Exploiting Redundant Computation in Communication-Avoiding Algorithms for Algorithm-Based Fault Tolerance

SIAM PP16

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April 15th, 2016

Roadmap

1. Introduction
   - Large-scale systems
   - Reliability
2. CAQR
3. FT-CAQR
   - FT-TSQR
   - FT-CAQR
4. Performance
5. Conclusion

Scalability

BIG machines

- How do we program them?
- Need for **scalable** algorithms

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What is scalability?

- How does the algorithm evolve when we add cores
- Two dimensions
  - Operations
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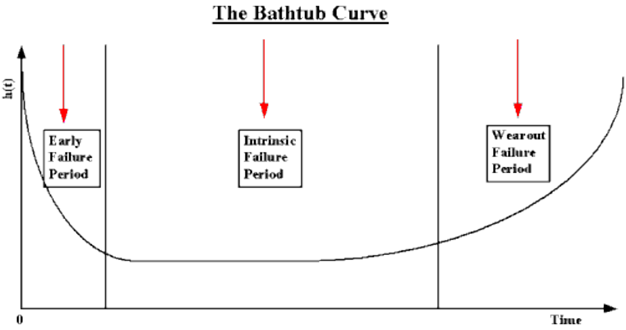
Computations vs communications

- Nodes are **fast**
- Communication speed is limited by physical constraints

Reliability of components

Life expectancy of an electronic component: the famous bathtub curve

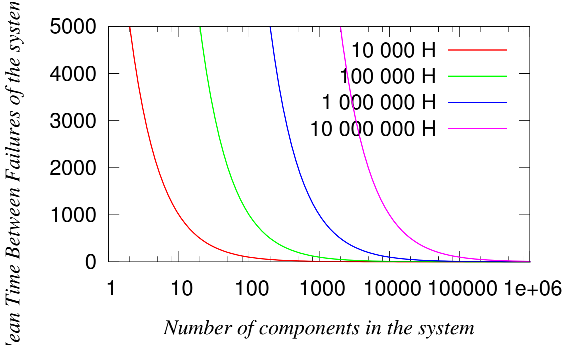


Reliability of a distributed system

Mean Time Between Failures

$$MTBF_{total} = (\sum_{i=0}^{n-1} \frac{1}{MTBF_i})^{-1} \quad (1)$$

→ The more components a system is made of, the more likely it is to have a failure.



Requirements for algorithms

Therefore, algorithms must be:

- **Scalable**
  - Scale with the number of processes
- **Fault tolerant**
  - Able to survive beyond failures

$\rightarrow$ communication-avoiding algorithms

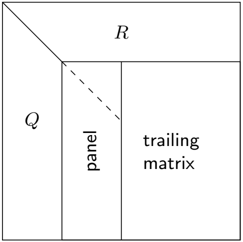
$\rightarrow$ User-Level Failure Mitigation for algorithm-based fault tolerance

Communication-Avoiding QR

Works by **panels** :

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} = Q_1 \begin{pmatrix} R_{11} & R_{12} \\ 0 & A_{22}^1 \end{pmatrix}$$

Then, recursively, work on A_{22}^1...



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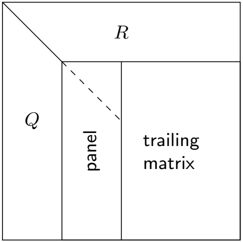

CAQR algorithm

1. Panel factorization:
$$\begin{pmatrix} A_{11} \\ A_{21} \end{pmatrix} = Q_1 \begin{pmatrix} R_{11} \\ 0 \end{pmatrix}$$
2. Compact representation:
$Q_1 = I - Y_1 T_1 Y_1^T$
3. Update the trailing matrix:
$$(I - Y_1 T_1 Y_1^T) \begin{pmatrix} A_{12} \\ A_{22} \end{pmatrix} = \begin{pmatrix} A_{12} \\ A_{22} \end{pmatrix} - Y_1 (T_1^T (Y_1^T \begin{pmatrix} A_{12} \\ A_{22} \end{pmatrix})) = \begin{pmatrix} R_{12} \\ A_{22}^1 \end{pmatrix}$$
4. Continue recursively on the trailing matrix A_{22}^1

Tall-and-Skinny QR

Panel factorization: cornerstone of the CAQR algorithm

$$\begin{pmatrix} A_{11} \\ A_{21} \end{pmatrix} = Q_1 \begin{pmatrix} R_{11} \\ 0 \end{pmatrix}$$

The matrix $\begin{pmatrix} A_{11} \\ A_{21} \end{pmatrix}$ is **tall and skinny** :

- number of lines $\gg$ number of columns

Specific algorithm to compute the QR factorization of a tall and skinny matrix: **TSQR**

TSQR algorithm

Goal: compute the QR factorization of a matrix A:

- $A = QR$
- A is tall and skinny

To compute it in parallel on P processes:

- M = number of lines, N = number of columns
- $M \geq NP$
  - $\rightarrow$ at least square matrices on each process

$$\begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{pmatrix} = Q_1 \begin{pmatrix} R_1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Update of the trailing matrix

Trailing matrix: denoted C, each C_i being on process i.

$$C_i = \begin{pmatrix} C_i' \\ C_i'' \end{pmatrix} = \begin{pmatrix} C_i[: N-1] \\ C_i[N :] \end{pmatrix}$$

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Operation to perform:

$$\begin{pmatrix} R_0 & C_0' \\ R_1 & C_1' \end{pmatrix} = \begin{pmatrix} QR & C_0' \\ & C_0' \end{pmatrix} = Q \begin{pmatrix} R & \hat{C}_0' \\ & \hat{C}_1' \end{pmatrix}$$

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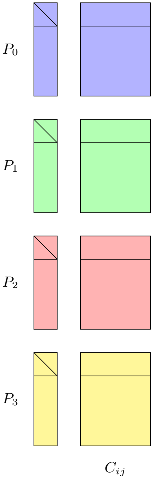
The compact representation becomes:

$$\begin{pmatrix} \hat{C}_0' \\ \hat{C}_1 \end{pmatrix} = \left(I - \begin{pmatrix} I \\ Y_0 \end{pmatrix} T^T \begin{pmatrix} I \\ Y_1 \end{pmatrix}^T \right) \begin{pmatrix} C_0' \\ C_1' \end{pmatrix}$$

Update of the trailing matrix: tree

Step 0



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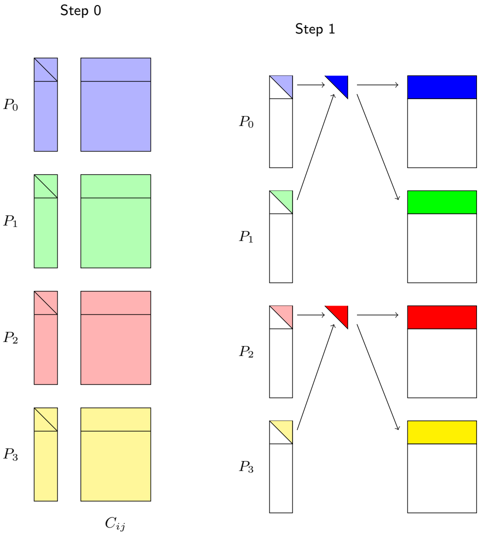

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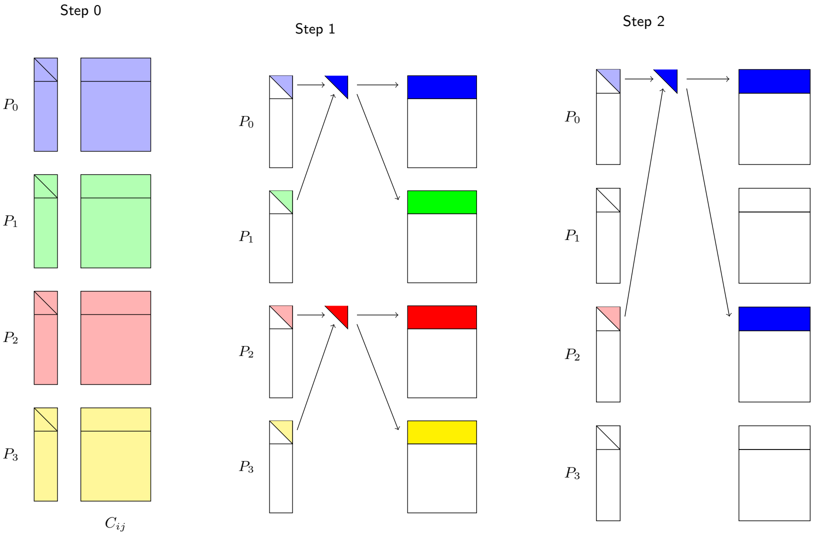
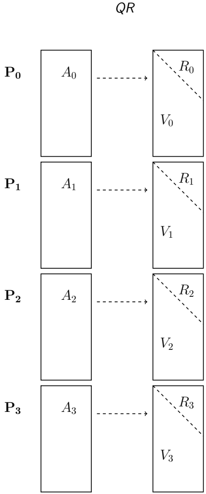


Fault tolerant TSQR

Let's look at TSQR in details



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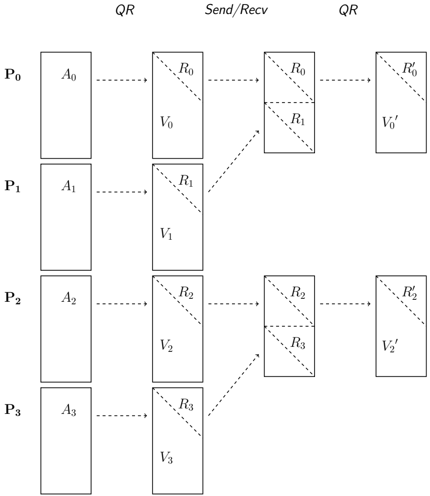

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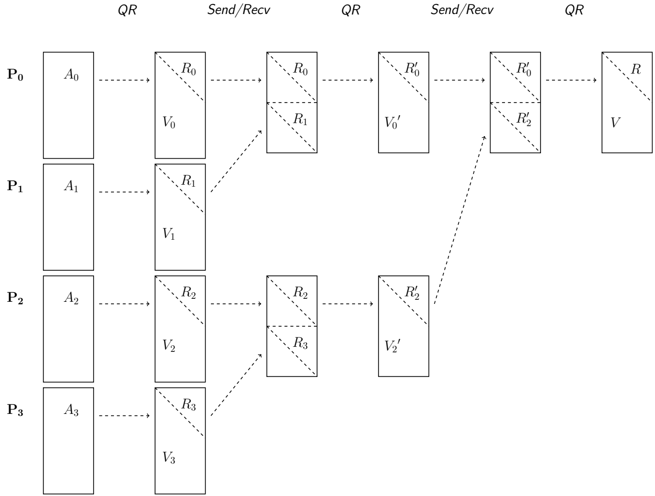

Fault tolerant TSQR

Let's look at TSQR in details

- P_0 works beginning $\rightarrow$ end
- P_2 works during the first two steps, then stops
- P_1 and P_3 work during the first step, then stops

Let's put these lazy dudes to work!

What do we expect from fault tolerance?

Have **one result** at the end

- No matter how many processes survive, one of them has the final answer
- Here: *Redundant TSQR*

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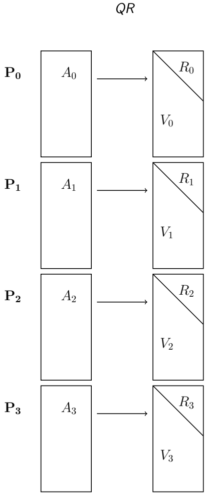
- Here: *Replace TSQR*

Have the result on the expected process and **all the processes are alive**

- Finish with a system that looks as if nothing bad happened
- Here: *Self-Healing TSQR*

Fault Tolerant TSQR: redundant TSQR

Introduce redundancy between processes: exchange between pairs.



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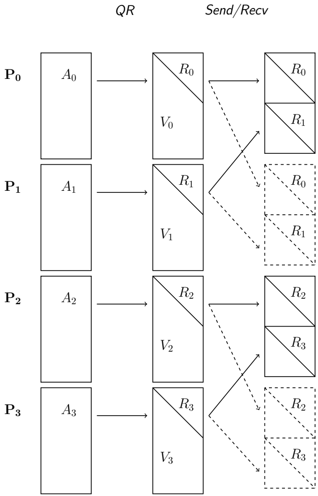

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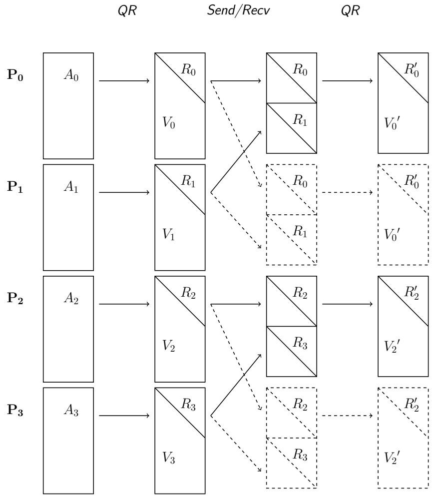

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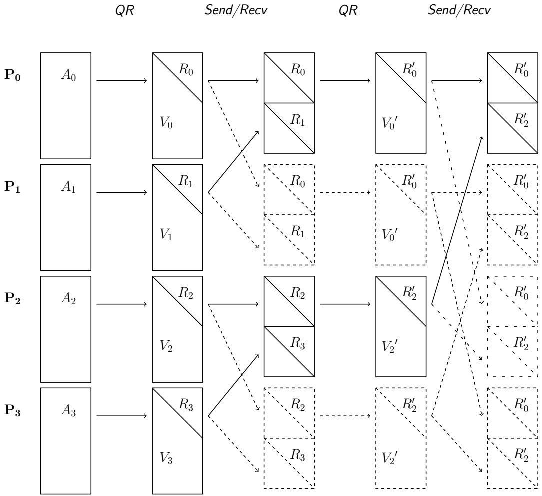

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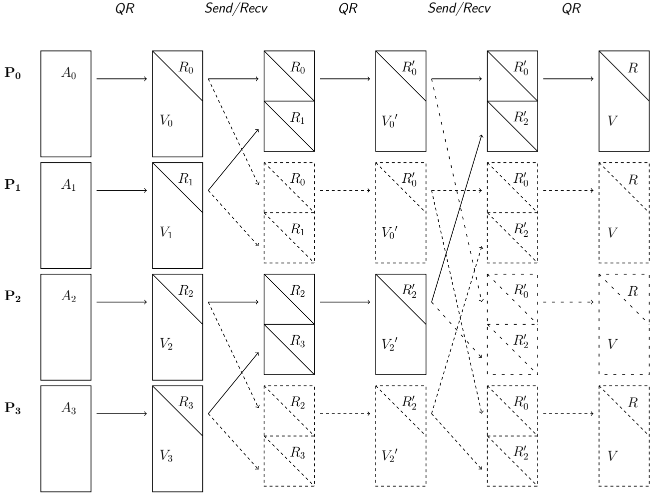
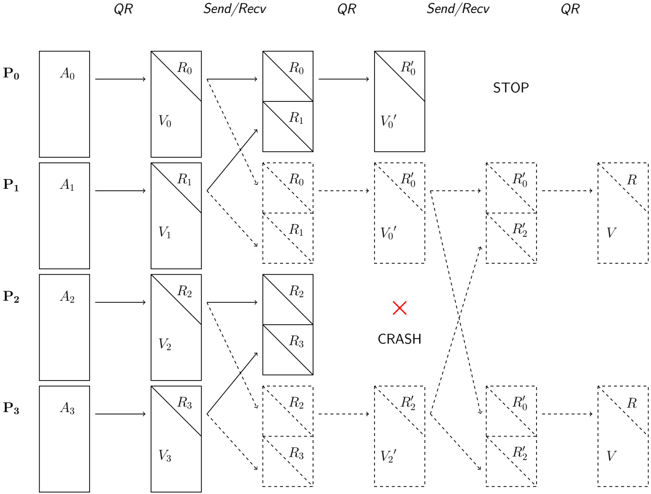


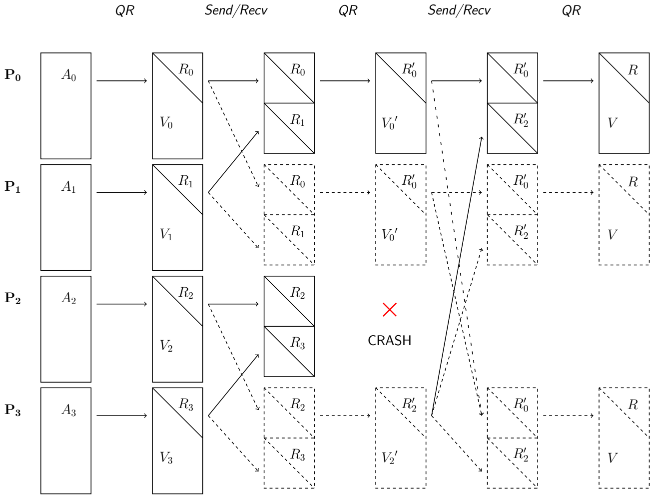
Redundant TSQR: failure

If a process fails: the other ones can continue, except those who need to communicate with the failed process.



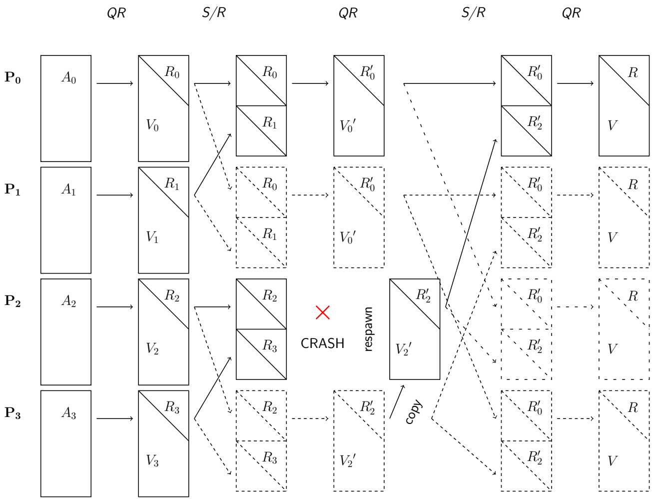
Fault Tolerant TSQR: Replace TSQR

When a process fails, another one takes its place: P_1 acts as P_2.



Fault Tolerant TSQR: Self-healing TSQR

Spawn a new process that recovers the data from a twin process



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1. P_0 sends its C'_0 to P_1 while P_1 computes T

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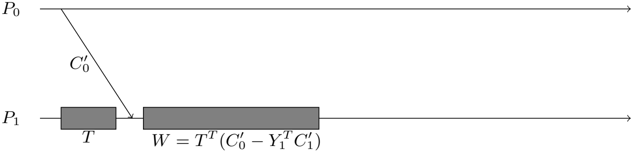

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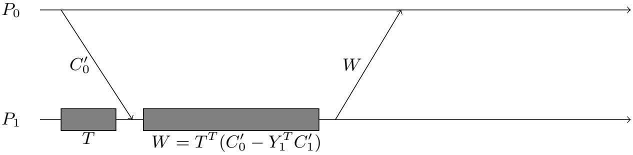

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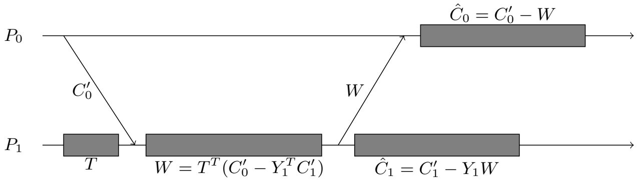

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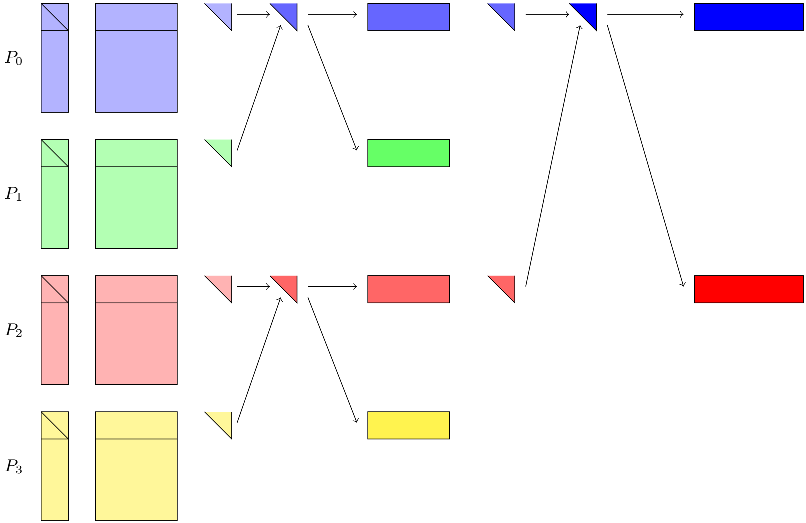
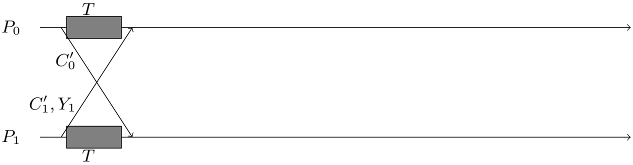
3. P_1 sends W to P_0
4. P_0 computes $\hat{C}'_0$ and P_1 computes $\hat{C}'_1$

Continue... by pairs of processes.

Update of the trailing matrix: tree



Doing the pairwise computation on both processes



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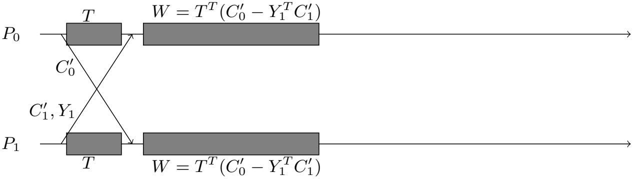

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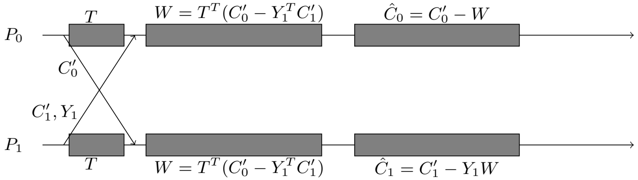

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Continue... by pairs of processes.

Failure recovery

At the end of a given step, between P_i and P_j:

- P_i has W, T, C'_i, C'_j, and $\hat{C}'_i$;
  - if P_j fails, P_i can send sufficient data for any process that has Y_j to recalculate $\hat{C}'_j = C'_j - Y_j W$

Variant: Exchange C'_x and $Y_x \rightarrow$ symmetric

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Performance evaluation

Performance evaluation: what do we measure?

- Overhead during fault-free execution
  - Very important!
  - Cost of the mechanisms put in place to make the FT possible
  - Here: additional communications
  - Same for the three algorithms

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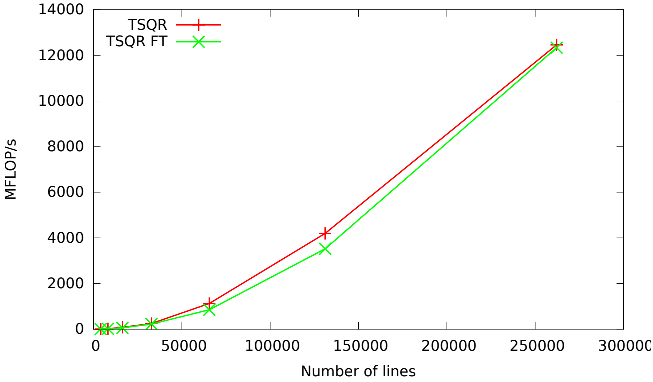
- Overhead during fault-free execution
  - Very important!
  - Cost of the mechanisms put in place to make the FT possible
  - Here: additional communications
  - Same for the three algorithms
- Recovery time
  - Depends on a lot of factors!
  - Failure detection (impossible with asynchronous communications)
  - Recovery made by the RTE (spawn and reconnect a new process)
  - Recovery protocol of the algorithm ← only interesting thing here, but hard to measure independently

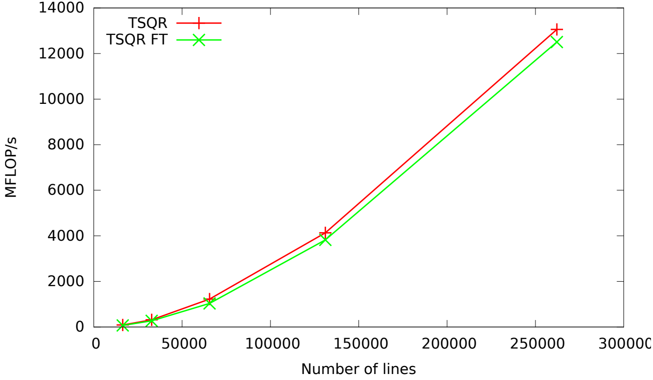
Performance overhead on TSQR

64 processes, 64 columns ($P = 64$, $N = 64$)



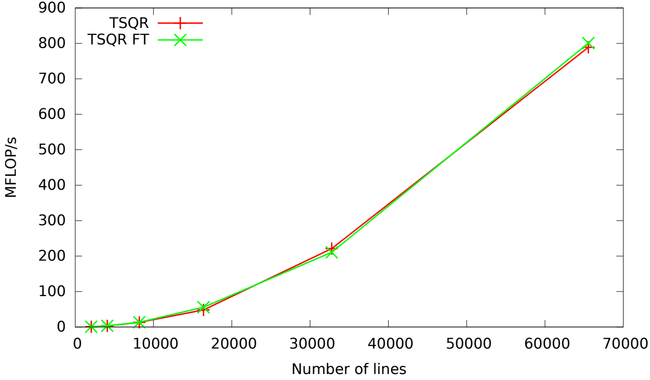
Performance overhead on TSQR

256 processes, 64 columns ($P = 256,\ N = 64$)



Performance overhead on TSQR

16 processes, 128 columns ($P = 16$, $N = 128$)



Conclusion

Three protocols for fault-tolerant QR factorization of tall-and-skinny matrices

- Cornerstone for general QR factorization
- Three recovery algorithms, one for each semantics

Algorithm for FT update of the trailing matrix

- Fault-tolerant QR for general matrices (R)

Scalable FT protocol based on scalable algorithms

Makes use of new features provided by the MPI-3 standard

- FT API now provided by MPI-3
- *User-Level Failure Mitigation*

Next step:

- Apply this to LU, Cholesky (the other *amigos*)
- Reconstruction of the Householder vectors (Q)
- Full performance analysis

References

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