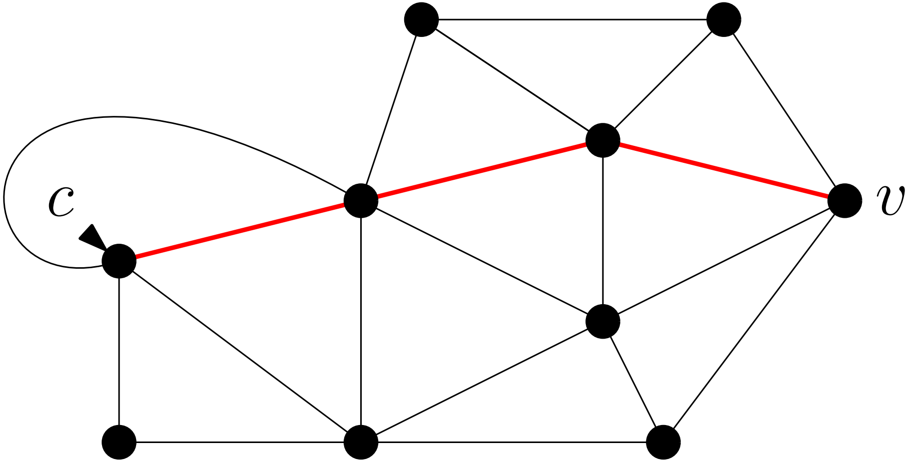

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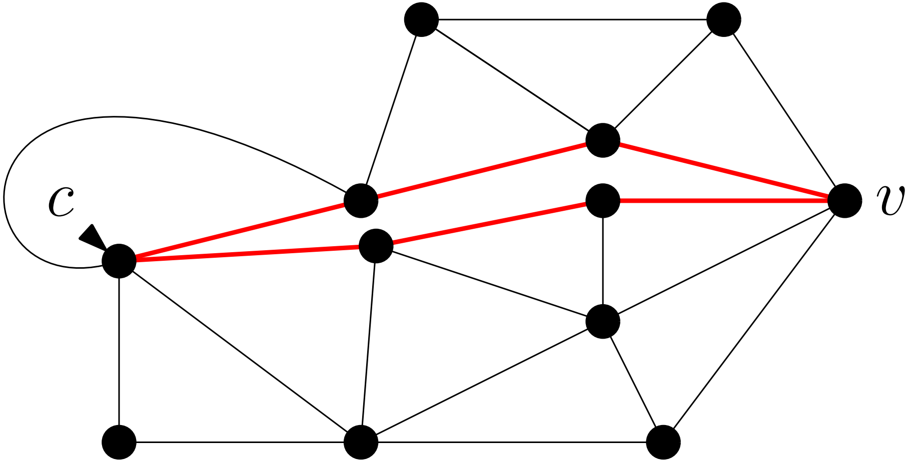

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Theorem (reformulation of Tutte's census of slicings, 1962)

The generating function R of planar bipartite maps with one marked edge and one marked vertex (i.e. pointed rooted maps) satisfies

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Let's see how we can rederive this using the slice decomposition.

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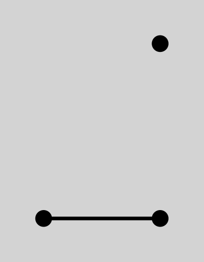

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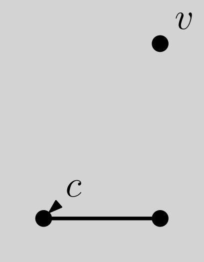

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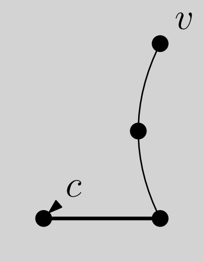

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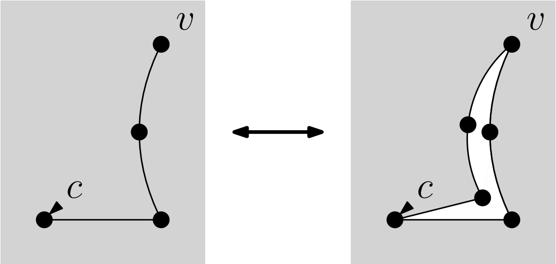

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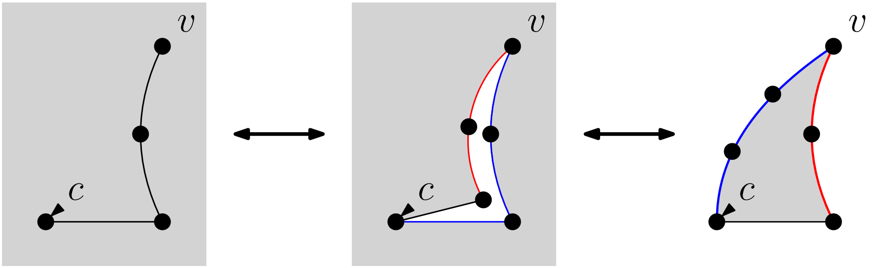

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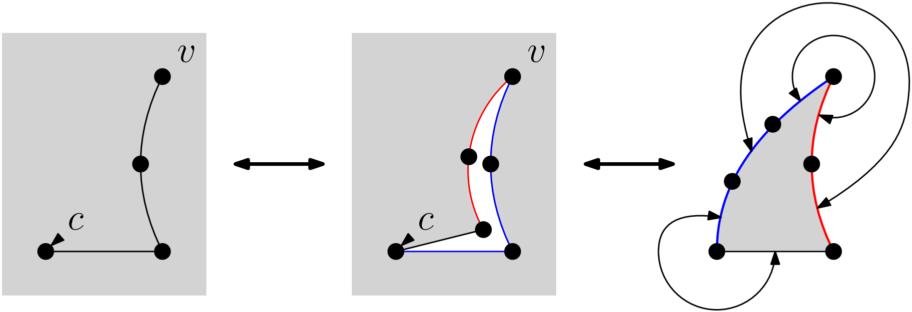
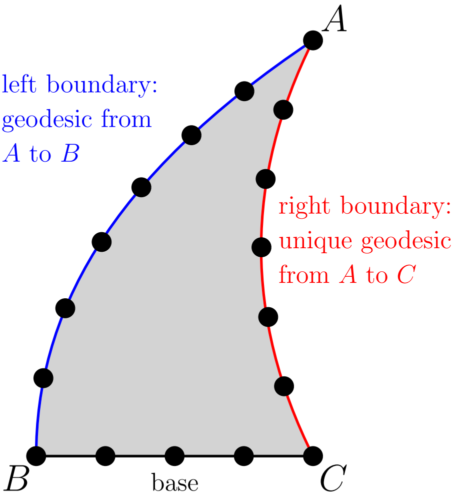


Slices: general definition



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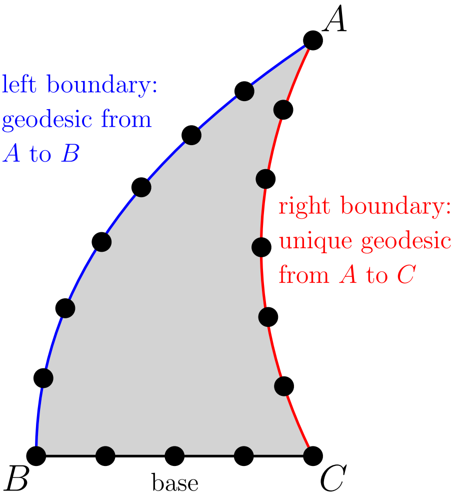


It is assumed that the left and right boundaries only meet at A.

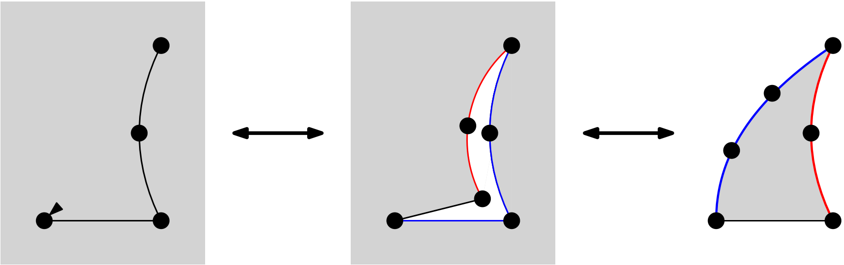
Terminology:

- *width*: length BC
- *depth*: length AB
- *tilt*: difference AB-AC

A slice of width 1 is said *elementary*. Its tilt is then ± 1, as we are in the bipartite case.

The only elementary slice of tilt -1 is the *trivial* slice reduced to a single edge (with $A = B \neq C$).

Pointed rooted maps are in bijection with elementary slices of tilt $+1$.



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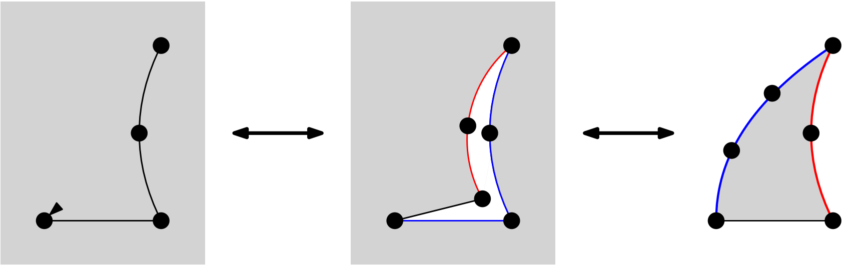

Thus, to recover Tutte's slicings formula, we should prove that the generating function R of elementary slices of tilt $+1$ satisfies

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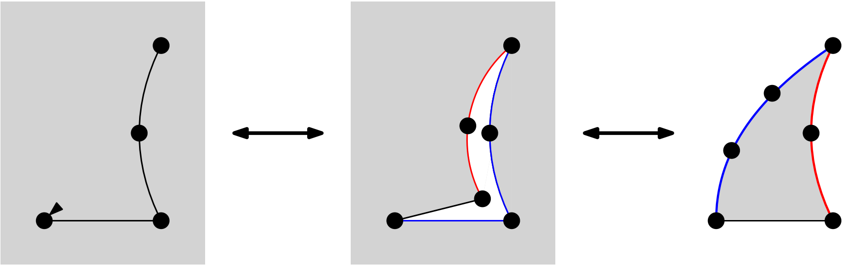

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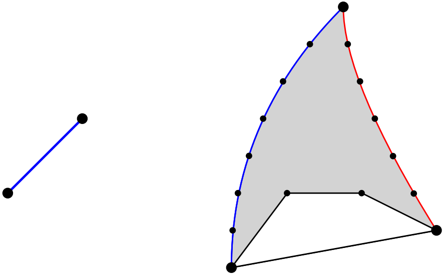
(NB: no weight for the outer face and the vertices on the right boundary.)

An elementary slice of tilt $+1$ is either reduced to a single edge, or is in bijection with a slice of arbitrary odd width and tilt $+1$.



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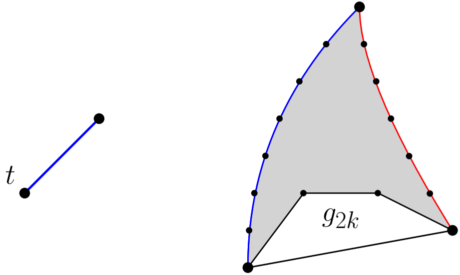

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$$R = t + \sum_{k \geq 1} g_{2k} C_{2k-1,1}$$

with $C_{\ell,i}$ the generating function of slices of width ℓ and tilt i.

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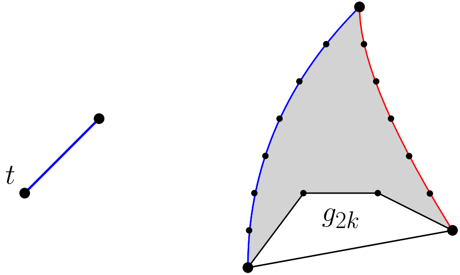

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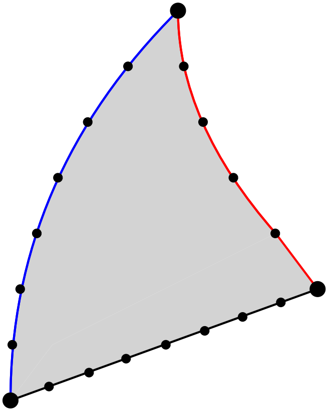
Claim: $$C_{\ell,i} = \begin{cases} \binom{\ell}{(\ell+i)/2} R^{(\ell+i)/2} & \text{if } \ell + i \text{ even,} \\ 0 & \text{otherwise.} \end{cases}$$

Cutting a general slice into elementary slices



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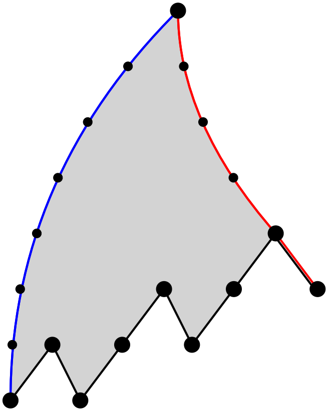

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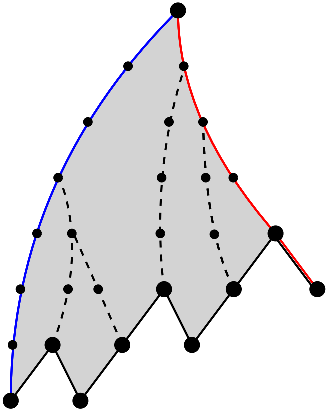

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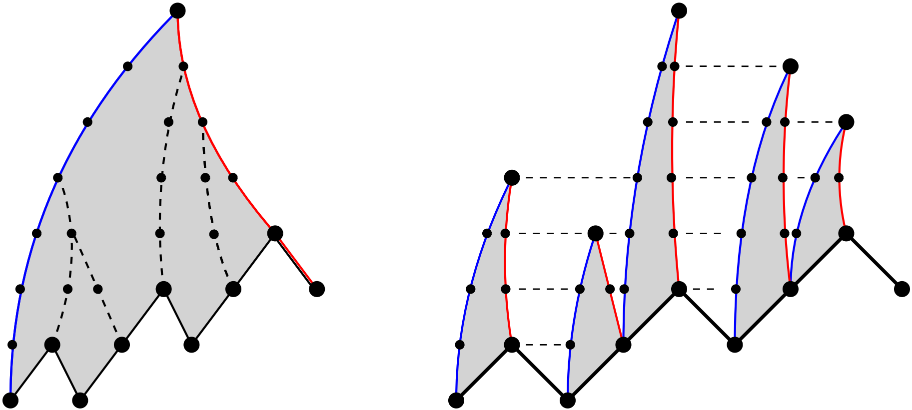

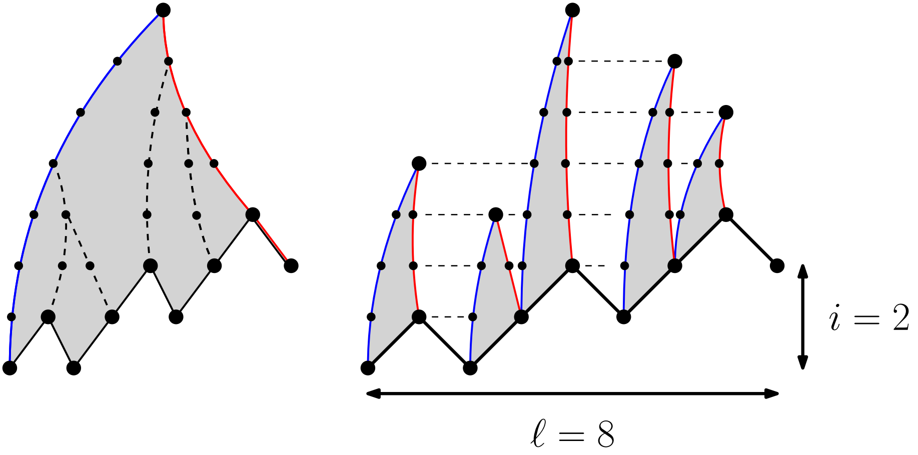

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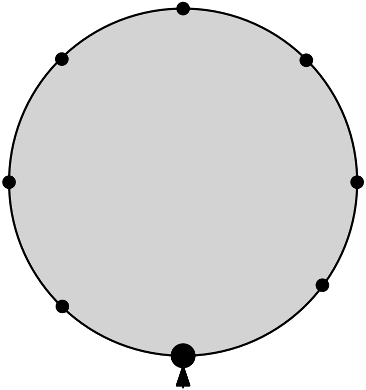

(Here $2p = 8$.)

Let F_{2p} denote the generating function of rooted maps with a root face of degree $2p$.

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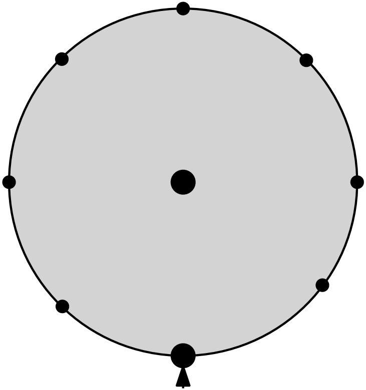

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$$F^{\bullet}_{2p} = \frac{d}{dt} F_{2p}$$

is the generating function of “pointed disks”.

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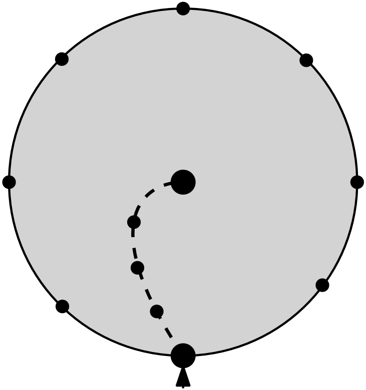

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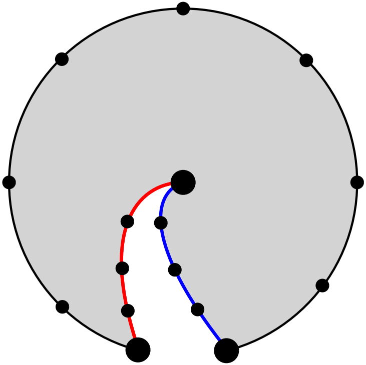

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The slice decomposition gives

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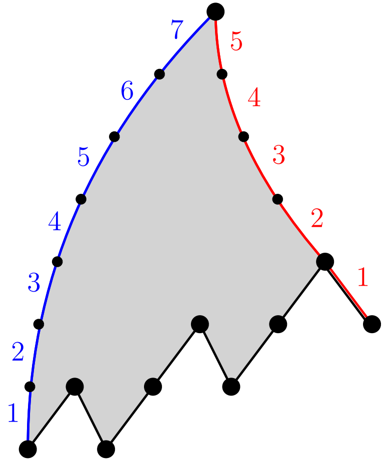

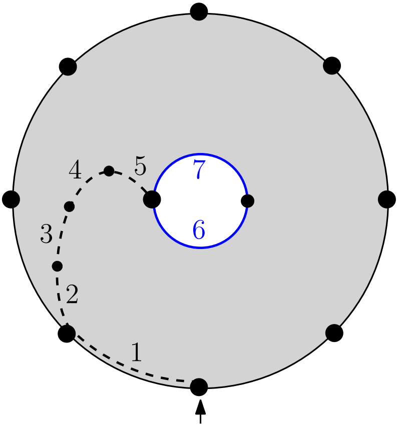

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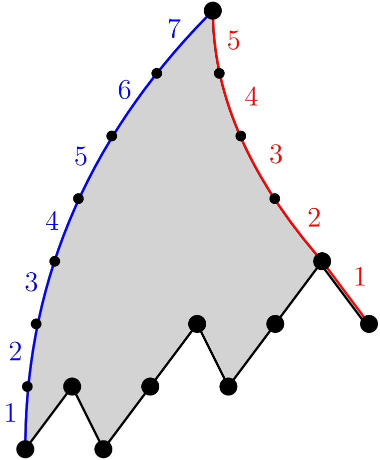

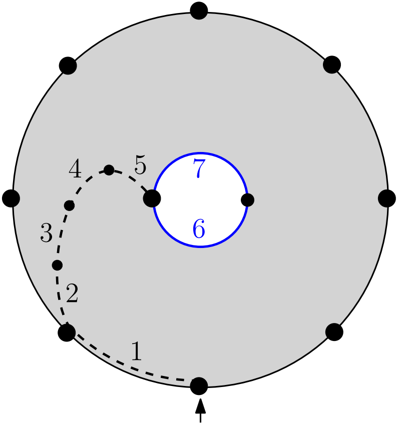

Theorem [B.-Guitter, 2014]

Slices of width ℓ and tilt $i \neq 0$ are in bijection with (ℓ, i)-funnels, i.e. annular maps whose marked faces have degree ℓ and $|i|$, the contour of the latter forming a minimal separating cycle, unique when $i < 0$.

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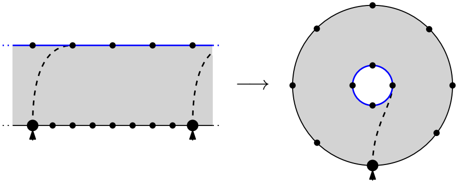
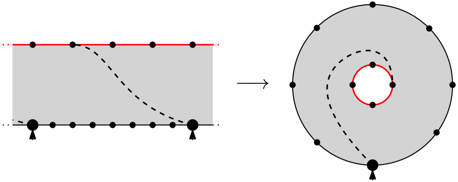


$z \mapsto \exp(2i\pi z)$



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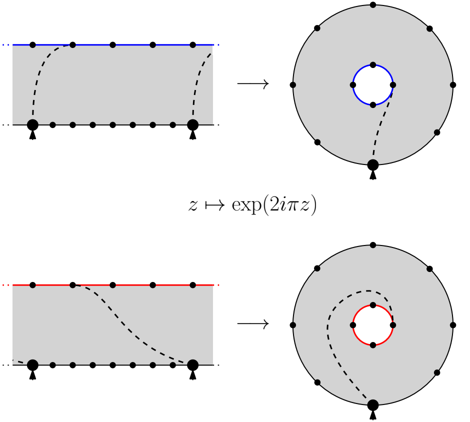

Key ideas: (modernized following BGM21)

- minimal separating cycles lift to infinite geodesics
- the Busemann function of an infinite geodesic γ:

$$d_\gamma(v) = \lim_{t \to \infty} \left(d(v, \gamma_t) - t \right)$$

- the leftmost geodesic is the leftmost path along which d_γ decreases. We ensure that it hits γ in a finite number of steps.

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We deduce the g.f. of annular maps whose marked faces have degrees ℓ and m ($\ell + m$ even), without minimality constraint:

$$A_{\ell,m} := \sum_{\substack{0 \leq i \leq \min(\ell,m) \\ \ell+i \text{ even}}} i C_{\ell,i} C_{m,-i}$$

$$= \frac{2}{\ell + m} \cdot \frac{\ell!}{\lfloor \frac{\ell}{2} \rfloor! \lfloor \frac{\ell-1}{2} \rfloor!} \cdot \frac{m!}{\lfloor \frac{m}{2} \rfloor! \lfloor \frac{m-1}{2} \rfloor!} \cdot R^{(\ell+m)/2}.$$

This formula also appears in Collet and Fusy (2012).

Outline

Setting

We now consider planar maps with three boundaries ("pairs of pants").
A boundary is a marked face or vertex, and its length is:

- 0 in the case of a vertex,
- its degree in the case of a face.

We assume that the three boundaries are distinct (no symmetries!).

A map is said essentially bipartite if each face other than a boundary has even length.

Theorem (Eynard, Collet-Fusy 2012)

Fix $a, b, c \in \mathbb{N}/2$ such that $a + b + c \in \mathbb{N}$. Then, the generating function of essentially bipartite planar maps with three boundaries of lengths $2a, 2b, 2c$ is equal to

$$P_{a,b,c} = n(a)n(b)n(c)R^{a+b+c}\frac{d \ln R}{dt} - t^{-1}\mathbf{1}_{a+b+c=0}$$

where $n(\ell) := \binom{2\ell-1}{\lfloor \frac{2\ell-1}{2} \rfloor}$ and where R is the series of pointed rooted maps:

$$R = t + \sum_{k\geq 1}\binom{2k-1}{k}g_k R^k.$$

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Eynard gave this formula in his book as an application of the framework of topological recursion, and Collet and Fusy (2012) gave an elementary bijective proof.

We say that a boundary is tight if it has minimal length in its homotopy class.

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Theorem (B.-Guitter-Miermont 2021)

Fix $a, b, c \in \mathbb{N}/2$ such that $a+b+c \in \mathbb{N}$. Then, the generating function of essentially bipartite planar maps with three tight boundaries of lengths $2a, 2b, 2c$ is equal to

$$T_{a,b,c} = R^{a+b+c} \frac{d \ln R}{dt} - t^{-1} \mathbf{1}_{a+b+c=0}$$

where R is the series of pointed rooted maps:

$$R = t + \sum_{k \geq 1} \binom{2k-1}{k} g_k R^k.$$

By cutting a general pair of pants along outermost minimal separating cycles, we get the relation

$$P_{a,b,c} = \sum_{a',b',c'} C_{a,a'} C_{b,b'} C_{c,c'} T_{a',b',c'}$$

where $C_{\ell,i}$ is the g.f. of (ℓ, i)-funnels.

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Thus, our formula is equivalent to the Eynard-Collet-Fusy formula. But, since the expression for $T_{a,b,c}$ is simpler, we want a direct bijective proof!

Generatingfunctionology

We want to prove (bijectively!) that

$$T_{a,b,c} = R^{a+b+c} \frac{d \ln R}{dt} - t^{-1} \mathbf{1}_{a+b+c=0}.$$

It is already known (bijectively!) that

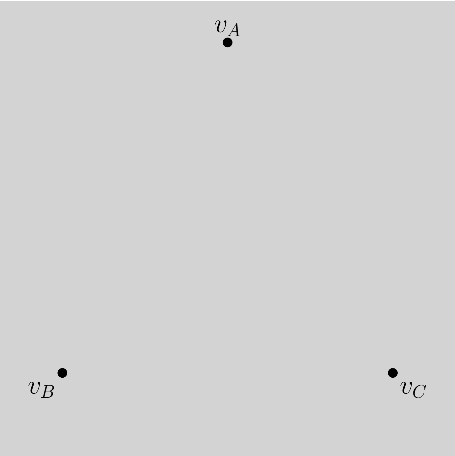
$$T_{0,0,0} = \frac{d \ln R}{dt} - t^{-1}$$

We will show (bijectively!) that

$$T_{a,b,c} = R^{a+b+c} \frac{X^3 Y^2}{t^6} - t^{-1} \mathbf{1}_{a+b+c=0}$$

with X, Y the g.f. of certain objects.

Warm-up: $T_{0,0,0} = X^3 Y^2 t^{-6} - t^{-1}$



Start from a planar map with three marked vertices.

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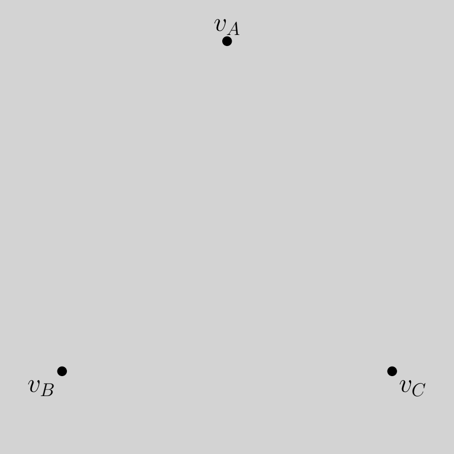

Start from a planar map with three marked vertices. Their distances can be written (see also B.-Guitter 2008)

$$d_{AB} = r_A + r_B$$
$$d_{BC} = r_B + r_C$$
$$d_{CA} = r_C + r_A$$

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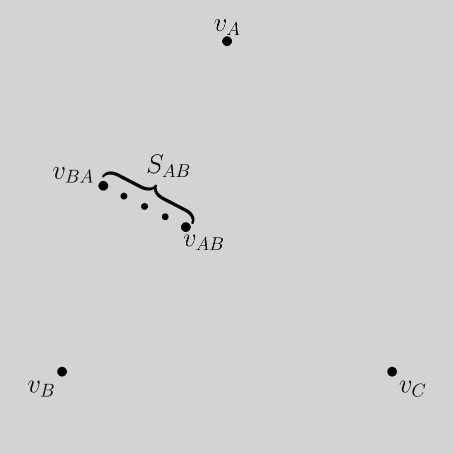

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The set S_{AB} of vertices at distance r_A from v_A and r_B from v_B has two extremal elements v_{AB} and v_{BA}.

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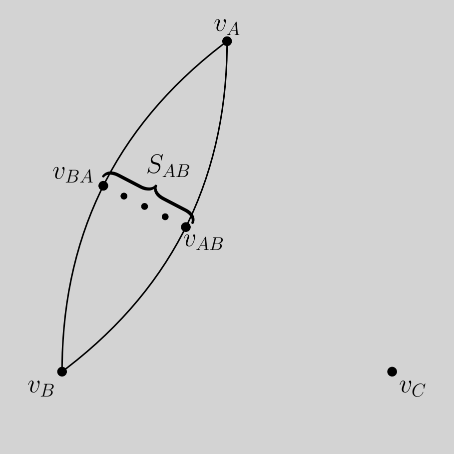

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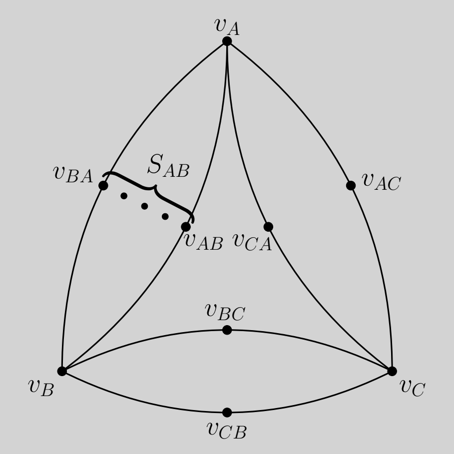

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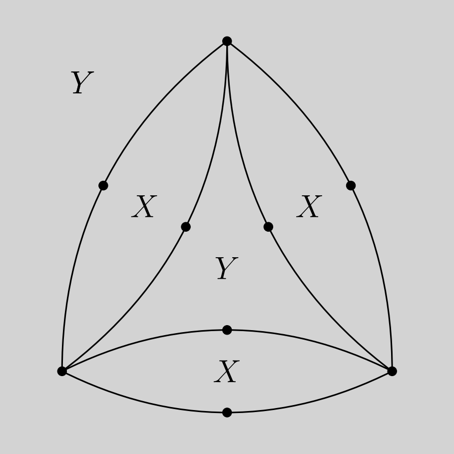

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This decomposes the map into three "balanced bigeodesic diangles" (X) and three "bigeodesic triangles" (Y), maybe reduced to single vertices (t).

To prove the relation

$$T_{a,b,c} = R^{a+b+c} \frac{X^3 Y^2}{t^6} - t^{-1} \mathbf{1}_{a+b+c=0}$$

for general a, b, c, i.e. to decompose a planar map with three boundaries, we need to consider the universal cover of a pair of pants.

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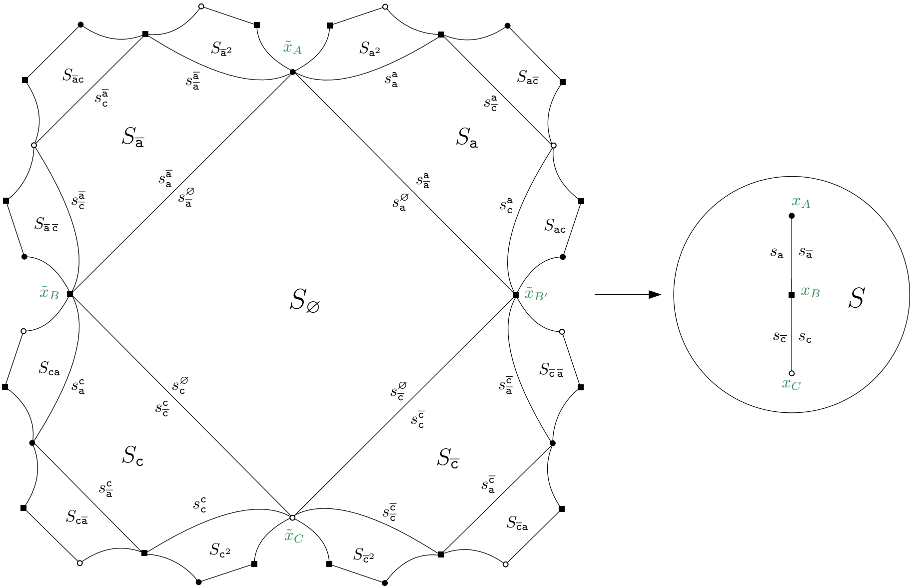
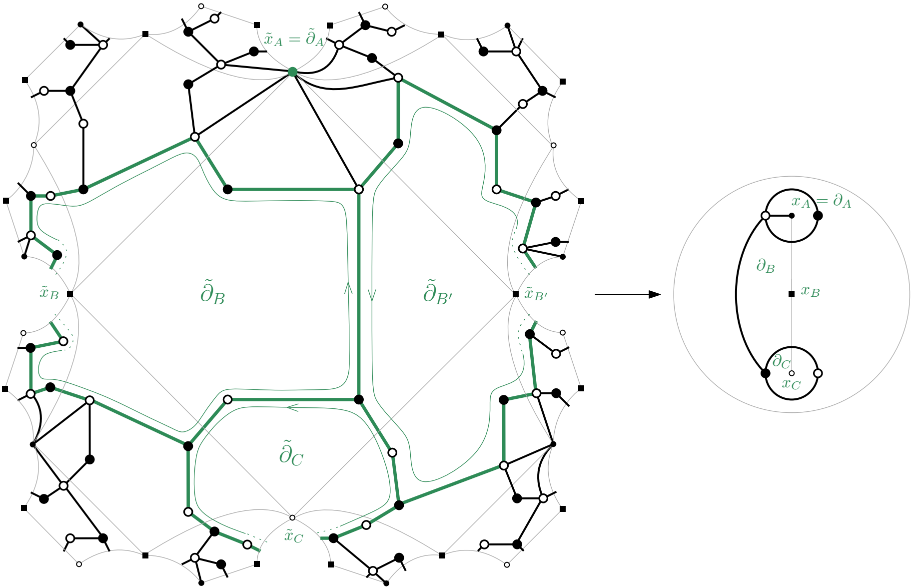

$S_{\bar{a}^2}$
$\tilde{x}_A$
S_{a^2}
$S_{\bar{a}c}$
$s^{\bar{a}}_{\bar{a}}$
s^{a}_{a}
$S_{a\bar{c}}$
$s^{\bar{a}}_{c}$
$s^{a}_{\bar{c}}$
$S_{\bar{a}}$
S_{a}
$s^{\bar{a}}_{a}$
$s^{\varnothing}_{\bar{a}}$
$s^{a}_{\bar{a}}$
$s^{\varnothing}_{a}$
$s^{\bar{a}}_{\bar{c}}$
s^{a}_{c}
$S_{\bar{a}\bar{c}}$
S_{ac}
$\tilde{x}_B$
$S_{\varnothing}$
$\tilde{x}_{B'}$
S_{ca}
$S_{\bar{c}\bar{a}}$
s^{c}_{a}
$s^{\varnothing}_{c}$
$s^{\varnothing}_{\bar{c}}$
$s^{\bar{c}}_{a}$
s^{c}_{c}
$s^{\bar{c}}_{c}$
S_{c}
$S_{\bar{c}}$
$s^{c}_{\bar{a}}$
$s^{\bar{c}}_{\bar{a}}$
s^{c}_{c}
$s^{\bar{c}}_{\bar{c}}$
$S_{c\bar{a}}$
$S_{\bar{c}a}$
S_{c^2}
$\tilde{x}_C$
$S_{\bar{c}^2}$
x_A
s_a
$s_{\bar{a}}$
x_B
S
$s_{\bar{c}}$
s_c
x_C

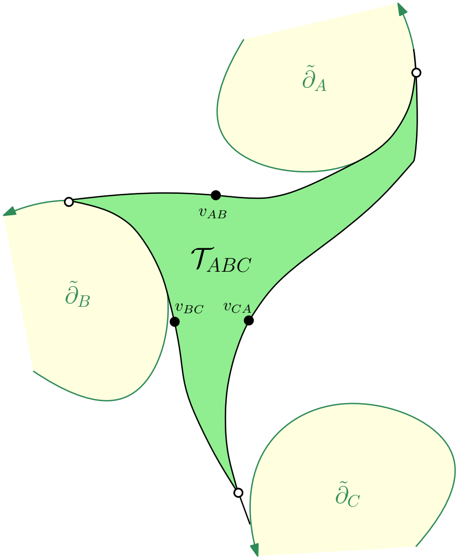

$\tilde{x}_A = \tilde{\partial}_A$
$\tilde{x}_B$
$\tilde{\partial}_B$
$\tilde{\partial}_{B'}$
$\tilde{x}_{B'}$
$\tilde{\partial}_C$
$\tilde{x}_C$
$x_A = \partial_A$
∂_B
x_B
∂_C
x_C

To prove the relation

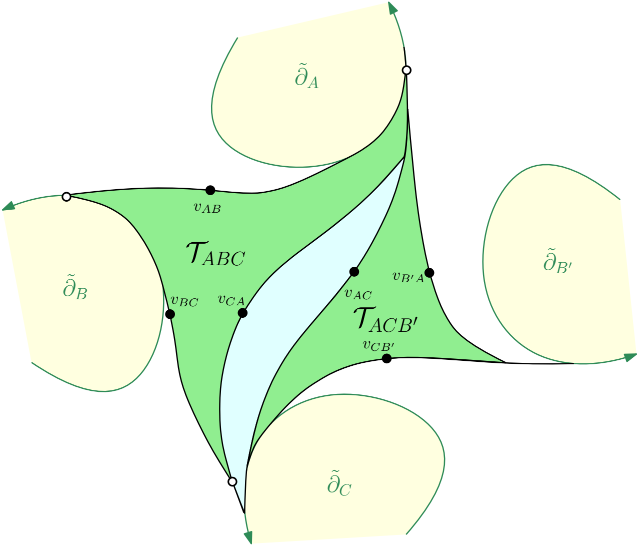
$$T_{a,b,c} = R^{a+b+c}\frac{X^3Y^2}{t^6} - t^{-1}\mathbf{1}_{a+b+c=0}$$

for general a, b, c, i.e. to decompose a planar map with three boundaries, we need to consider the universal cover of a pair of pants. It is slightly more complicated than that of a cylinder !

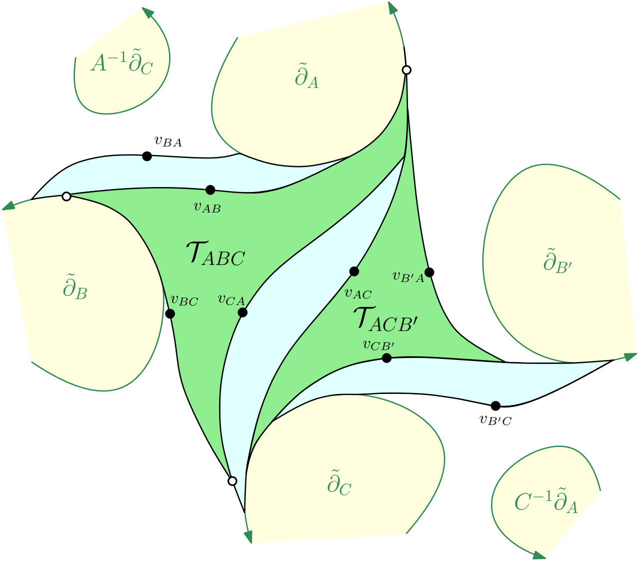
Still, we can proceed by combining ideas from the case of annular maps (Busemann functions, leftmost geodesics) and from the case of $T_{0,0,0}$.

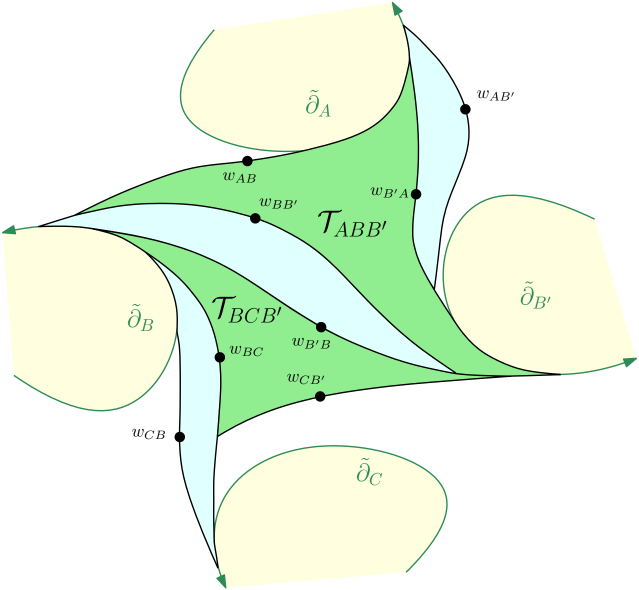


$\tilde{\partial}_A$
v_{AB}
$\mathcal{T}_{ABC}$
$\tilde{\partial}_B$
v_{BC}
v_{CA}
$\tilde{\partial}_C$



$\tilde{\partial}_A$
v_{AB}
$\mathcal{T}_{ABC}$
$\tilde{\partial}_B$
v_{BC}
v_{CA}
v_{AC}
$v_{B'A}$
$\mathcal{T}_{ACB'}$
$\tilde{\partial}_{B'}$
$v_{CB'}$
$\tilde{\partial}_C$


$A^{-1}\tilde{\partial}_C$
$\tilde{\partial}_A$
v_{BA}
v_{AB}
$\mathcal{T}_{ABC}$
$\tilde{\partial}_B$
$\tilde{\partial}_{B'}$
v_{BC}
v_{CA}
v_{AC}
$v_{B'A}$
$\mathcal{T}_{ACB'}$
$v_{CB'}$
$v_{B'C}$
$\tilde{\partial}_C$
$C^{-1}\tilde{\partial}_A$



$\tilde{\partial}_A$
$w_{AB'}$
w_{AB}
$w_{B'A}$
$w_{BB'}$
$\mathcal{T}_{ABB'}$
$\tilde{\partial}_{B'}$
$\tilde{\partial}_B$
$\mathcal{T}_{BCB'}$
$w_{B'B}$
w_{BC}
$w_{CB'}$
w_{CB}
$\tilde{\partial}_C$

Conclusion

- We have seen how to decompose planar maps with one, two and three boundaries into slices or related objects. The common idea is to cut along leftmost geodesics.
- Some probabilistic consequences: length of minimal separating cycles in random planar maps with two or three boundaries.
- Does this extend to other topologies: more boundaries, higher genus? This is work in progress.
- For planar maps with three boundaries, our construction is reminiscent of hyperbolic geometry, where a pair of pants can be decomposed into two ideal triangles.
- In view of Budd's recent work on random hyperbolic surfaces, we believe that this more than a coincidence...

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Thanks for your attention!