Hopf algebra and Lie group structures inherent in combinatorial field theories

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Abstract.
We consider two aspects of the product formula for formal power series applied to combinatorial field theories. Firstly, we remark that the case when the functions involved in the product formula have a constant term is of special interest as often these functions give rise to substitutional groups. The groups arising from the normal ordering problem of boson strings are naturally associated with explicit vector fields, or their conjugates, in the case when there is only one annihilation operator. We also consider one-parameter groups of operators when several annihilators are present. Secondly, we discuss the Feynman-type graph representation resulting from the product formula. We show that there is a correspondence between the packed integer matrices of the theory of noncommutative symmetric functions and these Feynman-type graphs. In particular, we obtain a new Hopf algebra structure over the space of matrix quasi-symmetric functions and a natural commutative Hopf algebra structure on the space of diagrams themselves. It is shown that this Hopf algebra originates from the formal doubling of variables in the product formula.