Exploiting Redundant Computation in Communication-Avoiding Algorithms for Algorithm-Based Fault Tolerance

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Roadmap

Communication-Avoiding Algorithms

Introduced in 2008 by Demmel *et al*

- Idea: minimize the number of communications
- Additional computations
- Communications are expensive, flops are not

$\rightarrow$ Compute more, communicate less

Exist for *los 3 amigos*: LU, QR, Cholesky

Communication-Avoiding QR

Works by **panels** :

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} = Q_1 \begin{pmatrix} R_{11} & R_{12} \\ 0 & A_{22}^1 \end{pmatrix}$$

Then, recursively, work on A_{22}^1...

Communication-Avoiding QR

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CAQR algorithm

1. Panel factorization:
   $\begin{pmatrix} A_{11} \\ A_{21} \end{pmatrix} = Q_1 \begin{pmatrix} R_{11} \\ 0 \end{pmatrix}$
2. Compact representation:
   $Q_1 = I - Y_1 T_1 Y_1^T$
3. Update the trailing matrix:
   $(I - Y_1 T_1 Y_1^T) \begin{pmatrix} A_{12} \\ A_{22} \end{pmatrix} = \begin{pmatrix} A_{12} \\ A_{22} \end{pmatrix} - Y_1 (T_1^T (Y_1^T \begin{pmatrix} A_{12} \\ A_{22} \end{pmatrix})) = \begin{pmatrix} R_{12} \\ A_{22}^1 \end{pmatrix}$
4. Continue recursively on the trailing matrix A_{22}^1

Tall-and-Skinny QR

Panel factorization: key piece of the CAQR algorithm

$$\begin{pmatrix} A_{11} \\ A_{21} \end{pmatrix} = Q_1 \begin{pmatrix} R_{11} \\ 0 \end{pmatrix}$$

The matrix $\begin{pmatrix} A_{11} \\ A_{21} \end{pmatrix}$ is **tall and skinny** :

- number of lines $\gg$ number of columns

Specific algorithm to compute the QR factorization of a tall and skinny matrix: **TSQR**

TSQR algorithm

Goal: compute the QR factorization of a matrix A:

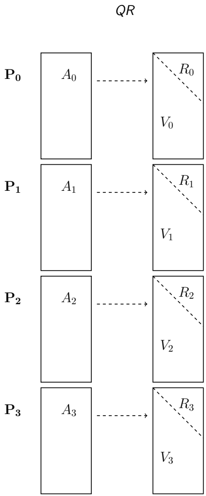
- $A = QR$
- A is tall and skinny

To compute it in parallel on P processes:

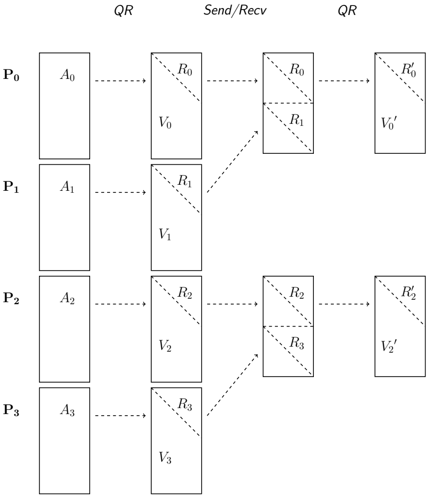
- M = number of lines, N = number of columns
- $M \geq NP$
  - $\rightarrow$ at least square matrices on each process

$$\begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{pmatrix} = Q_1 \begin{pmatrix} R_1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

TSQR algorithm



TSQR algorithm



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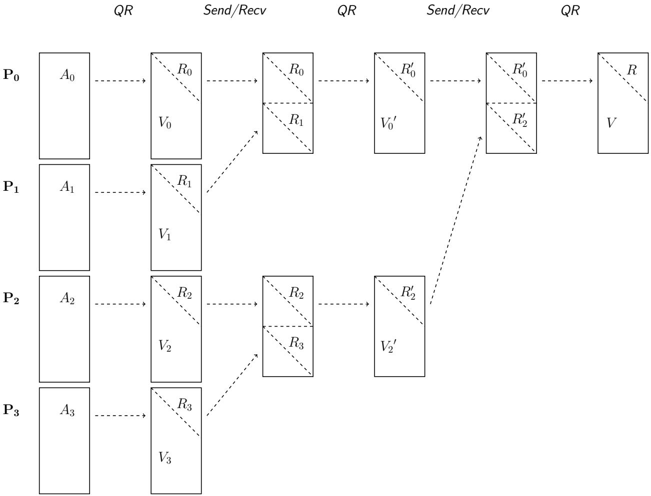


TSQR algorithm

Complexity of the TSQR algorithm:

- Matrix A: M lines, N columns ; P processes
- $\frac{4}{3}\frac{MN^2}{P} + \frac{3}{4}N^3 logP$ flops
- $logP$ communications

Complexity of a traditional QR factorization (ScaLAPACK):

- $\frac{4}{3}\frac{MN^2}{P}$ flops
- $NlogP$ communications

$\rightarrow$ Number of communications: save a factor N

$\rightarrow$ Flops: extra $\frac{3}{4}N^3 logP$ flops

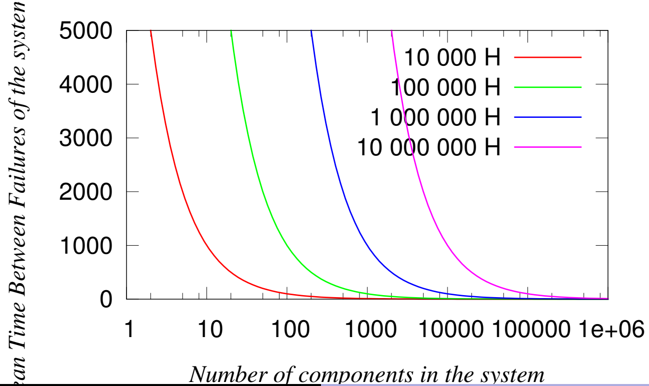
Compute more, communicate less!

Reliability of a distributed system

Mean Time Between Failures

$$MTBF_{total} = (\sum_{i=0}^{n-1} \frac{1}{MTBF_i})^{-1} \quad (1)$$

→ The more components a system is made of, the more likely it is to have a failure.



Approaches for fault tolerance: automatic

Automatic fault tolerance:

- Rollback recovery
- Distributed snapshots with coordinated checkpointing (Chandy-Lamport)
- Non-coordinated checkpointing with message-logging

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Automatic fault tolerance:

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Benefits:

- Completely automatic, transparent
- No modification in the code of the parallel program

Drawbacks:

- Performance overhead: when checkpoints are taken, when messages are logged
- Failure/restart: expensive
  - Coordinated checkpointing: all the processes roll back
  - Non-coordinated checkpointing: only the failed process rolls back, but subsequent synchronizations?

Approaches for fault tolerance: algorithm-based

Behavior upon failures: handled by the application itself

- Failure recovery and sustainability is handled by the parallel program
- Written by the programmer
- Data redundancy, diskless checkpointing
- Iterative checkpointing
- *User-Level Failure Mitigation* (MPI-3 standard)

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Benefits:

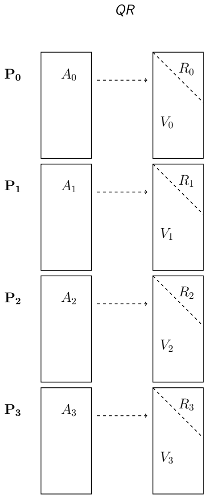
- FT mechanism adapted to the application
- Smaller checkpoints
- Adapted synchronizations

Drawbacks:

- Requires some work from the programmer
- Need for a parallel library and run-time environment that support the ABFT (FT-MPI, MPI-3)

Fault tolerant TSQR

Let's look at TSQR again



Fault tolerant TSQR

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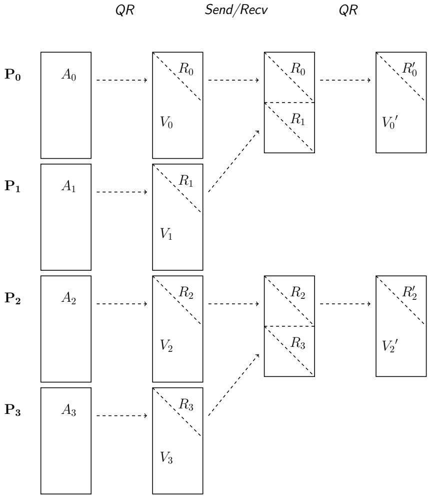

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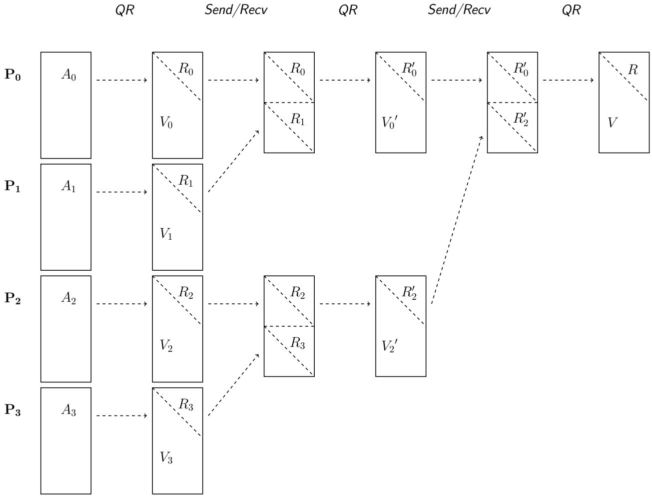

Fault tolerant TSQR

Let's look at TSQR again

- P_0 works beginning $\rightarrow$ end
- P_2 works during the first two steps, then stops
- P_1 and P_3 work during the first step, then stops

Let's put these lazy dudes to work!

What do we expect from fault tolerance?

Have **one result** and the end

- No matter how many processes survive, one of them has the final answer
- Here: *Redundant TSQR*

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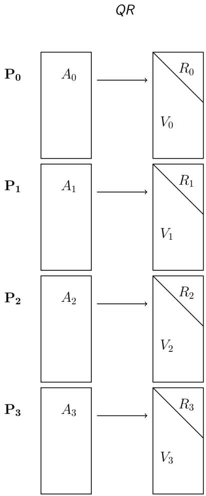
- Here: *Replace TSQR*

Have the result on the expected process and **all the processes are alive**

- Finish with a system that looks as if nothing bad happened
- Here: *Self-Healing TSQR*

Fault Tolerant TSQR: redundant TSQR

Introduce redundancy between processes: exchange between pairs.



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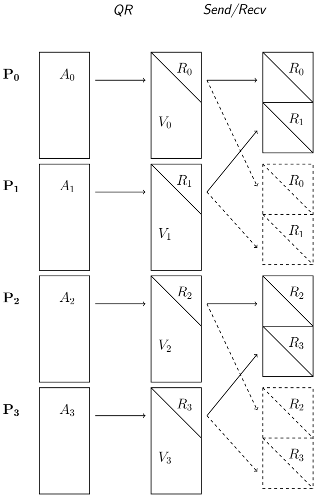

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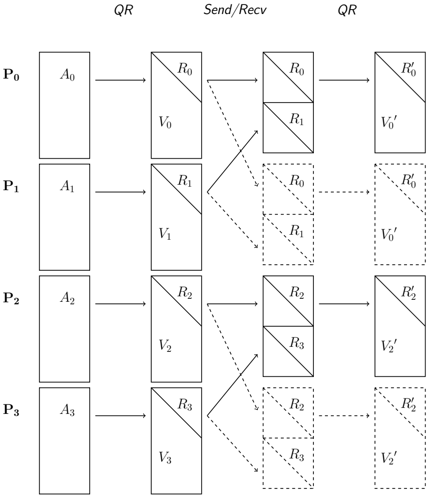

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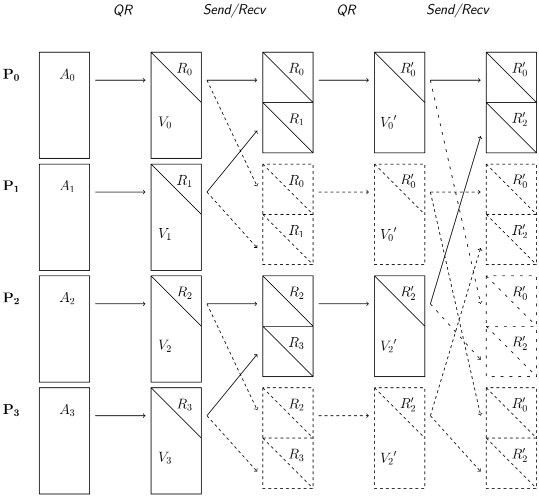

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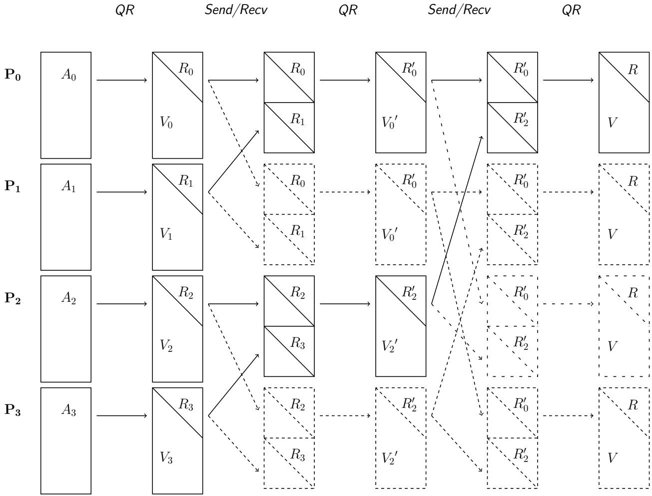
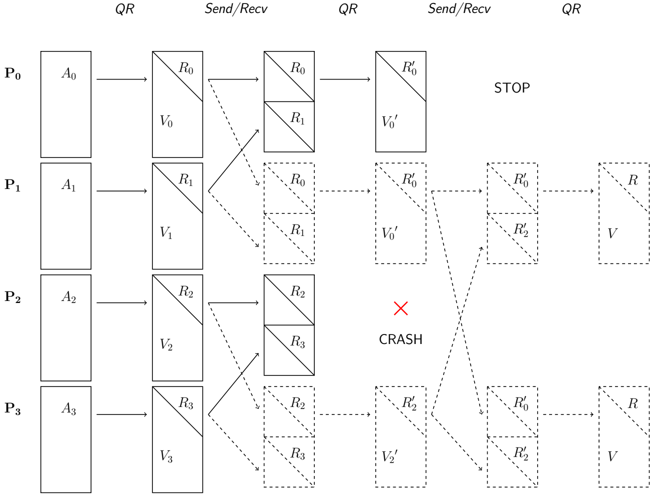


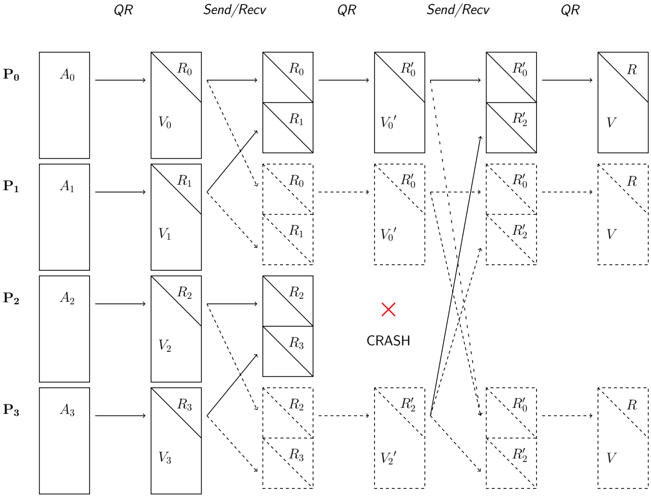
Redundant TSQR: failure

If a process fails: the other ones can continue, except those who need to communicate with the failed process.



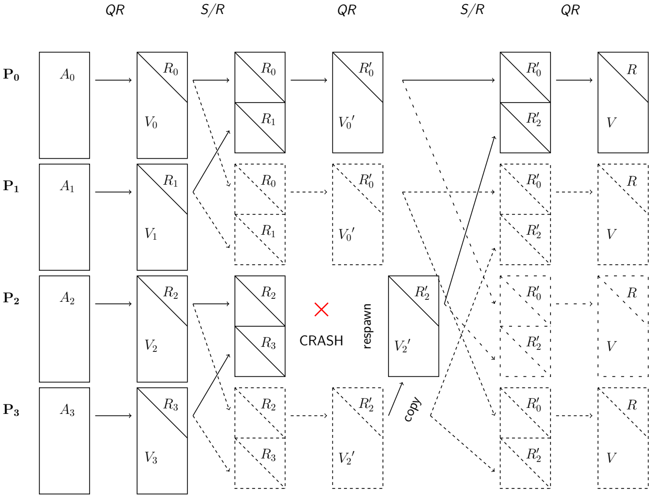
Fault Tolerant TSQR: Replace TSQR

When a process fails, another one takes its place: P_1 acts as P_2.



Fault Tolerant TSQR: Self-healing TSQR

Spawn a new process that recovers the data from a twin process



Performance evaluation

Performance evaluation: what do we measure?

- Overhead during fault-free execution
  - Very important!
  - Cost of the mechanisms put in place to make the FT possible
  - Here: additional communications
  - Same for the three algorithms

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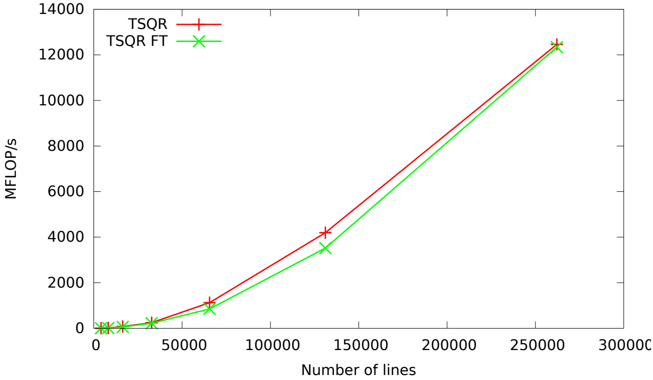
Performance evaluation: what do we measure?

- Overhead during fault-free execution
  - Very important!
  - Cost of the mechanisms put in place to make the FT possible
  - Here: additional communications
  - Same for the three algorithms
- Recovery time
  - Depends on a lot of factors!
  - Failure detection (impossible with asynchronous communications)
  - Recovery made by the RTE (spawn and reconnect a new process)
  - Recovery protocol of the algorithm $\leftarrow$ only interesting thing here, but hard to measure independently

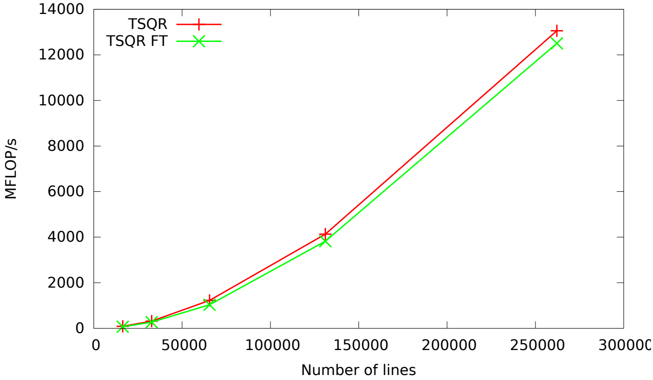
Performance overhead

64 processes, 64 columns ($P = 64$, $N = 64$)



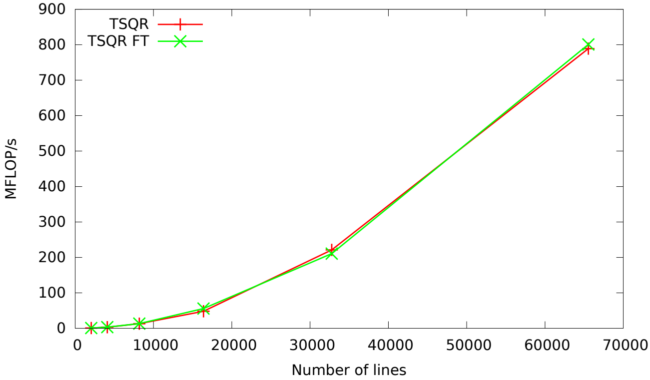
Performance overhead

256 processes, 64 columns ($P = 256,\ N = 64$)



Performance overhead

16 processes, 128 columns ($P = 16,\ N = 128$)



Conclusion

Three protocols for fault-tolerant QR factorization of tall-and-skinny matrices

- Cornerstone for general QR factorization
- Three recovery algorithms, one for each semantics

Scalable FT protocol based on scalable algorithms

Makes use of new features provided by the MPI-3 standard

- FT API now provided by MPI-3
- *User-Level Failure Mitigation*

Next step:

- Apply this to LU, Cholesky (the other *amigos*)
- FT CAQR for general matrices