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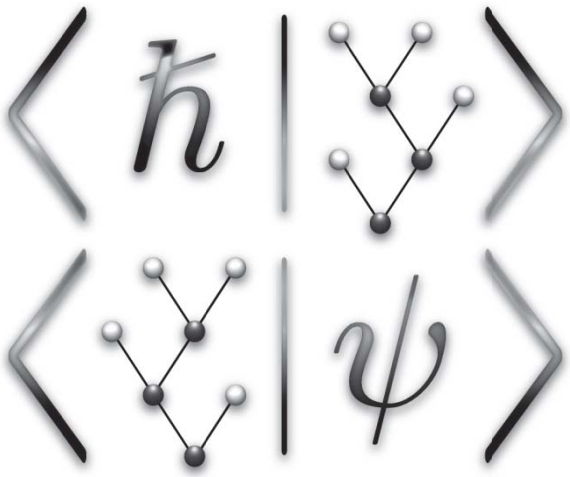
Combinatorial Physics

Book of Abstracts



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Author: ***J. Alvarez*** (Valladolid)

Title:

Abstract:

Author: ***P. Blasiak*** (Kraków)

Title: *Graphs for Quantum Theory*

Abstract:

Author: ***D. Broadhurst*** (Milton Keynes)

Title: *The combinatorics of renormalization, tamed by Hopf algebra*

Abstract:

Author: ***D. Chruściński*** (Toruń)

Title: *Permutations and quantum entanglement*

Abstract:

We construct a large class of quantum d $\otimes$ d states which are positive under partial transposition (so called PPT states). The construction is based on certain direct sum decomposition of the total Hilbert space which is governed by cyclic permutation from the symmetric group S_{d-1}. It turns out that partial transposition maps any such decomposition into another one corresponding to `complementary' permutation. This class contains many well known examples of PPT states from the literature and gives rise to a huge family of completely new states.

Author: ***G. Dattoli*** (Frascati)

Title: *Monomials and orthogonal polynomials*

Abstract:

Author: ***N. Destainville*** (Toulouse)

Title: *Flip dynamics and connectivity in 3-dimensional rhombohedra tilings*

Abstract:

We study single-flip dynamics in sets of three-dimensional rhombus tilings. This dynamics is likely to be slowed down by so-called cycles: such structures arise when tilings are encoded via the partition-on-tiling method and are susceptible to break connectivity by flips or at least ergodicity, because tilings are locally jammed in their neighbourhood. We first address the so-far open question of the connectivity of tiling sets by elementary flips. We prove exactly that sets of tilings of codimension one and two are connected for any dimension and tiling size. For higher-codimension tilings of dimension 3, the answer depends on the class of edge orientations. In most classes, we can prove connectivity despite the existence of cycles. In the few remaining classes, among which the icosahedral symmetry, the question remains open. We also tackle, by numerical means, the return to equilibrium of an observable indicative of the ability of tiles to find their average positions in the tiling. In this case, we find that the dynamics is slowed down and stuck in out-of-equilibrium states. We propose that this slowing-down might be due to cycles.

Author: ***K. Ebrahimi-Fard*** (Bonn) with D. Manchon

Title: *On the pre-Lie Magnus and Fer expansion*

Abstract:

The so-called Magnus and Fer expansion constitute well-known tools to construct approximate solutions of non-autonomous systems of linear ordinary differential equations. We provide a refined approach, unveiling new structure by using the language of dendriform and pre-Lie algebras. (Preprint: arXiv:0707.0607v2 [math.CO])

Author: ***D. Foata*** (Strasbourg)

Title: *Statistical distributions on words and q-calculus on permutations*

Abstract:

Calculations of distributions of multivariable statistics on permutations, plain or signed, often require three steps: an initial result on statistics on words, then a standardization of those words by permutations, finally q-calculus methods to derive the desired generating functions. As an illustration of this three-step approach, the generating polynomial for the symmetric group by a four-variable statistic involving number of fixed points, number of descents and number of excedances will be given.

Author: ***L. Foissy*** (Reims)

Title: *Combinatorial Dyson-Schwinger equations and Faà di Bruno subalgebras in the Hopf algebra of rooted trees*

Abstract:

In the (commutative) Hopf algebra H of rooted trees introduced by Connes and Kreimer,with its 1-cocyle B^+, we consider the following Dyson-Schwinger equation:

$$X=B^+(f(X)),$$

where f(h) is a formal series, with coefficients in the base field.

This equation admits a unique solution X, existing in a completion of H. The coefficients of X generate a subalgebra H_f of H. We characterize here the formal series f such that H_f is a Hopf subalgebra. We obtain in this way a family of Hopf subalgebras, separated into three isomorphism classes:

1) The subalgebra generated by the root tree.

2) The ladder subalgebra, isomorphic to the Hopf algebra of symmetric functions.

3) An infinite family of subalgebras, all isomorphic to the Faà di Bruno algebra.

We give a similar result in the case of the non commutative Hopf algebra of planar rooted trees, and also discuss the case of the free Faà di Bruno algebra on N variables.

Author: ***J-P. Gazeau*** (Paris)

Title: *A non-commutative reading of the complex plane through Delaunay sequences of numbers*

Abstract:

Suppose we are given an infinite strictly increasing sequence of nonnegative real numbers $\{x_n\}_{n\in\mathbb{N}}$ such that $x_0 = 0$, with $x_{n+1} - x_n \leq r > 0$ for any $n\in\mathbb{N}$ ("Delaunay sequence"). To this sequence of numbers correspond the sequence of "factorials" $x_n! = x_1 x_2 \ldots x_n$ with $x_0! = 1$, the "exponential"

$$E(t) = \sum_{n=0}^{+\infty} \frac{t^n}{x_n!},$$

the sequence of ``moment'' integrals

$$x_n!\mu_n = \int_0^{+\infty} \frac{t^n}{E(t)} dt,$$

and the ``renormalized'' sequence

$$\tilde{x}_n := \frac{\mu_n}{\mu_{n-1}} x_n.$$

In the present study we consider various examples of sequences for which it is proved, analytically or numerically, that the ratios $\mu_n / \mu_{n-1} \to 1$. A nice example is the sequence $\{ x_n = b_n , n \geq 0 \}$ of positive tau-integers, i.e. all these positive real numbers which are polynomial in powers of the golden mean $\tau = (1+\sqrt{5})/2$ when they are written in "basis" τ with the usual greedy algorithm. These tau-integers form a quasiperiodic sequence with two possible adjacent differences $b_{n+1} - b_n = 1$ or $1/\tau$.

We will explain how such sequences of numbers allow to implement a specific non-commutative (or "quantum") reading of the complex plane, different of the canonical quantization associated with the standard integers.

Author: ***Y. Hassouni*** (Rabat)

Title:

Abstract:

Author: ***J. Katriel*** (Haifa) with V. Bužek (Bratislava)

Title: *2-coloured binary trees and the classification of multipartite Einstein-Podolski-Rosen states*

Abstract:

The Einstein-Podolsky-Rosen (EPR) wavefunction is a two-particle wavefunction that is a simultaneous eigenfunction of the two-particle relative coordinate and center of mass momentum. The Fock space representation of this wave function is easily obtained as a product of an eigenfunction of the relative coordinate and an eigenfunction of the center of mass momentum. Straightforward generalization to an arbitrary number of particles is shown to be feasible in terms of the Jacobi coordinates. The well-known classification of Jacobi coordinates in terms of binary trees is extended to a classification of multipartite entangled states in terms of two-colored binary trees, the two colors corresponding to position and momentum, respectively.

Author: ***R. Kerner*** (Paris)

Title: *Combinatorial analysis of icosahedral viral capsid assembly pathways*

Abstract:

A combinatorial and statistical analysis of all possible icosahedral capsids is presented. The triangular number T is related to the total number of differentiated proteins necessary to encode capsid's size and type. The isomers are pointed out and classified, and the combinatorial rules strongly suggesting possible mutation pathways and evolutionary trends are discussed.

We also analyze chemical and energetic bariers responsible for proteins' agglomeration and optimizing the yield during capsid construction phase.

Author: ***R.C. King*** (Southampton)

Title: *Rational representations and their characters*

Abstract:

Although it is unnecessary to invoke rational, as opposed to polynomial, representations when dealing with such groups as SU(3) and SU(6), rational representations arise naturally in dealing with the quark model of baryons and mesons. Working with such a model involves the specification of characters, the determination of group-subgroup branching rules and the decomposition of tensor products. Motivated by this we show how the usual results on polynomial representations may be extended to the case of rational

representations. If time permits, some of the problems encountered in extending these ideas to the supersymmetric case will be discussed.

Author: ***A. Lascoux*** (Marne-la-Vallée)

Title: *Kazdhan-Lusztig bases of Temperley-Lieb spaces*

Abstract:

Kazdhan and Lusztig defined bases of the Hecke algebra, which give, as a special case, bases of path representations of the Temperley-Lieb algebras. We study the link of these constructions with classical Knizhnik-Zamolodchikov equations, Demazure characters and divided differences.

Author: ***A. Maciołek*** (Stuttgart)

Title: *Exact results for the effective forces induced by fluctuating interference*

Abstract:

We present exact derivations of the effective capillary wave fluctuation induced forces resulting from pinning of an interface between two coexisting phases at two points separated by a distance *r*. In two dimensions the Ising ferromagnet calculations based on the transfer matrix approach give an attractive force decaying as *1/r* for large distances. In three dimensions mapping of the body-centered solid-on-solid model onto the 6-vertex model allows for exact solution using the bosonization analysis of the equivalent *XXZ* Heisenberg quantum chain. The exact result gives the attractive force which decays asymptotically as *1/(r/log r)*.

Author: ***J-M. Maillard*** (Paris)

Title: *Lattice statistical mechanics and enumerative combinatorics: Functional equations, Tutte-Beraha numbers, and Bose-Mesner algebra*

Abstract:

We will revisit lattice statistical mechanics as problems of enumerative combinatorics, for which remarkable functional equations play a crucial symmetry role. Along this line we will recall the so-called inversion relation of

the two-dimensional Ising model, of the two-dimensional standard scalar Potts model, and the three-dimensional Ising and Potts models. We will show that these inversion relations, together with the lattice symmetries, generate an infinite set of birational symmetries of the parameter space of the lattice model. The relevance of multivalued functions for lattice statistical mechanics and the generating functions of enumerative combinatorics will be underlined on the checherboard lattice. We will show how selected values of the number of state of the Potts model, the Tutte-Beraha numbers occurring in chromatic polynomial problems can be understood as remarkable situations for these functional equations and for the previous birational symmetries. We will finally sketch other spin-edge models introducing Bose-Mesner algebra related to the so-called distance regular graphs, thus giving a new light on the structures that are really relevant for the integrability of lattice statistical mechanics or for enumerative combinatorics problems.

Author: ***M. Méndez*** (Caracas)

Title: *Set operads and incidence Hopf algebras*

Abstract:

Many years ago the author introduced a general way of constructing families of partially ordered sets from a set operad, and related incidence bialgebras. Recently Chapoton and Livernet proved that the full reduced incidence Hopf algebra of the family of posets associated to the operad of rooted trees is isomorphic to the Connes and Kreimer Hopf algebra. We present this result and a similar one obtained by the author in relation with the operad of pointed graphs.

Author: ***R. Mosseri*** (Paris)

Title: *Geometrical approach to SU(2) navigation with Fibonacci anyons*

Abstract:

Topological Quantum Computation (TQC) uses subtle properties of topological phases of matter to provide in principle, thank to topological protection, an original implementation for quantum computation somewhat immune to decoherence. Its main ingredients are anyonic excitation displaying non-abelian braiding statistics.

In this talk, we analyse a model with three Fibonacci anyons, and ask how their manipulation (upon braiding) can appropriately approximate the action of generic SU(2) unitay transformations. As known for some time now, such a system ca in principle allow for quantum computation, thank to its associated non abelian braid group representation which is dense in SU(2). A next step was then achieved by showing how to construct a universal set of quantum gates by appropriately braiding the Fibonacci anyons. An important ingrediant here relies on the efficiency of approaching a given SU(2) matrix upon braiding the 3 Fibonacci anyons.The method followed in former work was to split the braid search into two distinct parts : first a brute force search among all braids up to a given length, to generate the closest matrix to the target one; and then a refinement step done by iteratively implementing the (so-called) Solovay-Kitaev algorithm.

Here we propose an alternative approach, of rather different anture, in order to fulfil the same task. Instead of first insisting on the dense SU(2) covering generated by the Fibonacci braid group generators, we start by analysing how good they can approximate the generators of binary polyhedral SU(2) subgroups. It comes out that the subgroup of higher order, the binary icosahedral group Y with 120 elements, can indeed be very efficiently approached. Recalling the isomorphism between SU(2) and the 3 dimensional sphere S^3, this already allows a fine grained description of SU(2) into the 14400 fundamental regions of the symmetry group G of the regular polytope {3,3,5} associated with Y

We further show how to iteratively gets finer and finer meshes on S^3 by generating the so-called "geodesic hyperdomes", the analogues with one dimension more, of the celebrated families of geodesic domes which provide fine discrete approximations of the usual sphere S^2.

In a final part, a more "disordered" version of the latter step is described, which already provide an efficient speedup for "brute-like" search.

Author: ***L.M. Nieto*** (Valladolid)

Title:

Abstract:

Author: ***G. Oshanin*** (Paris)

Title: *Random walks and patterns generated by random permutations*

Abstract:

We survey recent results on some one- and two-dimensional patterns generated by random permutations of natural numbers. In the first part, we discuss properties of random walks, evolving on a one-dimensional regular lattice in discrete time n, whose moves to the right or to the left are induced by the rise-and-descent sequence associated with a given random permutation. We determine exactly the probability of finding the trajectory of such a permutation-generated random walk at site X at time n, obtain the probability measure of different excursions and define the asymptotic distribution of the number of "U-turns" of the trajectories - permutation "peaks" and "troughs". In the second part, we focus on some statistical properties of surfaces obtained by randomly placing natural numbers 1,2,3, >...,L on sites of a 1d or 2d square lattices containing L sites. We calculate the distribution function of the number of local "peaks" - sites the number at which is larger than the numbers appearing at nearest-neighboring sites - and discuss some surprising collective behaviour emerging in this model.

Author: ***C. Quesne*** (Brussels)

Title: *Deformed parafermions in quantum mechanics*

Abstract:

Standard, q-deformed and generalized deformed parafermionic (GDPF) algebras are reviewed. It is shown how the last ones may be used as spectrum generating algebras for some exactly solvable one-dimensional Schroedinger equations. The quadratic associative algebras, which act as symmetry algebras for any superintegrable two-dimensional Schroedinger equations with second-order integrals of motion, are then considered and the construction of their finite-dimensional unitary irreducible representations based upon GDPF algebras is presented. This method is finally illustrated with a position-dependent mass Schroedinger equation.

Author: ***S. Sabphapandit*** (Orsay)

Title: *Integer partitions and exclusion statistics: Limit shapes of Young diagrams*

Abstract:

I will start by talking about the correspondence between the integer partitions and exclusion statistics. I will then present our results on the limit shapes of the Young diagrams of the minimal difference partitions and provide a simple physical interpretation for the limit shapes.

Author: ***M. Schork*** (Frankfurt)

Title: *A few remarks on normal ordering multi-mode boson operators*

Abstract:

Some combinatorial aspects of normal ordering annihilation and creation operators of a multi-mode boson system are discussed.

Author: ***S. Severini*** (Waterloo)

Title: *A widefield picture of combinatorics in quantum information and nearby regions*

Abstract:

There is lots combinatorial situations when dealing with Hilbert space of small dimension and discrete quantum mechanical dynamics. I will take a widefield picture of some recent uses of combinatorics in quantum information and nearby regions.

Author: ***A.I. Solomon*** (Milton Keynes)

Title:

Abstract:

Author: ***J-Y. Thibon*** (Marne-la-Vallée)

Title: *Tree expanded series in some combinatorial Hopf algebras*

Abstract:

Tree expanded series occur in QFT as formal solutions of complicated functional equations. However, tree expansions of the solutions of very simple differential equations already lead to interesting combinatorics when lifted to certain Hopf algebras. For example, the trivial fact that the geometric series $x(t)=1/(1-t)$ satisfies $x'(t)=x^2$ has nontrivial consequences, such as the Björner-Wachs q-hook length formulas for decreasing binary trees. The algebraic mechanism underlying such identities can then be traced back to some dendriform structure, which allows for new generalizations.

Author***: A. Vourdas*** (Bradford)

Title: *Galois fields in quantum mechanics*

Abstract:

Galois quantum system where the position and momentum take values in the field GF(p^{ℓ}) are considered. The Heisenberg-Weyl group of displacements and the symplectic Sp(2,GF(p^{ℓ})) group are studied. The Frobenius transformations is a discrete symmetry in these systems (which has no analogue in the harmonic oscillator case).

Author: ***E. Zipper*** (Katowice) with M.Kurpas, J.Dajka

Title: *Entanglement of distant flux qubits*

Abstract:

Entanglement of qubits, the key feature of quantum registers, is crucial for information transmission and processing. We discuss two solid state flux qubits each interacting with a single mode electromagnetic field produced in a separate quantum cavity. This interaction leads to an entangled state of the qubit photon pairs. Then by performing the Bell state measurements on the electromagnetic field modes the entanglement is transferred to the qubits. We quantify this entanglement by calculating the linear entropy.
