

# On Kashiwara-Vergne Lie algebra and double shuffle Lie algebra in mould theory

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## Abstract

In 2012, Schneps ([?]) showed that there exists an embedding between the double shuffle Lie algebra introduced by Racinet ([?]) and the Kashiwara-Vergne Lie algebra introduced by Alekseev and Torossian ([?]). In the proof of this embedding, a relation called the *senary relation* is used, which is a notion introduced in Ecalle's study ([?]) of multiple zeta values using mould theory. On Lie algebras and embeddings above, bigraded or elliptic versions have also been studied ([?], [?]). In my talk, I will explain the above topics and recent topic ([?]) as much as time permits.

## References

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