Rowmotion on fences

Bruce Sagan
Michigan State University
www.math.msu.edu/~sagan
joint with Sergi Elizalde, Matthew Plante, and Tom Roby

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Université Paris Nord

Rowmotion

Fences

Self-dual posets

Comments and open questions

Outline

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$$\frac{\operatorname{st} \mathcal{O}}{\# \mathcal{O}} = c.$$

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Ex. When $n = 4$ and $k = 2$ there are two orbits

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1100	4	1010	3
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The inversion statistic is $k(n-k)/2$-mesic for rotation on $S_{n,k}$.

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Note that homomesy implies homometry, but not conversely.

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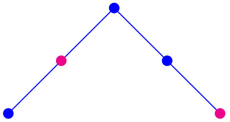
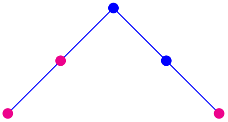
$A =$  $\overset{\Delta}{\mapsto}$ $I =$ 

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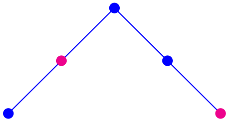
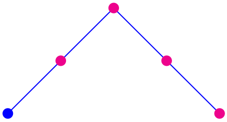
$A =$  $\overset{\nabla}{\mapsto}$ $U =$ 

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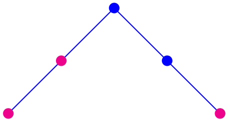
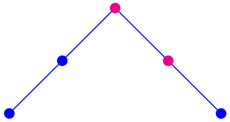
$I =$  $\overset{c}{\mapsto}$ $U =$ 

Rowmotion on antichains of poset P is $\rho : \mathcal{A}(P) \to \mathcal{A}(P)$ where

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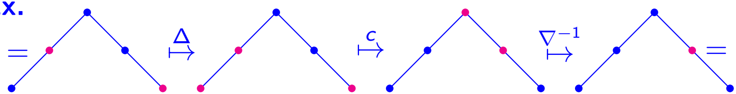
$A =$  $= \rho(A)$

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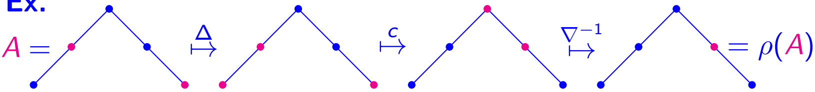

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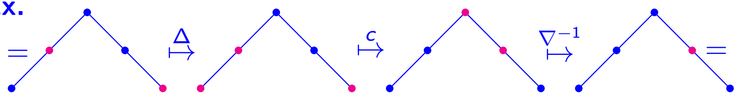
$A =$ 

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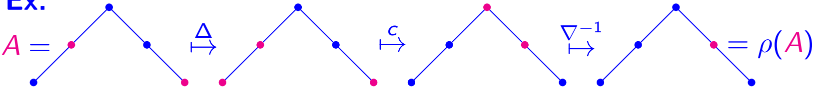

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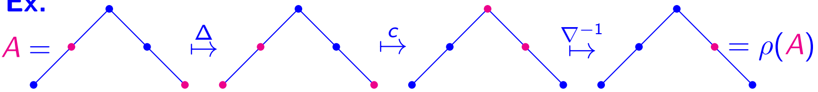

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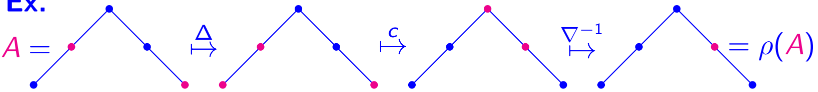

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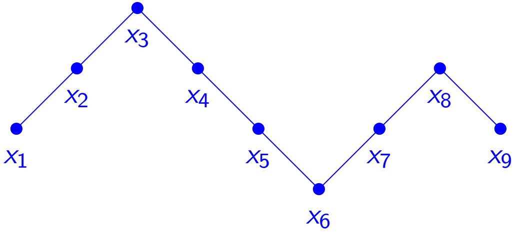
$F =$ 

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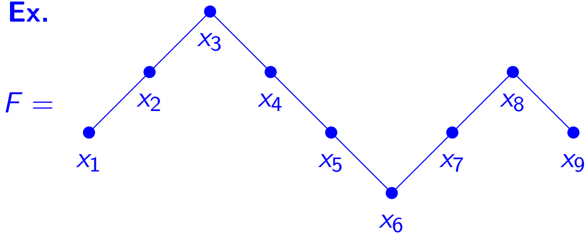

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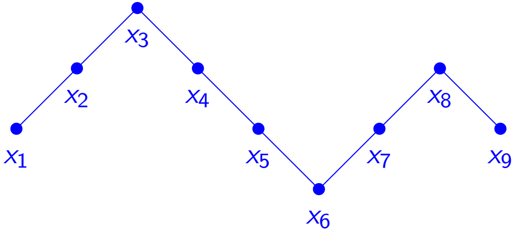
$F =$ 

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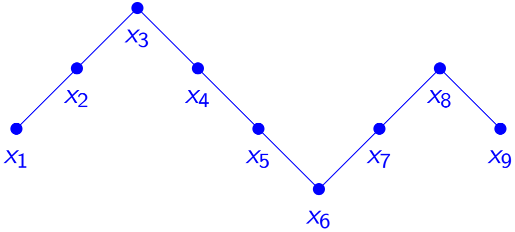
$F =$ 

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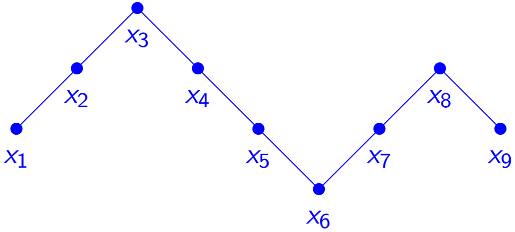
$F =$ 

4 segments

Fences have important connections with cluster algebras, q-analogues, unimodality, and Young diagrams. The maximal chains of F are called *segments*. Elements on two segments are called *shared*.

A *fence* is a poset with elements $F = \{x_1, x_2, \ldots, x_n\}$ and covers

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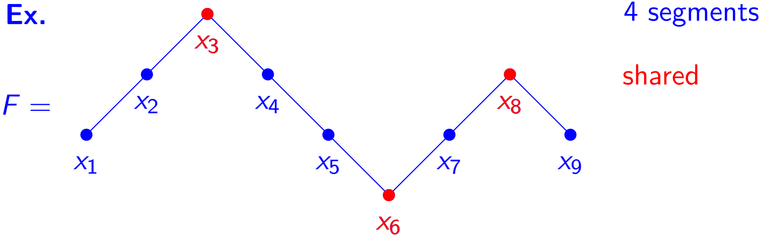

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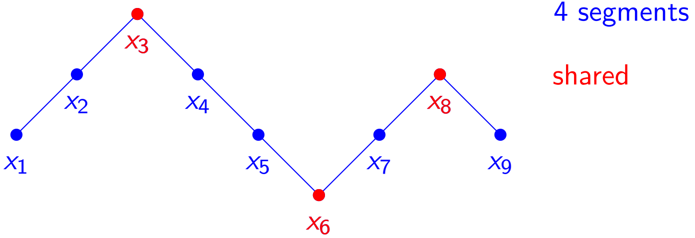

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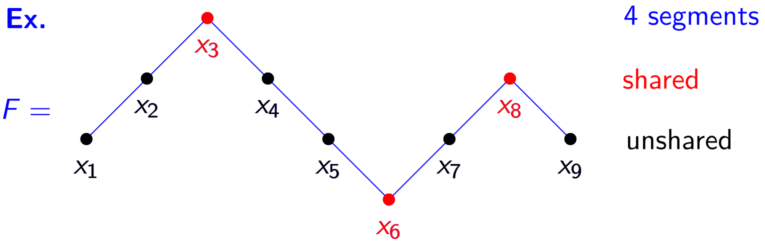

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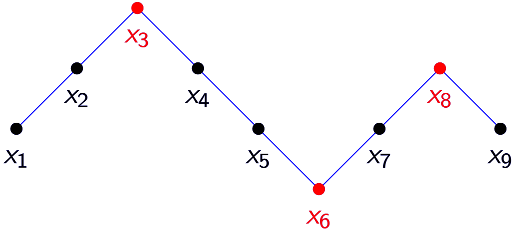

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unshared

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Ex. $\check{F}(3,3,2,2)$

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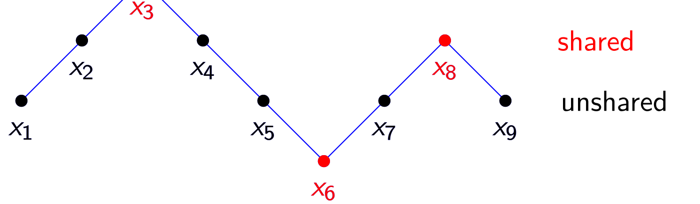

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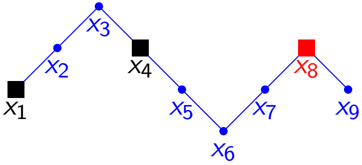
As an example of rowmotion on antichains in a fence, consider F below and $A = \{x_1, x_4, x_8\}$ indicated by squares.



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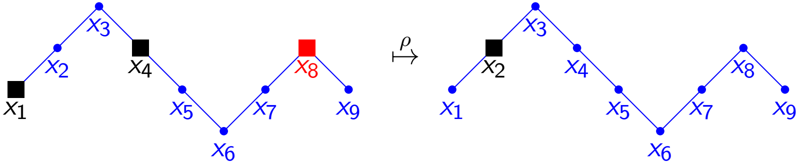

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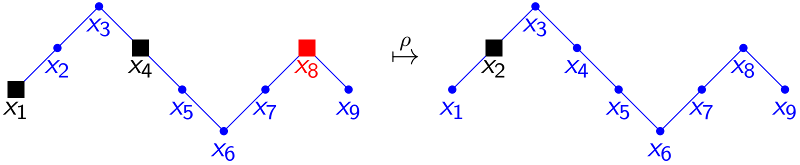

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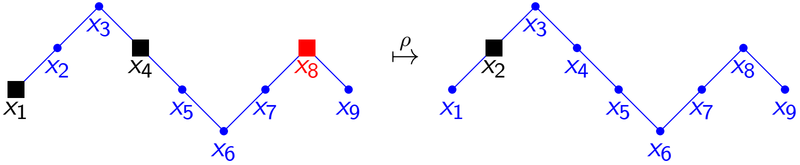

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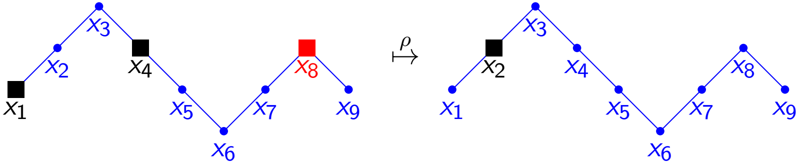

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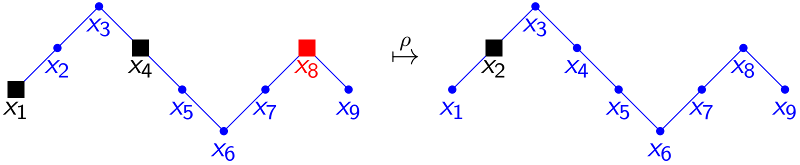

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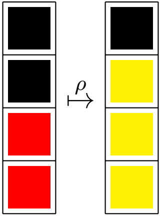

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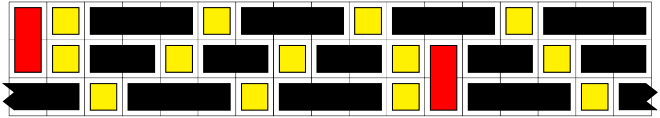

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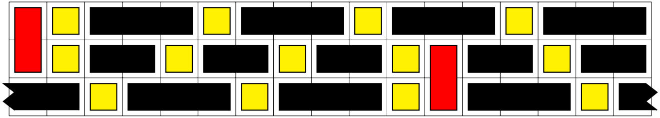

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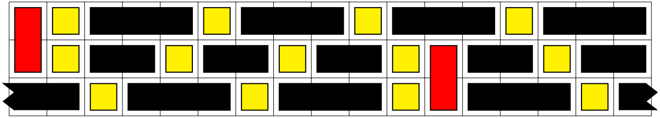

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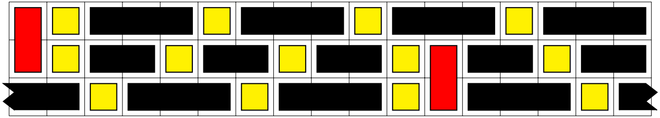

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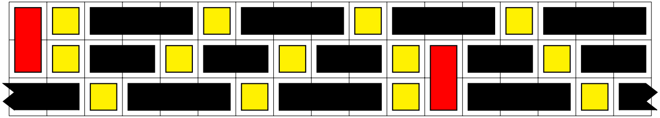

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(b) There is a red tile in a column covering rows i and $i + 1$ if and only if either the next column contains two yellow tiles in those two rows when i is odd, or the previous column contains two yellow tiles in those two rows when i is even.

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Outline

Rowmotion

Fences

Self-dual posets

Comments and open questions

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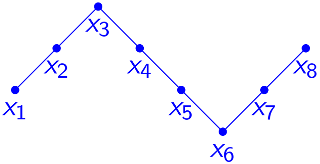

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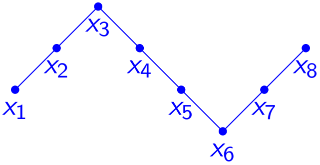

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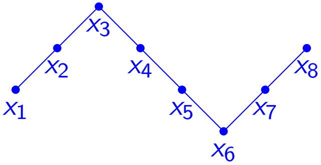

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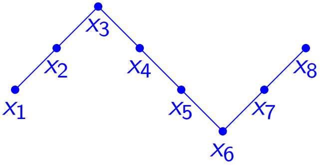

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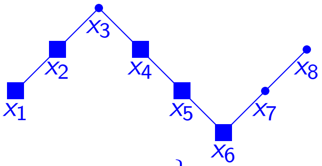

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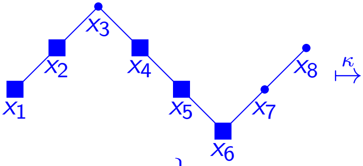

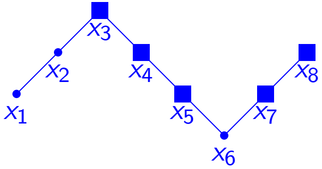

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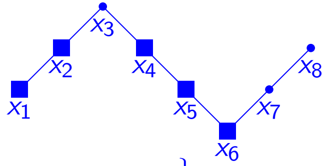

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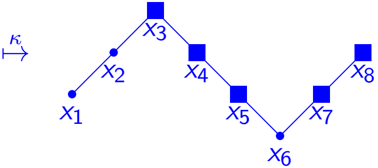

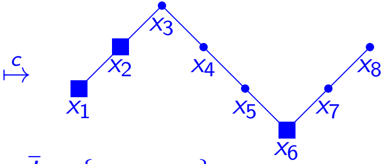

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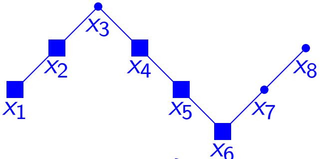

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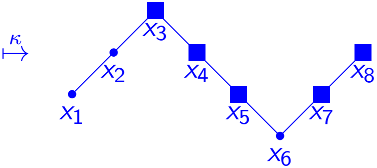

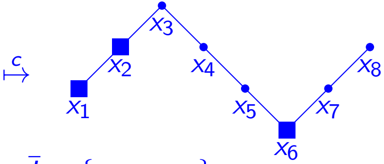

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Theorem (EPRS)

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Corollary (EPRS)

If P is self-dual with $n = \#P$ then $\hat{\chi}$ is $(n/2)$-mesic on superorbits.

Outline

Rowmotion

Fences

Self-dual posets

Comments and open questions

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For $\hat{\chi}$ one can not use our results on self-dual posets since I and $\bar{I}$ are not always in the same orbit.

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Question

Let $F = \check{F}(\alpha)$ with α palindromic. Find necessary and/or sufficient conditions on α for the black or the red tile sequences to be palindromic for all rowmotion orbits.

MERCI POUR

VOTRE (HOMOMÉSIQUE?)

ATTENTION!