Algebra for Automata II

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(Joint work with Nicoletta Sabadini)
This was a continuation of the lecture by N. Sabadini on the algebra of spans and cospans of graphs [KSW97], [KSW00].

I began by giving a mathematical motivation for the introduction of spans and cospans of systems. The most fundamental operations on (state spaces of) systems are colimits (sequential) and limits (parallel composition). The category cospan(**A**) with an appropriate algebraic structure permits the *compositional* calculation of finite colimits in category **A**, and similarly span(**A**) permits the compositional calculation of finite limits. The appropriate algebraic structure on these categories is that they are wscc (well-supported compact closed) categories [CW87]. In more detail, it was proved in [RSW04] that cospan(FiniteGraphs/G) (restricted to discrete graphs as objects) is the free wscc category on the graph G. Further, given a category **A** with finite colimits, then the colimit construction induces a functor

colimit: cospan(FiniteGraphs/**A**) -> cospan(**A**).

But functors out of a free category are always evaluations, and hence finite colimits may be calculated by evaluation of wscc expressions in cospan(**A**) (limits by evaluation in span(**A**)).

I then described a general distributive law relating parallel and sequential operations; the formulation involves considering a reflexive graph shaped diagram of spans of systems, and its colimit in systems.

The remainder of the lecture consisted of examples of the use of spans and cospans of graphs and the distributive law in the theory of operating systems, including a examples involving semaphores, and raising some problems [W06] with A. Tanenbaum's widely circulated solution to the Sleeping Barber problem. In fact, Tanenbaum [T06] will be removing the example from the new edition of his book on operating systems due to lack of space.

References:
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[RSW04] Rosebrugh R., Sabadini N., Walters R.F.C., Generic commutative separable algebras and cospans of graphs, Proceedings CT04, Vancouver 2004.
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