Combinatorics and Arithmetic for Physics
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Abstracts.–

Nicolas Behr

Speaker: Nicolas Behr (CNRS, Université Paris Cité, IRIF)
Title: Decomposition Spaces in Combinatorics
Abstract: Motivated by joint work with Joachim Kock (UAB Barcelona & U Copenhagen), I plan to present an introduction to the powerful machinery of decomposition spaces (also known as 2-Segal spaces) from the viewpoint of its applications to combinatorics and its computational properties.
Keywords: Objective combinatorics, categorical foundations, incidence (co)algebras.
Domains: Combinatorics, Category Theory, Logic in Computer Science.

Joseph Bengeloun

Speaker: Joseph Bengeloun (LIPN - Paris XIII)
Title: Computational complexity of column sums of symmetric group character tables and counting of surfaces.
Abstract: The character table of the symmetric group S_n, of permutations of n objects, is of fundamental interest in theoretical physics, combinatorics as well as computational complexity theory. We investigate the implications of an identity, which has a geometrical interpretation in combinatorial topological field theories, relating the column sum of normalised central characters of S_n, to a sum of structure constants of multiplication in the centre of the group algebra of S_n. The identity leads to the proof that a combinatorial computation of the column sum belongs to complexity class ♯ P. The sum of structure constants has an interpretation in terms of the counting of branched covers of the sphere. This allows the identification of a tractable subset of the structure constants related to genus zero covers. We use this subset to prove that the column sum for a conjugacy class labelled by partition λ is non-vanishing if and only if the permutations in the conjugacy class are even. This leads to the result that the determination of the vanishing or otherwise of the column sum is in complexity class P.

Keywords: Symmetric group, characters, computational complexity, permutation topological field theory, matrix/tensor models, permutation factorization.
Domains: Representation Theory (math.RT) Combinatorics (math.CO); Group Theory (math.GR); High Energy Physics - Theory (hep-th).

Lara Bossinger

Speaker: Lara Bossinger (IM UNAM, Oaxaca & IAS, Princeton)
Title: Cluster structures on kinematic spaces.
Abstract: The kinematic spaces modeling massless particle scattering can be parametrized in different ways such as via the spinor helicity formalism or using momentum twistor coordinates. Either case yields an algebraic variety that we call the spinor helicity variety, respectively the momentum twistor variety. In a general set up (not assuming neither planarity nor dual conformal symmetry in the model) both varieties are isomorphic to (quotients of) partial flag varieties and therefore inherit a cluster structure. I will present results indicating that these cluster structures are relevant in the bootstrap for the amplitude and also exhibit embeddings of the relevant partial flag varieties into Grassmannian respecting the cluster structure. Based on joint works with James Drummond, Ross Glew, respectively with Jianrong Li.
Keywords: Cluster algebras, scattering amplitude, flag varieties.
Domains: High Energy Physics - Theory (hep-th), Algebraic Geometry (math.AG).

Marek Bożejko

Speaker: Marek Bożejko (Institute of Mathematics Wroclaw University).
Title: Deformations and q-Convolutions. Old and New Results.
Abstract: This talk is dedicated to the survey of some of our results related to q-deformations of the Fock spaces and related to q-convolutions for probability measures on the real line R. The main idea is done by the combinatorics of moments of the measures and related q-cumulants of different types. The main and interesting q-convolutions are related to classical continuous (discrete) q-Hermite polynomial. Among them are classical (q = 1) convolutions, the case q = 0, gives the free and Boolean relations, and the new class of q-analogue of classical convolutions done by Carnovole, Koornwinder, Biane, Anshelovich, and Kula.
The related paper contains many questions and problems linked to the positivity of that class of q-convolutions. The main result is the construction of Brownian motion related to q-Discrete Hermite polynomial of type I. Keywords— Ortogonal polynomials, Measures convolution, Khintchine inequality, q-Gaussian operators.
For more details, see: Marek Bozejko, Wojciech Bozejko,
(dedicated to Professor Jan Stochel on the occasion of his 70th birthday).
Deformations and q-Convolutions. Old and New Results, Complex Analysis and Operator Theory (2024)

https://doi.org/10.1007/s11785-024-01572-8
Keywords: Fock spaces, q-Hermite polynomials, q-convolutions, Khinchine inequalities, factors of von Neumann algebras.
Domains: Orthogonal polynomials, Measures convolution, Khintchine inequality, q-Gaussian operators.

Stéphane Dartois

Speaker: Stéphane Dartois (Univ. Paris Saclay, CEA)
Title: On the injective norm of random tensors and quantum states.
Abstract: The injective norm is a natural generalization to tensors of the operator norm of a matrix. In quantum information, the injective norm is one important measure of genuine multipartite entanglement of quantum states, where it is known as the geometric entanglement. In this talk, we give a high-probability upper bound on the injective norm of real and complex Gaussian random tensors, corresponding to a lower bound on the geometric entanglement of random quantum states, and to a bound on the ground-state energy of a particular multispecies spherical spin glass model. For some cases of our model, previous work used ε-net techniques to identify the correct order of magnitude; in the present work, we use the Kac-Rice formula to give a one-sided bound on the constant which we believe to be tight.
Keywords: random tensors, injective norm, Kac-Rice formula, random matrices, geometric measure of entanglement, spin glasses

Jehanne Dousse

Speaker: Jehanne Dousse (Univ. de Genève).
Title: Partition identities of the Andrews-Gordon type: commutative algebra and combinatorial proofs.
Abstract: A partition of a positive integer n is a non-increasing sequence of positive integers, called parts, whose sum is n. A partition identity is a theorem stating that for all n, the number of partitions of n satisfying some conditions (often congruence conditions on the parts) equals the number of partitions of n satisfying some other conditions (often difference conditions between the parts). The Andrews-Gordon identities, which generalise the Rogers-Ramanujan identities, are among the most famous and widely studied partition identities. Using techniques from commutative algebra, Pooneh Afsharijoo conjectured in 2020 a companion to these identities (i.e. a partition identity with the same congruence conditions but other difference conditions). We will explain the origins of this conjecture, give a combinatorial proof using new combinatorial dissections of Young diagrams and q-series identities, and show how another type of combinatorial proof can lead to other (new and known) identities of the same type.
This is joint work with Pooneh Afsharijoo, Frédéric Jouhet, Isaac Konan and Hus-

sein Mourtada and on
https://doi.org/10.1016/j.aim.2023.108946 and
https://arxiv.org/abs/2403.05414 .
Keywords: Young diagrams, integer partitions, partition identities, q-series.
Domains: math.CO, math.AC, math.NT

Gérard H. E. Duchamp

Speaker: Gérard H. E. Duchamp (LIPN, Univ. Paris Nord)
Title: Eilenberg-Schützenberger machines, States, Σ-modules and applications.
Abstract: The behavior of multiplicity automata is computable by means of the star of a matrix with noncommutative coefficients taken within a semiring (commutative or noncommutative). Our purpose here is to review applications of this unifying concept (Sweedler's duals, Topological algebras, Infinite iterated integrals). In passing, we indicate how to extend holomorphic-valued shuffle characters as, for example, polylogarithms [1]. In the end of the talk, we will describe a very simple two-state transducer producing the Collatz function. This transducer is the seed of an award-winning recent paper [2].
Work in progress, joint with Didier Caucal (G. Eiffel Lab, Marne-la-Vallée), Nihar Gargava (IRMA, Strasbourg) and Pierre Simonnet (Univ. Corsica).
.[1] Gérard H.E. Duchamp, Quoc Huan Ngô and Vincel Hoang Ngoc Minh, Kleene stars of the plane, polylogarithms and symmetries, TCS 800 (2019).
.[2] D. Caucal and C. Rispal, *On the Powers of the Collatz Function*, Best Paper Award of MCU 2024, to be published in LNCS series by Springer Verlag.
Keywords: Transfer matrices, words, noncommutative series, noncommutative differential equations, transducers.
Domains: Symbolic Computation (cs.SC), Combinatorics (math.CO), Operator Algebras (math.OA), Complex Variables (math.CV).

Vladimir Fock

Speaker: Vladimir Fock (IRMA, Strasbourg)
Title: From knot invariants to Schur-Weyl duality.
Abstract: We will show that HOMFLY knot invariant can be used to study representation theory of quantum groups and Hecke algebras and other combinatorial problems. The work is an interpretation of the talks by D.Gurevich at this conference.
Keywords: Skein relations, Symmetric group.
Domains: Representation theory (math.RT).

Darij Grinberg

Speaker: Darij Grinberg (Drexel Univ.)
Title: Monomial identities in the Weyl algebra.
Abstract: The Weyl algebra (or Heisenberg-Weyl algebra) is the free algebra with two generators D and U and single relation $DU - UD = 1$. As a consequence of this relation, certain monomials are equal, such as $DUUD$ and $UDDU$. We characterize all such equalities over a field of characteristic 0, describing them in several ways: operational (by a combinatorial equivalence relation generated by certain moves), computational (through lattice path invariants) and in terms of rook theory. We also enumerate the equivalence classes and several variants thereof and discuss possible extensions to other algebras.
Joint work with Tom Roby, Stephan Wagner, Mei Yin; inspired by a question of Richard P. Stanley.
Keywords: Weyl algebra, words, lattice paths, rook placements, Ferrers boards, Dyck words, monoid kernel, bond percolation, PBW bases, down-up algebra, non-commutative algebra, rings, combinatorics, finite fields.
Domains: (MSC 2020 Classifications) 12H05, 16S32, 05A15, 68R15.

Preprint: `https://arxiv.org/abs/2405.20492`

Dimitry Gurevich

Speaker: Dimitry Gurevich (IITP)
Title: Reflection Equation Algebras vs Quantum Groups.
Abstract: I plan to compare the roles of Reflection Equation Algebras and Quantum Groups in different problems of Combinatorics and Mathematical Physics. A special attention will be paid to the Quantum version of the Capelli identity.
Keywords: r-matrix Poisson structure, quantum algebras, immanants, Capelli identity.
Domain: Mathematical Physiscs.

Yuki Kanakubo

Speaker: Yuki Kanakubo (Ibaraki Univ.)
Title: Inequalities defining polyhedral realizations and monomial realizations of crystal bases.
Abstract: Crystal bases $B(\infty)$, $B(\lambda)$ are powerful tools to study representations of Lie algebras and quantum groups. We can get several essential information of integrable highest weight representations or Verma modules from $B(\lambda)$ or $B(\infty)$. To obtain such information from crystal bases, we need to describe them by combinatorial objects. The polyhedral realizations invented by Nakashima-Zelevinsky are combinatorial descriptions for $B(\infty)$ in terms of the set of integer points of a convex cone, which coincides with the string cone when the associated Lie alge-

bra is finite dimensional simple. It is a fundamental and natural problem to find an explicit form of this convex cone.
The monomial realizations introduced by Kashiwara and Nakajima are combinatorial expressions of crystal bases $B(\lambda)$ as Laurent monomials in double indexed variables.
In this talk, we give a conjecture that the inequalities defining the cone of polyhedral realizations can be expressed in terms of monomial realizations of fundamental representations.
Keywords: Crystal bases; Combinatorics; Quantum groups; Lie algebras; Polyhedral realizations; Monomial realizations.
Domain: Quantum Algebra.

Arthemy Kiselev

Speaker: Arthemy Kiselev (University of Groningen, NL)
Title: New identities for differential-polynomial structures built from Jacobian determinants.
Abstract: The Nambu-determinant Poisson brackets on $\mathbb{R}^d$ are expressed by the formula

$$\{f,g\}_d(\boldsymbol{x}) = \varrho(\boldsymbol{x}) \cdot \det\big(\partial(f,g,a_1,...a_{d-2})/\partial(x^1,...,x^d)\big),$$

where a_1, …, a_{d-2} are smooth functions and x^1, …, x^d are global coordinates (e.g., Cartesian), so that $\varrho(\boldsymbol{x}) \cdot \partial_{\boldsymbol{x}}$ is the top-degree multivector.

For an example of Nambu–Poisson bracket in classical mechanics, consider the Euler top with $\{x,y\}_3 = z$ and so on cyclically on $\mathbb{R}^3$.

Independently, Nambu's binary bracket $\{-,-\}_d$ with Jacobian determinant and $d-2$ Casimirs a_1, …, a_{d-2} belong to the Nambu (1973) class of N-ary multi-linear antisymmetric polyderivational brackets $\{-,\dots,-\}_d$ which satisfy natural N-ary generalizations of the Jacobi identity for Lie algebras.

In the study of Kontsevich's infinitesimal deformations of Poisson brackets by using 'good' cocycles from the graph complex, we detect case-by-case that these deformations preserve the Nambu class, and we observe new, highly nonlinear differential-polynomial identities for Jacobian determinants over affine manifolds. In this talk, several types of such identities will be presented.
(Work in progress, joint with M. Jagoe Brown, F. Schipper, and R. Buring; special thanks to the Habrok high-performance computing cluster.)

Maxim Kontsevich

Speaker: Maxim Kontsevich (IHES)
Title: Matrices filled by Variables, from Posets to Coxeter Groups, and Beyond.
Abstract: Last year, at CAP2023, I gave a talk about a class of matrices filled by variables, whose eigenvalues are linear forms in the variables. The construction

was based on finite posets. I'll speak about a generalization to convex sets in general reflection groups, and even to the case where there is no group at all.

Gleb Koshevoy

Speaker: Gleb Koshevoy (IITP, Moscow)
Title: Higher Bruhat orders and higher operads.
Abstract: We define higher d-operads, $d \geq 1$. For $d = 1$, 1-operads are nc-operads.
We show that higher Bruhat orders on the discrete Grassmannians $\binom{[n]}{d}$, $n \geq d$, form a d-operad.
This is joint work with Vadim Schechtman.

Vincent Lahoche

Speaker: Vincent Lahoche (Univ. Paris Saclay, CEA)
Title: Functional renormalization group for tensorial field theories: a pedagogical introduction.
Abstract: In this pedagogical presentation we discuss the construction of the non-perturbative renormalization group in the Wetterich formalism for tensor field theories at the melonic approximation. These theories introduced in the framework of group field theories approaches to quantum gravity are characterized by the specific non-locality of their interactions. The breaking of the global U(N) symmetry by the kinetic term also introduces non-trivial Ward identities, which must be taken into account in the construction of the renormalization group. We show in particular that at the melonic approximation these identities introduce an exact relation between the beta functions of the renormalizable sector, and that the only fixed point solution is the Gaussian point at this approximation.
Keywords: Nonperturbative renormalization group, nonlocal field theory, quantum gravity, random geometry
Domain(s): High energy physics.

Toshiki Nakashima

Speaker: Toshiki Nakashima (Sophia University, Tokyo)
Title: Crystal Structure of Localized Quantum Unipotent Coordinate Category.
Abstract: For a monoidal category $\mathcal{T}$, if there exists a "real commuting family $(C_i, R_{C_i}, \phi_i)_{i\in I}$", we can define a localization $\widetilde{\mathcal{T}}$ of $\mathcal{T}$ by $(C_i, R_{C_i}, \phi_i)_{i\in I}$. Let $R = R(\mathfrak{g})$ be the quiver Hecke algebra(=KLR algebra) associated with a symmetrizable Kac-Moody Lie algebra $\mathfrak{g}$ and $\mathscr{C}_w$ the subcategory of R-gmod(=the category of graded finite-dimensional R-modules) associated with a Weyl group element

w, which is a monoidal category with a real commuting family $(C_i, R_{C_i}, \phi_i)_{i\in I}$. Thus, we get its localization $\widetilde{\mathscr{C}_w}$, which is called a "localized quantum unipotent coordinate category" associated with w. In the former half of the talk, we shall present that for a (semi-)simple $\mathfrak{g}$ and the longest element w_0, the family of self-dual simple modules in $\widetilde{\mathscr{C}_{w_0}} = \widetilde{R\text{-gmod}}$ holds a crystal structure and is isomorphic to the cellular crystal $\mathbb{B}_{i_1\dots i_N}$ where $i_1 \dots i_N$ is an arbitrary reduced word of w_0. Furthermore, in the latter half of the talk, the latest result, which is a joint work with M. Kashiwara, will be presented that for a general symmetrizable Kac-Moody Lie algebra $\mathfrak{g}$ and a general Weyl group element w, the family of self-dual simple modules in the localized category $\widetilde{\mathscr{C}_w}$ also holds a crystal structure, and it is isomorphic to the cellular crystal $\mathbb{B}_{i_1\dots i_m}$ associated with a reduced word $i_1 \cdots i_m$ of w.

Keywords: Crystal, quiver Hecke algebra, categorification, localization.
Domains: math.CO, math.QA, math.RT.

Mohamed Ouerfelli

Speaker: Mohamed Ouerfelli (Univ. Paris Saclay, CEA)
Title: Sum of tensor trace invariants for spin glass landscapes optimisation.
Abstract: Spin glass models have been a interesting research subject due to the multiple valuable insights it brought to various fields such as statistical physics and machine learning. The spherical p-spin glass model in particular has been proven an excellent candidate to investigate the landscape of such models. On the other side, theoretical tools for the study of random tensors have been developed in the field of high energy physics by Razvan Gurau, Vincent Rivasseau and their collaborators. These tools have been successfully been used to address some questions related to the spherical p-spin glass. They have been used in (Evnin, 2020) to study the ground state of the spherical p-spin glass Hamiltonian. Subsequently, (Gurau, 2020) provided a theoretical study on a function based on an infinite sum of tensorial trace invariants for studying the p-spin glass with a planted spike. This function allows the detection of the presence of the planted spike above a given threshold. However, evaluating it involves computing an integral over a n-dimensional space, which may not be possible in a polynomial time. [Ouerfelli et al., 2024] showed that random tensor theory techniques can also provide a general framework for computationable algorithms. In this talk, we investigate an approach based on the random tensor theory framework that aims to progress towards the theoretical study of [Gurau, 2020] by studying the optimal way to sum tensor trace invariants in order to find local maxima of this landscape. This is based on a joint work with Parham Radpay, Mohamed Tamaazousti and Vincent Rivasseau.
Keywords: random tensor, spin glass, optimization of rough landscapes.

Karol A. Penson

Speaker: Karol A. Penson (LPTMC, Sorbonne Université, 75005 Paris)
Title: Algebraicity beyond Beukers-Heckman and Bober: emerging patterns.
Abstract:
Keywords:
Domains: We consider positive integer sequences rho(n), n=0,1..., expressible through the ratios of products of factorials, or of ratios of products of factorials along with Gamma functions. Admitting certain forms of these ratios, the generating functions (gf) of $\rho(n)$'s become generalized hypergeometric functions (gf), which turn out to be algebraic. Detailed conditions for the aforementioned algebraicity were given in [1]. In the spirit of [1], in [2] three parametrized families of, as well 52 parameter-free factorial ratios were given, all having algebraic gf's. In this work we initiate the study of factorial ratios beyond the classification of [1] and [2], for which we demonstrate by construction the algebraicity of their gf's. They include several families of shifted binomials, as well as sequences derived from recent integrality criteria put forward in [3]. For certain parametrized $\rho(n)$'s we obtained closed-form parametric expressions for their algebraic equations (algeqs) for gf's. In a parallel approach we conceived the $\rho(n)$'s in question as power moments of certain weight functions with finite support. The weights are solutions of the Hausdorff moment problem, obtained via inverse Mellin transform. In many instances the weights obey explicit algeqs that mirror those of gf's. This correspondence can be made neatly explicit, especially when elementary functions in form of radicals are involved.
Joint work with G. H. E. Duchamp, M. Kontsevich, and G. Koshevoy.
.[1]: F. Beukers and G. Heckman, Invent. Math. 95, 325-354 (1989);
.[2]: J. W. Bober, arXiv: 0709.1977v1;
.[3]: A. Adolphson and S. Sperber, arXiv: 2001.03296, and private communication (2024).

Gleb Pogudin

Speaker: Gleb Pogudin (École Polytechnique, Institute Polytechnique de Paris)
Title: Poincaré-type series for the arc space of a fat point.
Abstract: Fat point is a scheme defined by an ideal whose solution set is a single point (but the ideal is not necessarily maximal, so it may have multiplicity). For an algebraic variety, the arc scheme can be thought of as the scheme of all possible formal trajectories on the variety (in other words, power series solutions of the corresponding equations). This scheme is defined by an ideal in an infinite dimensional polynomial ring obtained by the original equations by formal differentiation. The original multiplicity structure of a fat point "propagates" to its arc scheme in a nontrivial and intriguing way (for example, it is capable of encoding nontrivial partition identities).

One way to describe this is multiplicity structure is to build a Poincaré-type series from the multiplicities of the truncations of varying orders. In the talk I will describe some recent results showing that these series are geometric series for a fat point on a line and some higher-dimensional cases and report results of computational experiments exploring more complicated fat points.

The talk is based on joint works with Rida Ait El Manssour.

Keywords: arc spaces, differential algebra, Poincaré series, Gröbner basis.
Domains: math.AG, math.AC, cs.SC.

Markus Reineke

Speaker: Markus Reineke (Ruhr University Bochum)
Title: Functional equations for motivic generating series of Kronecker moduli.
Abstract: Kronecker moduli are algebraic varieties parametrizing linear algebra data up to base change. We consider generating series of their Euler characteristic and/or Betti numbers, and discuss their algebraicity and more general functional/q-difference equations defining them.
Keywords:Kronecker moduli, Betti numbers, generating series, algebraicity.
Domains: math.AG, math.RT

Ioannis Vlassopoulos

Speaker: Ioannis Vlassopoulos (Athena Research Center)
Title: Directed metric spaces, alcoved polytopes and Large Language Models.
Abstract: Large Language Models are neural networks which are trained to produce a probability distribution on the possible next words to given texts in a corpus, in such a way that the most likely word predicted, is the actual word in the training text.

We will explain what is the mathematical structure defined by such conditional probability distributions of text extensions.

Changing the viewpoint from probabilities to -log probabilities, we observe that the data of text extensions are encoded in a directed (non-symmetric) metric structure defined on the space of texts $\mathcal{L}$. We then consider the space $P(\mathcal{L})$, of non-expansive functions on $\mathcal{L}$ which turns out to be a directed metric, alcoved polytope, in which $\mathcal{L}$ is isometrically embedded as generators of certain special extremal rays. Each such generator encodes extensions of a text along with the corresponding probabilities. Moreover $P(\mathcal{L})$ is $(\min, +)$ (i.e. tropically) generated by the text extremal rays.

$P(\mathcal{L})$ encodes semantic information about the language. We study this space and in particular explain a duality theorem relating the space generated by text extensions and that generated by text restrictions.

The metric space $\mathcal{L}$ can equivalently be considered as an enriched category and then the embedding into $P(\mathcal{L})$ is the Yoneda embedding into its category

of presheaves. In fact all constructions have categorical meaning (in particular generalizing the familiar view of language as a monoid or as a poset with the subtext order).

This is joint work with Stéphane Gaubert.