Tout ce que je sais sur

Banderier

Def: Let σ be an involution on a finite alphabet. Then a word w is a σ-palindrome if

$$w = \sigma(\tilde{w}).$$

${}_\sigma\mathrm{Pal}(w)$: set of σ-palindrome factors of w

Note: If $\sigma = \mathrm{Id}$, this corresponds to usual palindromes, in which case we write $\mathrm{Pal}(w)$

Example: Let σ be the involution defined by

$$\sigma: B \leftrightarrow L \; ; \; E \leftrightarrow E \; ; \; R \leftrightarrow T \; ; \; S \leftrightarrow S \,.$$

Then BERSTEL is a σ-palindrome.

Reconstruction problem

Let P be a finite set of σ-palindromes in A^*, and factorially closed.

Describe the set of words in A^* whose σ-palindromes are contained in P.

Examples

$P \subseteq Pal(A^*)$:

(i) $P = \{ \varepsilon, a, b \}$

(ii) $P = \{ \varepsilon, a, b, c \}$

$P \subseteq Pal_\sigma(A^*)$:

$P = \{ \varepsilon, ab \}$

Reconstruction problem

Let P be a finite set of σ-palindromes in A^*, and factorially closed.

Let Q be the set of minimal elements of $Pal_\sigma(A^*)-P$
(minimilaty taken with respect to the partial factorial order)

Thm: The maximal language whose σ-palindromes are contained in P is given by

$$X_P = A^* - A^* Q A^*$$

Computation of Pal(w)

LPS$_u$(w): longest Palindromic Suffix of w unioccurrent

Computation of LPS$_u$(w):

w		I	S	A	W	I	W	A	S	I	B	O	B
|LPS$_u$|	0	1	1	1	1	*	3	5	7	9	1	1	3

A lacuna

more statistics on a word

$D(w)$: number of lacunas of w

$C_w(n)$: number of distinct factors of length n of w

$P_w(n)$: number of palindromic factors of length n of w

w		B	A	N	D	E	R	I	E	R			
$\|LPS_u\|$	0	1	1	1	1	1	1	1	*	*			

Thm: D(BANDERIER) = 2

A remarkable identity suggested by

B A N D E R I E R

$$2D(w) = \sum_{n=0}^{k} C_w(n+1) - C_w(n) + 2 - P_w(n) - P_w(n+1).$$

n	0	1	2	3	4	5	6	7	8	9	10	11	
C_w	1	7	7	7	6	5	4	3	2	1	0	0	
P_w	1	7	0	0	0	0	0	0	0	0	0	0	
T_w	0	-5	2	1	1	1	1	1	1	1			

2D(BANDERIER) = 2 × 2 = 9 − 5.

Def: Call genial a word without lacunas.

- Christoffel words are genial
- MAIRESSE and DUCHAMP are genial
- BASSINO, BODINI, JACQUOT, ROSSIN, SORIA, VALLÉE, and some others are genial as well

but

- BANDERIER is a good friend
- DENISE and FERNIQUE as well.

Infinite words: periodic case

w		I	S	A	W	I	W	A	S	I	B	O	B	I	S	A	W	I	W
$\|LPS_u\|$	0	1	1	1	1	*	3	5	7	9	1	1	3	5	7	9	11	13	1

w		B	A	N	D	E	R	I	E	R	B	A	N	D	E	R	I	E
$\|LPS_u\|$	0	1	1	1	1	1	1	1	*	*	*	*	*	*	*	*	*	*

Some important results

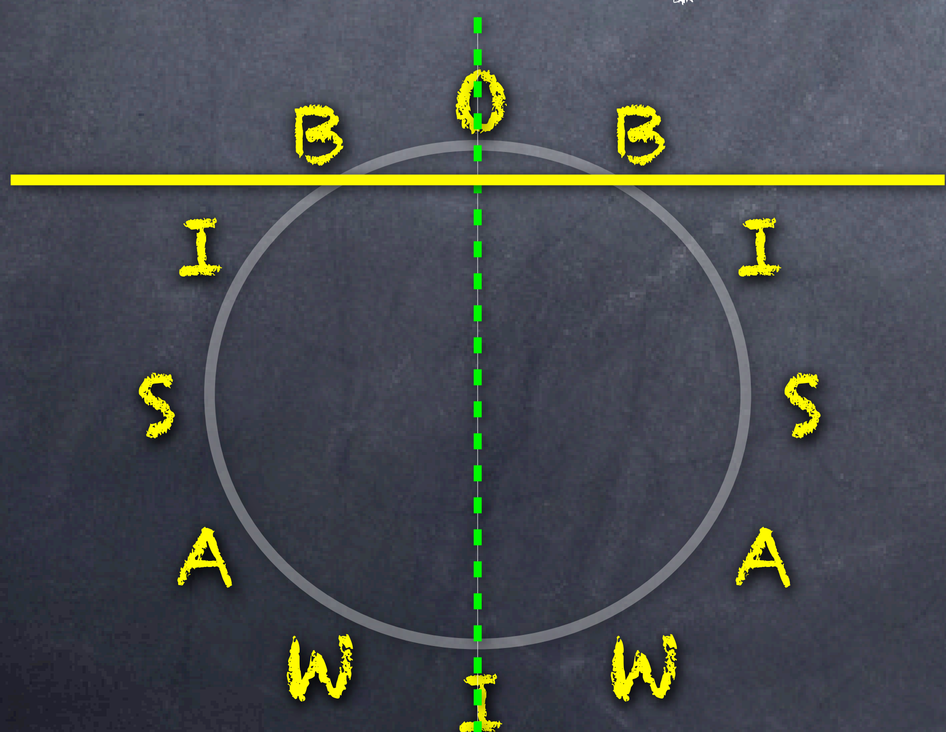
1. The following conditions are equivalent :

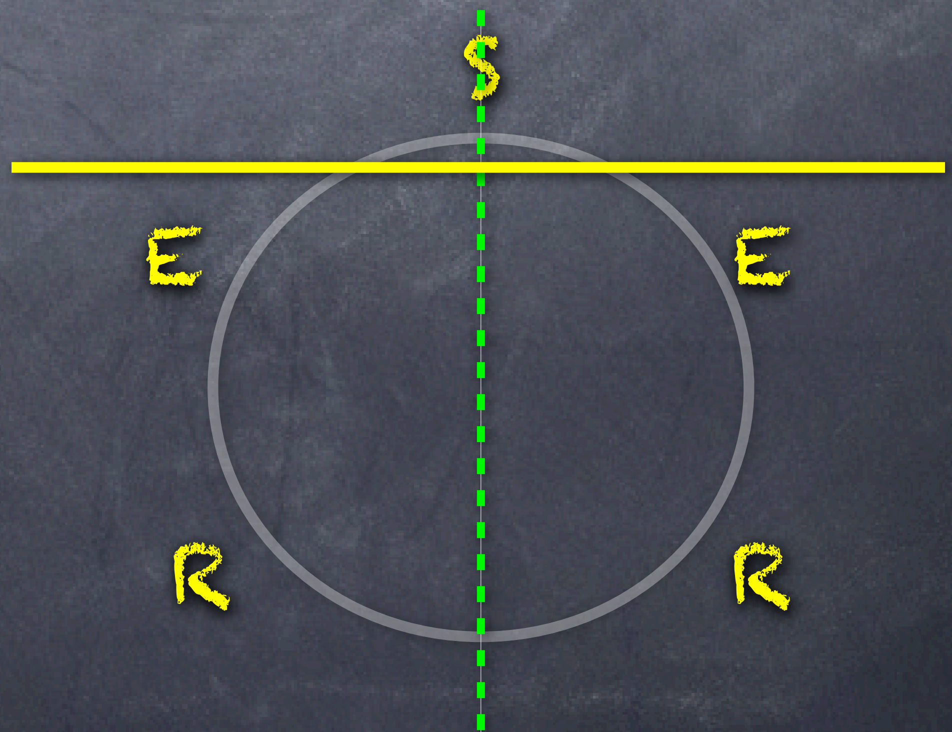
(i) $|Pal(w^{\omega})| = \infty$;

(ii) $w = u.v$, where u, v are palindromes ;

(iii) w is conjugate either to an even palindrome or to a word of the form $a.p$ with $a \in A$ and $p \in Pal(w)$;

(iv) the conjugacy class $[w]$ has an axial symmetry .





Computation of the lacunas

2. $D(w^\omega) = D(w.x)$ where $|x| = |\,(|u| - |v|)/3\,|$

3. $D(w^\omega) = D(w')$ for some $w' \in [w]$

The bound given in 2) is attained. Immediate consequences are

$D(w^\omega) = 0 \iff D(w.x) = 0$ where $|x| = |\,(|u| - |v|)/3\,|$

$\iff D(w^2) = 0$

$\iff D(w^k) = 0$ where $k \geq 1.333333...$

Determining the lacunas of a periodic word is easy.

Def: Words that are product of two palindromes are called **symmetric**.

Exercise: give an algorithm to determine whether a word is symmetric or not.

Here is one showing that BANDERIER is not symmetric

≠

...... the infinite case

- Thue-Morse M is not genial.
- The lacunas of M are not recognizable.
- $(\text{BANDERIER})^{\omega}$ is not genial but the lacunas are recognizable.
- SERRE is genial and so is $(\text{SERRE})^{\omega}$.
- BASSINO is genial but $(\text{BASSINO})^{\omega}$ is not. This is the case for many others, including BRLEK .....
- Fibonacci word and all Sturmian ones are genial.

The remarkable identity satisfied by BANDERIER extends to some infinite words.

Thm: Let w be an infinite word with language closed under reversal, then

$$2D(w) = \sum_{n=0}^{\infty} C_w(n+1) - C_w(n) + 2 - P_w(n) - P_w(n+1).$$

Examples: Thue-Morse, Sturmian, all periodic words, Oldenburger (closed under reversal?)

Conjecture: Let W be a fixpoint of a primitive morphism. If $D(W)$ is positive and finite, then W is periodic.

- Disproved by the following example

  a-> aabcacba ; b-> aa ; c -> a

  W = aabcacba.aabcacba.aa.a.aabcacba. ......

  $D(W) = 1$

- Still holds for two letter alphabets.

Another viewpoint on

B A N D E R I E R

Let σ be the involution defined by

$$\sigma\colon B \longleftrightarrow D\ ;\ E \longleftrightarrow R\ ;\ I \longleftrightarrow I\ ;\ A \longleftrightarrow N\ .$$

Then, BANDERIER is not a σ-palindrome but is conjugate to a σ-palindrome

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New notation

$\sigma Pal(w)$: set of σ-palindromic factors of w

$L\sigma PS_u(w)$: longest σ-palindromic suffix of w unioccurrent

$D_\sigma(w)$: number of σ-lacunas of w

$\sigma P_w(n)$: number of σ-palindromic factors of length n

Computations with BANDERIER

ω		B	A	N	D	E	R	I	E	R		
$\lvert L_\sigma PS_u \rvert$	0	*	*	2	4	*	2	1	3	5		

n	0	1	2	3	4	5	6	7	8	9	10	
C_ω	1	7	7	7	6	5	4	3	2	1	0	
${}_\sigma P_\omega$	1	1	2	1	1	1	0	0	0	0	0	
T_ω	6	-1	-1	-1	-1	0	1	1	1	1		

Some new important results:

Prop: For any finite word w, $|\sigma Pal(w)| \leq |w| + 1 - t$.

Thm: For any finite word w, the (BR) identity holds

$$2D_\sigma(w) = \sum_{n=0}^{k} C_w(n+1) - C_w(n) + 2 - \sigma P_w(n) - \sigma P_w(n+1).$$

and for infinite periodic words

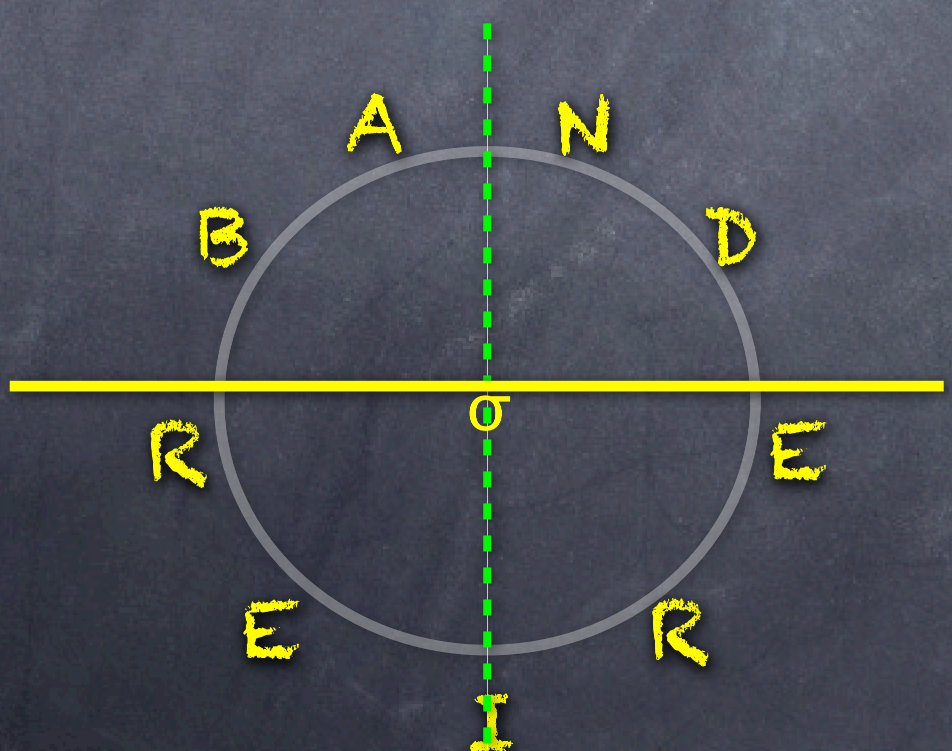
w		B	A	N	D	E	R	I	E	R	B	A	N	D	E	R	I	E
$\lvert LPS_u \rvert$	0	*	*	2	4	*	2	1	3	5	7	9	11	13	15	17	19	21

. . . .

1. $|\sigma Pal(w^{\omega})| = \infty \iff w = u.v$, with u, v σ-palindromes

2. $D_{\sigma}(w^{\omega}) = D_{\sigma}(w^{2}) = D_{\sigma}(w.x)$ where $|x| = |\,(|u| - |v|)/3\,|$

Def: Words that are product of two σ-palindromes are called σ-symmetric.



Thm: [BANDERIER] is σ-symmetric.

Proof:

Wednesday, April 2, 2014

Thm: Let w be an infinite word with language closed under σ-reversal, then

$$2D_\sigma(w) = \sum_{n=0}^{\infty} C_w(n+1) - C_w(n) + 2 - \sigma P_w(n) - \sigma P_w(n+1).$$

Examples: Thue-Morse, Oldenburger (not known if closed under σ-reversal)

Fact: Sturmian words satisfy the (BR) identity but are not closed under σ-reversal.

Def: Let $w \in A^*$. If there exists an involution σ such that $D_\sigma(w^\omega)$ is finite, then w is called **almost genial**.

Example: BANDERIER is almost genial.

Proof: Indeed $D_\sigma(\,(\mathrm{BANDERIER})^\omega) = 3$.

(DENISE and FERNIQUE as well !)

Def: Let $w \in A^*$. If there is no involution σ such that $D_\sigma(w^\omega)$ is finite, then w is called **inherently not genial**.

Example

has recognizable lacunas

and hence is

not genial

but from another viewpoint is

very close friend

THE END