Scalable, Robust, Fault-Tolerant Parallel QR Factorization

Camille Coti

LIPN, CNRS UMR 7030, SPC, Université Paris 13, France

DCABES
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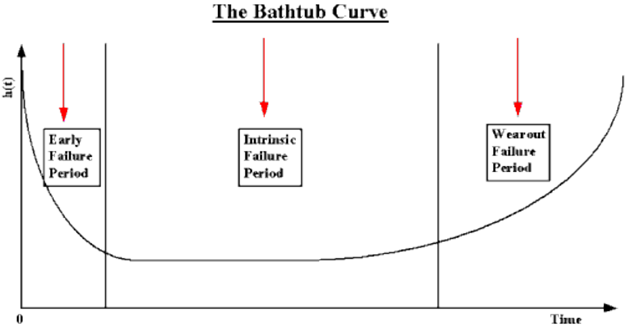
Roadmap

Large-scale systems are volatile

Life expectancy of an electronic component: the famous bathtub curve

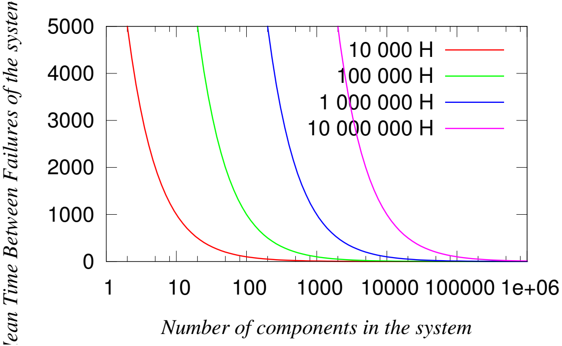


Reliability of a distributed system

Mean Time Between Failures

$$MTBF_{total} = (\sum_{i=0}^{n-1} \frac{1}{MTBF_i})^{-1} \quad (1)$$

→ The **more components** a system is made of, the **more likely** it is to have a failure.



QR Factorization

Matrix A: $A = QR$

- R is upper-triangular
- Q is orthogonal

Unicity:

- Yes, if the diagonal elements of R are positive
- No otherwise

Several algorithms:

- Dense linear algebra: algorithms based on unitary transformations.
- (Modified) Gram-Schmidt, Givens rotation, Householder projection...

QR Factorization

Panel-update approach

- Factorize a panel
- Update the trailing matrix
- Repeat recursively on the trailing matrix

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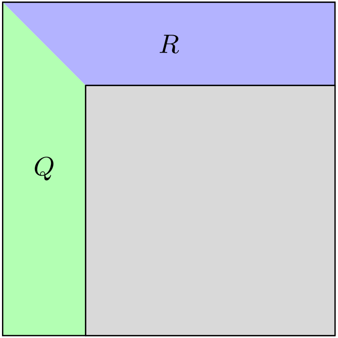
$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}$$
$$= Q_1 \begin{pmatrix} R_{11} & R_{12} \\ 0 & A_{22}^1 \end{pmatrix}$$



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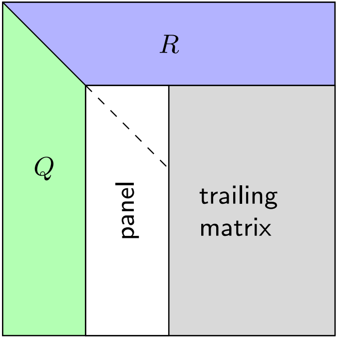

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   - Failures
   - QR Factorization
2. Communication-avoiding QR
   - Panel factorization
   - Trailing matrix update
3. Fault-tolerant QR
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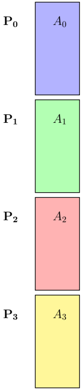
   - Resilience to soft errors
4. Conclusion

Panel factorization

QR factorization of a panel: particular shape

- $M >> N$: the matrix is **tall-and-skinny**
- Specific algorithm: **TSQR**

$$\begin{pmatrix} A_{11} \\ A_{21} \end{pmatrix} = Q_1 \begin{pmatrix} R_{11} \\ 0 \end{pmatrix}$$



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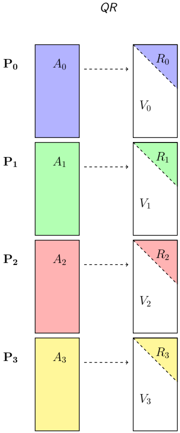

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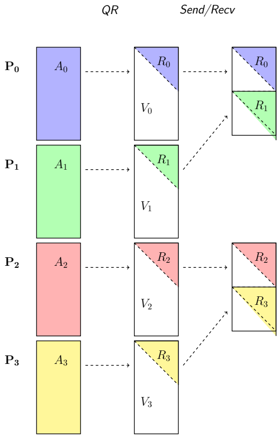

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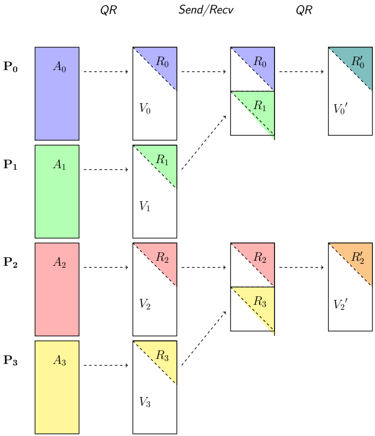

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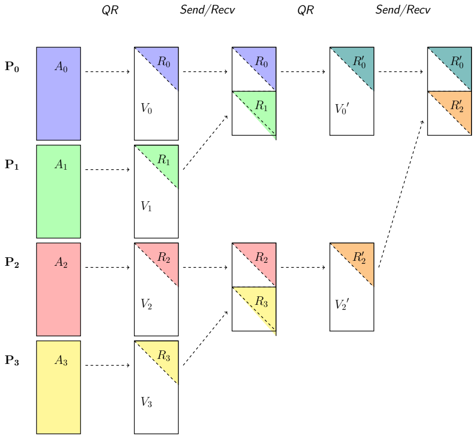

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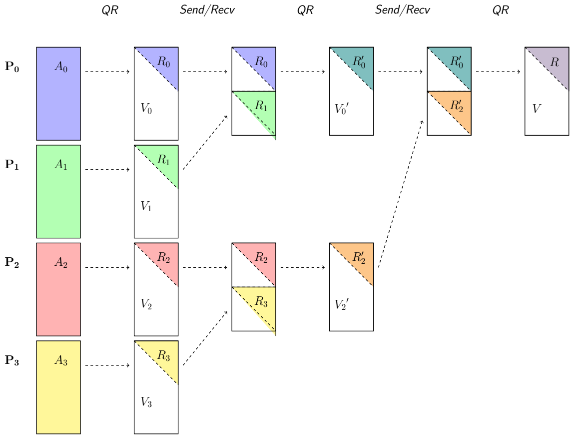


Trailing matrix update

After the panel factorization:

- Compute the **compact representation** of Q_1: $Q_1 = I - Y_1 T_1 Y_1^T$
- Use the Y_1 and T_1 matrices to update the trailing matrix:

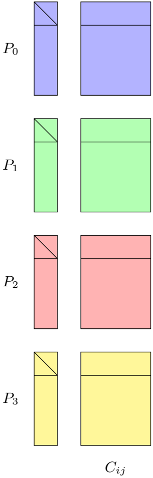
$$(I - Y_1 T_1 Y_1^T) \begin{pmatrix} A_{12} \\ A_{22} \end{pmatrix} = \begin{pmatrix} A_{12} \\ A_{22} \end{pmatrix} - Y_1 (T_1^T (Y_1^T \begin{pmatrix} A_{12} \\ A_{22} \end{pmatrix})) = \begin{pmatrix} R_{12} \\ A_{22}^1 \end{pmatrix}$$

- R_{12} : upper part of the R
- A_{22}^1 : new trailing matrix.

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} = Q_1 \begin{pmatrix} R_{11} & R_{12} \\ 0 & A_{22}^1 \end{pmatrix}$$

Trailing matrix update

Step 0



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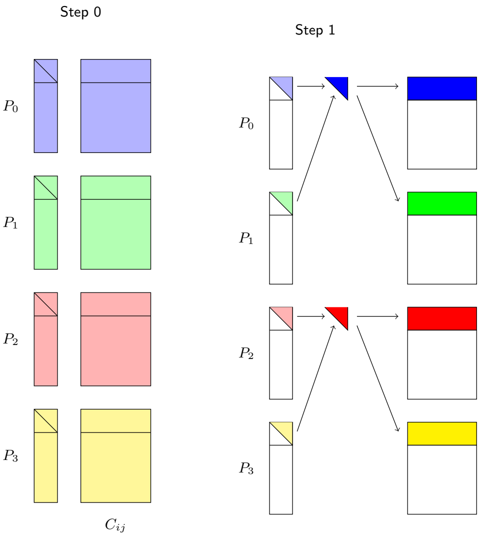

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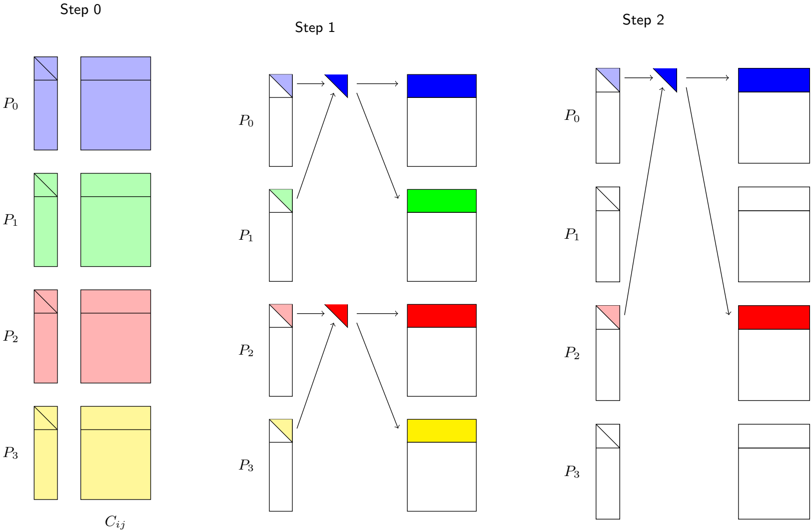

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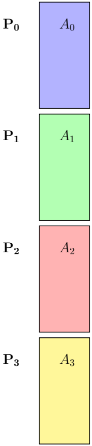

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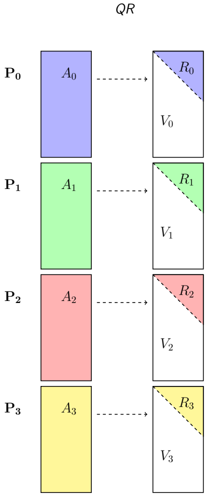

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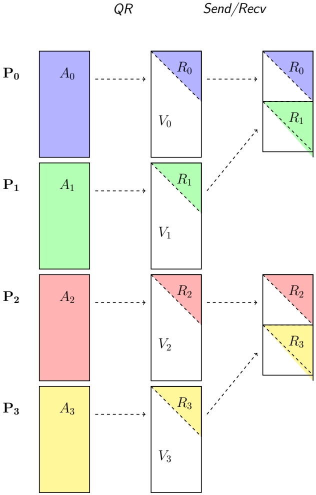

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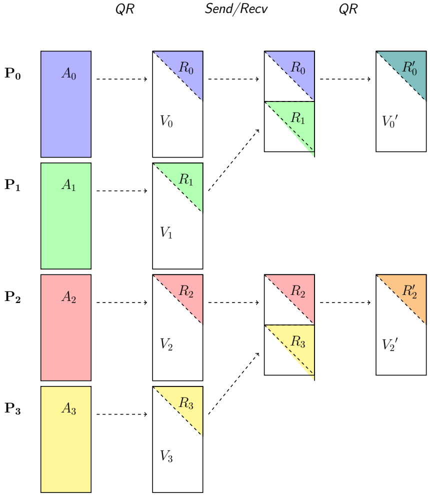

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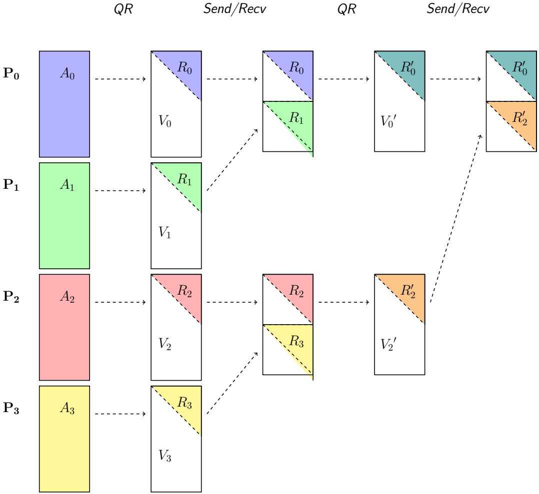

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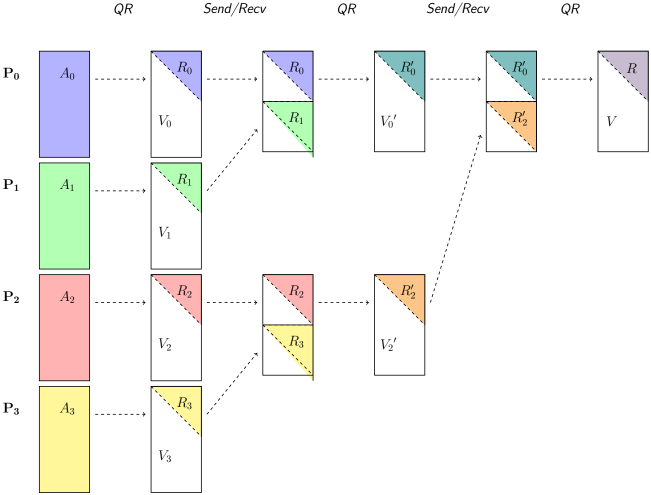
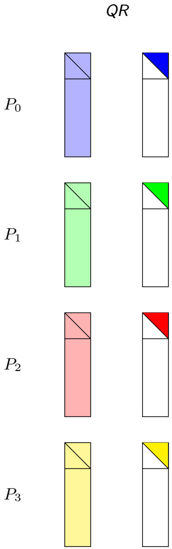


Panel factorization

- P_0 works beginning $\rightarrow$ end
- P_2 works during the first two steps, then stops
- P_1 and P_3 work during the first step, then stops

Let's put these lazy dudes to work!

Fault-tolerant TSQR



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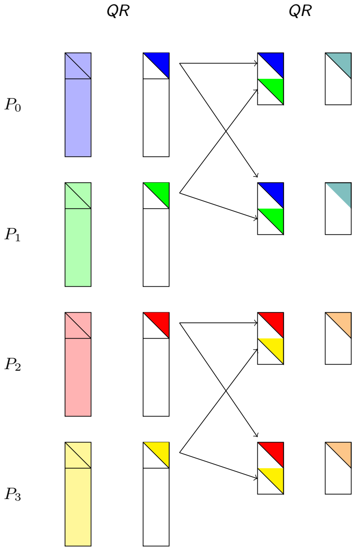

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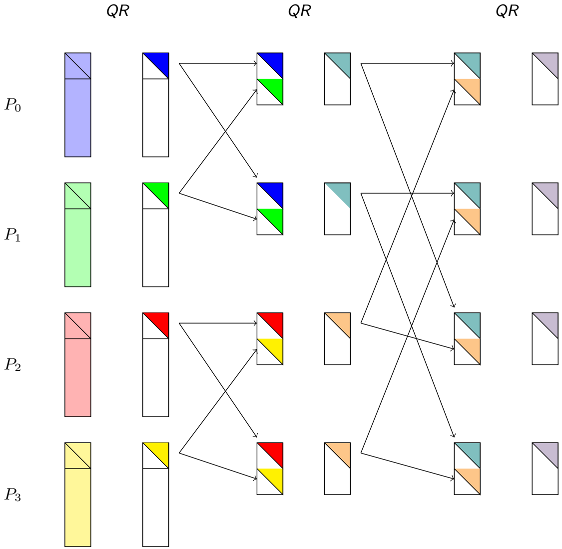


Trailing matrix update

Update of the trailing matrix

- Triggered by the **tree of the panel factorization**
- Uses the intermediate Y and T matrices
- **Pairwise operation** $\rightarrow$ tree

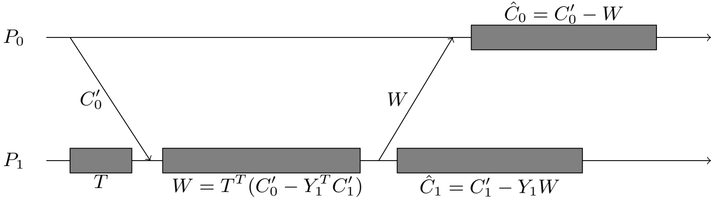
$$A = Q \begin{pmatrix} R & \hat{C}'_0 \\ & \hat{C}'_1 \end{pmatrix}$$

Decompose the blocks of the trailing matrix:

$$C_i = \begin{pmatrix} C'_i \\ C''_i \end{pmatrix} = \begin{pmatrix} C_i[: N-1] \\ C_i[N :] \end{pmatrix}$$

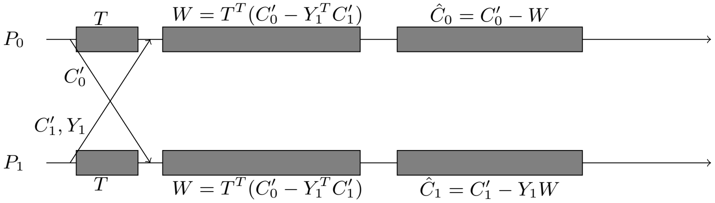
Compute the update :

$$\begin{pmatrix} R_0 & C'_0 \\ R_1 & C'_1 \end{pmatrix} = \begin{pmatrix} QR & C'_0 \\ & C'_1 \end{pmatrix}$$



Fault-tolerant update

Change the order of the operations



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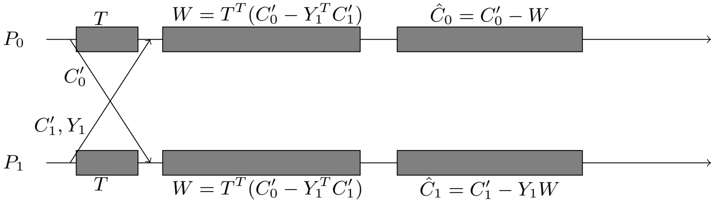
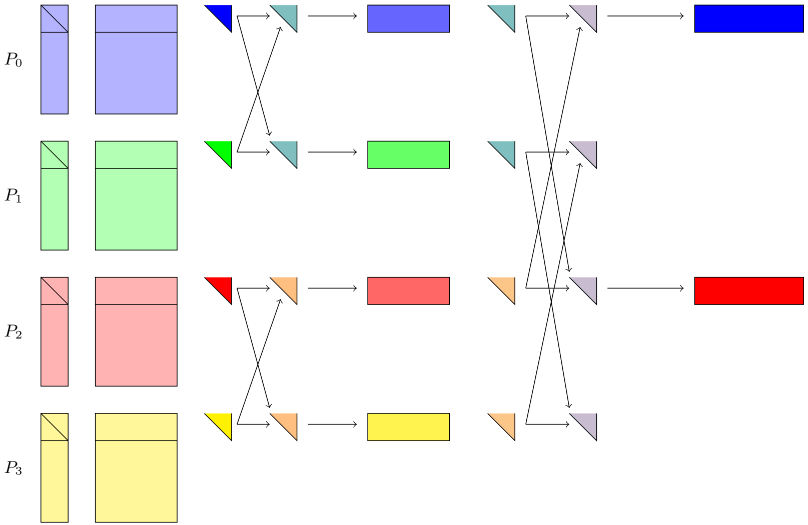


If a process fails

- Both process have the W matrix
- Computation recovered from the W matrix and the initial submatrix

Communication scheme



Soft errors

Soft errors :

- Bit flips causing numerical errors
- Caused by radiations, cosmic rays, various electric disturbances...

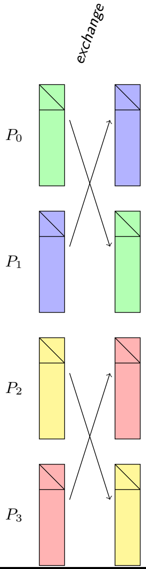
Approaches to tackle soft errors:

- Triple modular redundancy
- ECC memory
- Multiple executions
- ...
- At several levels: hardware, system, application

Idea here:

- **Take advantage of partial redundancy** to detect errors
- Recompute only the erroneous step

Soft errors



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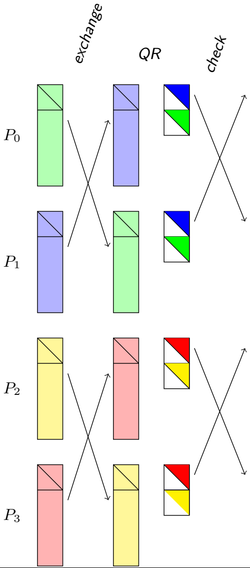

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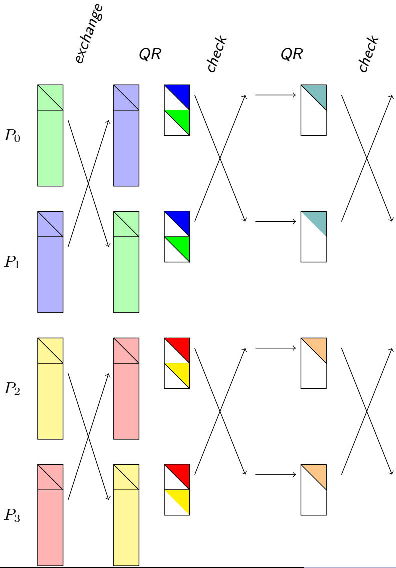

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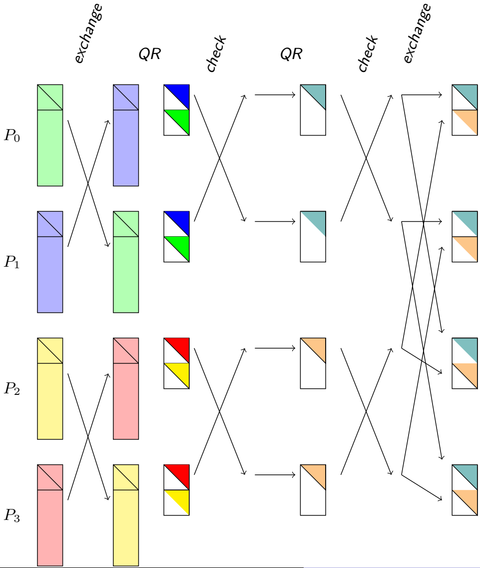

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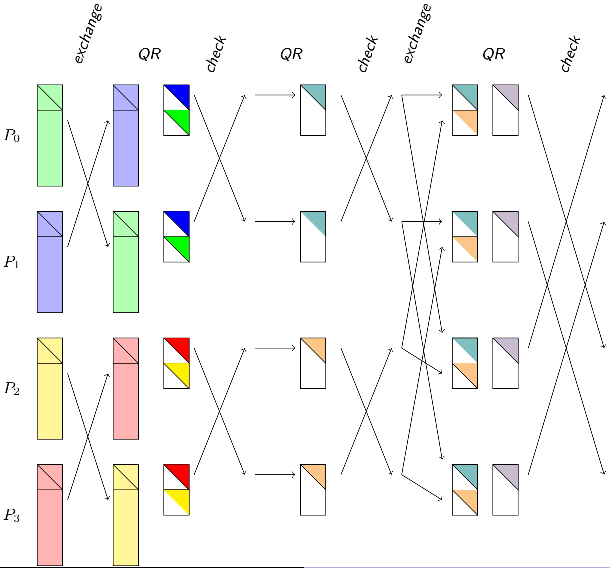

Conclusion

Algorithm-based fault-tolerance

- Failure recovery is handled by the application
- Behavior upon failures: **defined in the algorithm**
- Goal:
  - Minimal overhead during failure-free executions
  - As few recomputations as possible after failures

Fault-tolerant algorithms for QR factorization

- Use **intrinsic redundancy** or idle processes
- **Operation reordering** based on algebraic properties

Small modifications in the critical path

- No additional computations
- Data exchange instead of send/recv
- Partial results already ready for recovery

Appendix: performance