Combinatorial aspects of renormalization in QFT

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Introduction

- Renormalization $\leftrightarrow$ physics, combinatorics, algebra, number theory,...
- Particles physics described by renormalizable quantum field theory (Standard Model).
- Interpretation: physical constants depend on the observation scale.

Noncommutative space:

- definition of a new class of renormalization group (harmonic term).
- Topical problem of physics: compatibility between quantum physics and general relativity.

$\Rightarrow$ At high energy scale, space-time could be noncommutative.

$\rightarrow$ Existence of renormalizable noncommutative QFT is a crucial question.

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2. Power counting
3. Renormalization
4. Hopf algebra interpretation
5. Noncommutative QFT

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Definition of the theory

- Action with parameters m and λ:

$$S[\phi] = \int d^D x \left(\frac{1}{2}(\partial_\mu \phi)^2 + \frac{m^2}{2}\phi^2 + \lambda \phi^4\right)$$

- Feynman graphs: arbitrary graphs whose vertices are of coordination 4 (internal) or 1 (external).
- 1PI graphs: connected and still connected after cutting any internal line.

Amplitudes of the graphs:

- Each line carries an oriented impulsion $k \in \mathbb{R}^D$.
- Conservation of impulsion for every vertex.
- Remaining internal impulsions are integrated over in the amplitude.
- Contribution of a vertex: λ.
- Contribution of an internal line: $\frac{1}{k^2+m^2}$.

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Physical quantities

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- Particles interpretation: Feynman graphs represent particles of a certain impulsion propagating along the lines and interacting at the vertices.
- Some coefficients of λ are divergent. Example: the tadpole.

$$\int \mathrm{d}^D k \frac{1}{k^2 + m^2}$$

is quadratically divergent for $D = 4$ in the UV sector ($|k| \to \infty$).

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Superficial degree of divergence

Let G be a 1PI Feynman graph with V vertices, L loops and N external legs.

- Amplitude:

$$A_G(p_1, .., p_N) = \delta(p_1 + .. + p_N) \int \prod_{i=1}^{L} \mathrm{d}k_i \; I_G(p_2, .., p_N, k_1, .., k_L)$$

- Euler characteristic $\Rightarrow L = V + 1 - \frac{N}{2}$.
- Scale transformation: $p_i \mapsto \rho p_i$ and $k_i \mapsto \rho k_i$

$$A_G^{(\rho)} \propto \rho^{\omega(G)}.$$

- Superficial degree of divergence of the theory:

$$\omega(G) = D + (D-4)V + (2-D)\frac{N}{2}.$$

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Renormalizability

A graph G is said primitively divergent if $\omega(G) \geq 0$.

Theorem

The amplitude of a graph G is absolutely convergent if and only if G and each of its 1PI subgraphs are not primitively divergent.

$$\omega(G) = D + (D-4)V + (2-D)\frac{N}{2}.$$

- $D > 4$: $\forall N$, $\exists V$, $\omega(G) \geq 0$: non-renormalizable.
- $D < 4$: finite number of (N, V) such that $\omega(G) \geq 0$: super-renormalizable.
- $D = 4$: $N = 2, 4 \Leftrightarrow \omega(G) \geq 0$: renormalizable.

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Subtraction scheme

- Dimensional regularization: analytic continuation $D \in \mathbb{C}$. Singularity of the amplitudes for $D = 4$.
- Subtraction operator: Taylor

$$\tau A_G(p_1, .., p_N) = \delta(p_1 + .. + p_N) \sum_{j=0}^{\omega(G)} \frac{1}{j!} \frac{\mathrm{d}^j}{\mathrm{d}t^j} A_G(tp_2, .., tp_N)|_{t=0}$$

- G: prim. div. graph without prim. div. subgraph

$$A_G^R = (1 - \tau) A_G: \text{ renormalized amplitude}$$

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General formula

- Contracted graph: let g be a subgraph of G. G/g: graph G where g is contracted to a point.
- G: graph with only one prim. div. subgraph g

$$A_G^R = A_G - \tau A_G - (\tau A_g)(A_{G/g} - \tau A_{G/g})$$

- General case:

$$A_G^R = A_G - \tau A_G - \sum_{g \subset G} (\tau A_g) A_{G/g}^R$$

where g is sumed over the 1PI prim. div. subgraphs of G.

→ Recursive method.

- Solution of the recursive equations: forest formula (Zimmermann).

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BPHZ renormalization

Properties:

- The renormalized amplitudes are convergent for $D \to 4$.
- Locality: All the divergent counterterms $c_G = A_G - A_G^R$ are of the form of the action, so that they can be included in the constants: $\lambda \mapsto \lambda_R$, $m \mapsto m_R \ldots$
- The correlation function $\Gamma_N(p_1, \ldots, p_N)$ is the sum of the renormalized amplitudes of all 1PI Feynman graphs with N external legs carying the impulsions p_i for the renormalized constants.

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Hopf algebra of graphs

- Complex vector space associated to 1PI Feynman graphs.
  Empty graph$=\mathbb{1}$ (unit).
- Product μ: (disconnected) juxtaposition of graphs.

$\rightarrow$ $\mathcal{H}$: generated algebra. Graded by number of loops.

- Counit is trivial: $\varepsilon : \mathcal{H} \to \mathbb{C}$, $\varepsilon(\mathbb{1}) = 1$.
- Coproduct: $\Delta : \mathcal{H} \to \mathcal{H} \otimes \mathcal{H}$

$$\Delta G = G \otimes \mathbb{1} + \mathbb{1} \otimes G + \sum_{g \subset G} g \otimes G/g$$

  where the sum is over the 1PI prim. div. subgraphs g of G.
- Antipode: $S(G) = -G - \sum_g S(g)(G/g)$, $S(\mathbb{1}) = \mathbb{1}$.

Theorem (Connes Kreimer)

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- Product μ: (disconnected) juxtaposition of graphs.

→ $\mathcal{H}$: generated algebra. Graded by number of loops.

- Counit is trivial: $\varepsilon : \mathcal{H} \to \mathbb{C}$, $\varepsilon(\mathbb{1}) = 1$.
- Coproduct: $\Delta : \mathcal{H} \to \mathcal{H} \otimes \mathcal{H}$

$$\Delta G = G \otimes \mathbb{1} + \mathbb{1} \otimes G + \sum_{g \subset G} g \otimes G/g$$

where the sum is over the 1PI prim. div. subgraphs g of G.

- Antipode: $S(G) = -G - \sum_g S(g)(G/g)$, $S(\mathbb{1}) = \mathbb{1}$.

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Renormalized amplitudes

- $\mathcal{A}_\varepsilon$: algebra of Laurent series in ε.
- Amplitude $A : \mathcal{H} \to \mathcal{A}_\varepsilon$ is a homomorphism.
- Taylor operator is a projection $\tau : \mathcal{A}_\varepsilon \to \mathcal{A}_\varepsilon$.
- Convolution product: if $f, g \in Hom(\mathcal{H}, \mathcal{A}_\varepsilon)$,

$$f * g := \mu_{\mathcal{A}_\varepsilon} \circ (f \otimes g) \circ \Delta.$$

- Counterterm: twisted antipode

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The Moyal space

- Space of Schwartz functions $f, g \in \mathcal{S}(\mathbb{R}^D, \mathbb{C})$.
- Deformed product:

$$(f \star g)(x) = \frac{1}{\pi^D \theta^D} \int d^D y d^D z \, f(x+y) g(x+z) e^{-2iy\Theta^{-1}z}$$

$$\Theta = \theta \Sigma, \qquad \Sigma = \begin{pmatrix} \begin{matrix} 0 & -1 \\ 1 & 0 \end{matrix} & 0 & \\ 0 & \begin{matrix} 0 & -1 \\ 1 & 0 \end{matrix} & \\ & & \ddots \end{pmatrix}$$

- For $\theta = 0$: $(f \star g)(x) = f(x) \cdot g(x)$.
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UV-IR mixing

- Action ϕ^4 on the Moyal space:

$$S[\phi] = \int d^D x \left(\frac{1}{2}(\partial_\mu \phi)^2 + \frac{m^2}{2}\phi^2 + \lambda\, \phi \star \phi \star \phi \star \phi\right)$$

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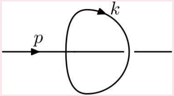
- UV-IR mixing for this theory (Minwalla *et al.* '00).
- Tadpole:



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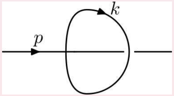

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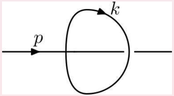

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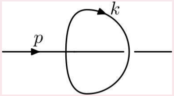

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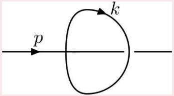

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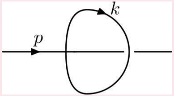

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- Addition of a harmonic term to the action:

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- Power counting ($D = 4$: renormalizable).
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$\Rightarrow$ Renormalizability of the theory to all orders ($D = 2, 4$) (Grosse Wulkenhaar '04).

- New properties of the flow (Disertori Gurau Magnen Rivasseau '06).
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- Ingredients of the renormalization: power counting and locality.
- BPHZ subtraction scheme has a Hopf algebra structure.
- Noncommutative field theory exhibits a new divergence: UV-IR mixing.
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