Dynamical Systems Synchronization (Complex Emergent Properties)

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Abstract

Complex systems research overlaps substantially with nonlinear dynamics research, but complex systems specifically consist of a large number of mutually interacting dynamical parts.

In this talk, using chaos synchronization tools, we demonstrate, via two examples of three-dimensional autonomous differential systems, that, regular behavior emerges from chaotic behavior and vice-versa.

In this presentation, I will not mention a number of interesting topics on complexity ! The selected topic (emergence) that I'll develop reflect only my recent own research interests and aims to share some ideas with you.

Outline

What is Complexity ?

An extremely difficult "**I know it when I see it**" concept to define.

"I think the next century (21th) will be the century of complexity" (Stephen Hawking)

Complexity is the opposite of simplicity .... oupppsss !

Complicated (intricated) OR complex ??

In the litterature, 2 notions are often dissociated :
- A complicated system can be reduced to be better understood;
- A complex system cannot be reduced without losing its intelligibility. We must consider the whole.

Complexity : examples

Examples of complex systems include :

- nervous systems,
- immune systems,
- ant-hills,
- Cellular Automata,
- telecomunication infrastructures ...

These things have little in common, hence that the term ”complex system” is vacuous!
However, all complex systems are held to have **behavioral and structural features in common**, which at least to some degree unites them as phenomena.

Complexity : definition ?

There are many definitions of complexity.
Intuitively, complexity is usually greater in systems whose components are arranged in some intricate *difficult-to-understand* pattern.

Definition ??? : a complex system is an animate or inanimate system composed of many interacting components whose behavior or structure is difficult to understand.

But, sometimes a system may be structurally complex, like a mechanical clock, and behaves very simply.

Complexity : definition

Although complexity is now a somewhat overused expression, it has a precise meaning within this talk, in which we embrace the following definition :

Complexity *is a scientific theory that asserts that some systems display behavioural phenomena is completely inexplicable by any conventional analysis of the systems' constituent parts.*

In fact, the great majority of natural or artificial systems are of a complex nature.
- Most biological systems are complex systems,
- Most humanly engineered systems are not (a car).

Complexity : features ?

Features of complex systems in nature :

- Relationships are non-linear
- Relationships contain feedback loops
- Complex systems are open : they are usually far from energetic equilibrium: but, there may be pattern stability.
- ...

Complexity : features ?

- Complex systems have a memory,
- Complex systems may be nested : the components of a complex system may themselves be complex systems.
- Boundaries are difficult to determine : It can be difficult to determine the boundaries of a complex system. The decision is ultimately made by the observer.
- ...

Complexity : characteristics

Scientists are finding that complexity itself is often characterized by a number of important characteristics :

- Non-Linearity
- Order/Chaos Dynamic
- Self-Organization
- Emergent Properties.

Nonlinearity

The term complex system formally refers to a system of many parts which are coupled in a nonlinear fashion.

Because they are nonlinear, complex systems are **more than the sum of their parts**,
(Linear system is subject to the superposition principle, and hence is literally the sum of its parts, while a nonlinear system is not.)

In practical terms, this means that a small perturbation may cause a large effect.
(In linear systems, effect is always directly proportional to cause.)

Self-Organization

Scientists are finding that :
- change occurs naturally and automatically in systems in order to increase efficiency and effectiveness, as long as the systems are complex enough as defined above.

- Elements that survive negative environmental feedback will automatically re-organize themselves and their interactions in order to better accomplish the system's goals.
Success at this then assures their continued existence by also protecting or reinforcing the structures of which the elements are a part.

Self-Organization : examples

where have we seen self-organization before ?

- Natural Selection
- The complex system of the central nervous system of animals :

(in this example, the brain cell networks that are the ones that most successfully help the animal survive are the one's that are the most used, and thus are the ones that grow the most in size and complexity.
In contrast, those brain cell networks that do not help the animal survive are less used, and thus grow less, and may even stop growing, atrophy, and disappear.
N.B. : Researchers have developed a computer programming technique based on this approach to solving problems (like survival) called **Neural Networks**.)

Self-Organization : examples

- In nature, cells make up organs in the body.

(Cells do not exist separate from the organ. Cells in fact make up the organ's very structure, and then perform different roles in the overall work of the organ which accomplishes it's overall purpose.
Researchers have developed a computer programming technique based on this approach to solving problems called **Cellular Automata**.)

Self-Organization : examples

- Thomas Schelling's segregation model

- Artificial life

- ...

Remark :
The self-organization of elements (cells for example) into complex interacting systems can be described using **nonlinear dynamics**, which includes the study of **chaos**.

Emergence

Emergence is the process of complex pattern formation from simpler rules.
Emergence refers to the appearance of higher-level properties and behaviours of a system that while obviously originating from the collective dynamics of that system's components -are neither to be found in nor are directly deducable from the lower-level properties of that system.

Emergent properties *are properties of the 'whole' that are not possessed by any of the individual parts making up that whole.*

Emergence : The Brain example

1- An “isolated” neuron is a complicated cell, but is not conscious : it is not a complex system !

- The Brain is conscious, it is a complex system.
(The human brain is composed of approximately ten billion neurons which interact by means of electrico-chemical signals through their synapses, producing a human brain capable of thought, even though the constituent neurons are not individually capable of thought.)

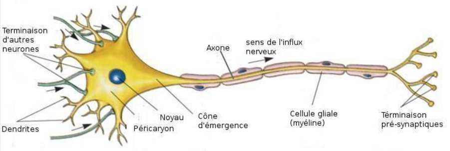


Figure: a neuron

Emergence : Immune system example

- A lymphocyte is a very complicated cell, but is not a complex system.
- The immune system is composed of approximately ten billion cells (lymphocytes) with a very large number of specificities which interact via molecular recognition.

Interactions between these billion cells give rise to a very complex immune system, with typically novel and unanticipated Emergent properties.

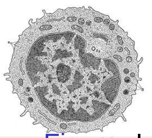
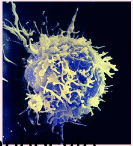



Figure: Lymphocyte

Emergence : Other examples

- Individual line of computer code, for example, cannot calculate a spreadsheet.

- An air molecule is not a tornado.

- Emergence in physics :
In physics, emergence is used to describe a property, law, or phenomenon which occurs at macroscopic scales (in space or time) but not at microscopic scales, despite the fact that a macroscopic system can be viewed as a very large ensemble of microscopic systems (example : color, friction, ...).

Chaotic dynamical systems

$$\begin{cases} \dot{x} = -\sigma(x - y) \\ \dot{y} = rx - y - xz \\ \dot{z} = xy - bz \end{cases} \tag{1}$$

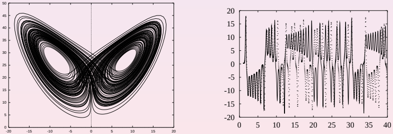


Figure: Lorenz System for $\sigma = 10.0$, $r = 28.0$, $b = 8/3$. And two time-series

Chaotic dynamical systems

$$\begin{cases} \dfrac{dx}{dt} = \alpha(y - x - bx + \frac{1}{2}(a-b)[|x+1| - |x-1|]) \\ \dfrac{dy}{dt} = x - y + z \\ \dfrac{dz}{dt} = -\beta y - \gamma z, \end{cases} \tag{2}$$

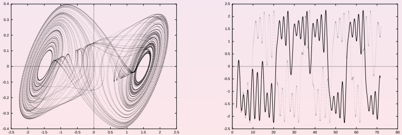


Figure: Chua system for $\alpha = 9.7633, \beta = 15.5709, \gamma = 0.0123, a = -1/7, b = 2/7$. And two time-series

Remark

One can say that this 3-D system is complex since in the case of a dynamical system, the outcome of a process is difficult to predict from its initial state (sensitive dependence on initial conditions).

Synchronization

- syn = common
- *chronos* = *time*

$\Longrightarrow$ to share the common time or to occur at the same time, that is correlation or agreement in time of different processes.

Thus, synchronization of two dynamical systems generally means that one system somehow traces the motion of another.

Periodic Oscillators

The original work on synchronization involved periodic oscillators.

It goes back at least as far as **C. Huygens** (1673), who, during his experiments on the development of improved pendulum clocks, discovered that two very weakly coupled pendulum clocks become synchronized in phase.

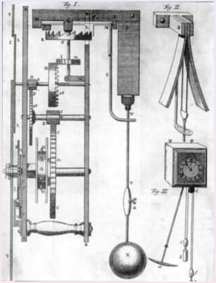


Figure: *Huygens' 'cycloïdal' pendulum clock (from "Horlogium Oscillatorium", 1673)*

Synchronization

Synchronization of chaotic systems is often studied for schemes of the form :

$$\begin{aligned}\frac{dX}{dt} &= F(X) + kN(X - Y) \\ \frac{dY}{dt} &= G(Y) + kM(X - Y)\end{aligned} \qquad (3)$$

where

- F and G act in R^n, $(X, Y) \in (R^n)^2$;
- M and N are coupling matrices belonging to $R^{n \times n}$;
- k a scalar.

Synchronization

Several different regimes of synchronization :

- **Identical Synchronization** (IS), which is defined as the coincidence of states of interacting systems, $\lim_{t\to\infty} ||X(t) - Y(t)|| = 0, \forall (X(0), Y(0)) \in B$.
- **Generalized Synchronization** (GS), which extends the IS phenomenon and implies the presence of some functional relation between two coupled systems ; if this relationship is the identity we recover the IS ;
- **Phase Synchronization** (PS), which means entrainment of phases of chaotic oscillators, wheras their amplitudes remain uncorrelated ;
- **Lag Synchronization** (LS), which appears as a coincidence of shifted-in-time states of 2 systems.

Synchronization and regular emergent properties

or order from chaos

A Lorenz-type dynamical system :

$$\begin{cases} \dot{x} &= -9x - 9y \\ \dot{y} &= -17x - y - xz \\ \dot{z} &= -z + xy \ . \end{cases} \tag{4}$$

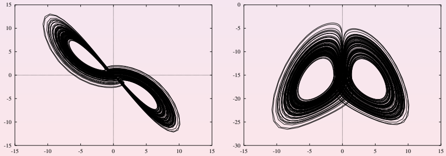


Figure: The chaotic attractor of system (4) : *xy* and *xz*-plane projections.

Bi-directionnel coupling

$$\begin{cases} \dot{x}_1 = -9x_1 - 9y_1 - k(x_1 - x_1) \\ \dot{y}_1 = -17x_1 - y_1 - x_1 z_1 \\ \dot{z}_1 = -z_1 + x_1 y_1 \\ \dot{x}_2 = -9x_2 - 9y_2 - k(x_2 - x_1) \\ \dot{y}_2 = -17x_2 - y_2 - x_2 z_2 \\ \dot{z}_2 = -z_2 + x_2 y_2 \end{cases} \quad (5)$$

Our numerical computations yield the optimal value $\tilde{k}$ for the synchronization : $\tilde{k} \simeq 2.50$,

Bi-directionnel coupling

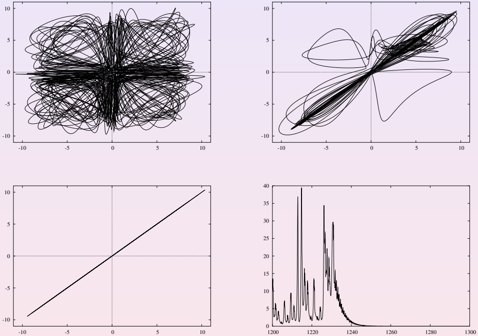
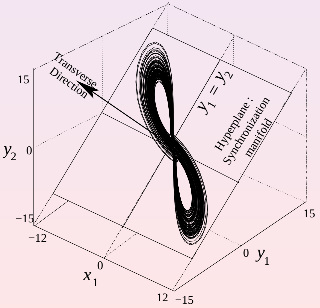


Figure: Illustration of the onset of synchronization of system (5). (a), (b) and (c) plot the amplitudes x_1 against x_2 for values of the coupling parameter $k = 0.5$, $k = 1.5$ and $k = 2.8$ respectively. The system synchronizes for $k \geq 2.5$.

Stable synchronization manifold

Geometrically, the fact that these subsystems, beyond synchronization, generate the same attractor as for system (4), implies that the attractors of these combined drive-response **6-dimensional** system are confined to a **3-dim** hyperplane (the *synchronization manifold*) defined by $Y = X$.

The motion of synchronized system takes place on a chaotic attractor which is embedded in the synchronization manifold, that is the hyperplane defined by $x_1 = x_2$, $y_1 = y_2$ and $z_1 = z_2$.



Stable synchronization manifold : Lyapunov exponents

This hyperplane is stable since small perturbations which take the trajectory off the synchronization manifold will decay in time: the largest conditional Lyapunov exponent is negatif.

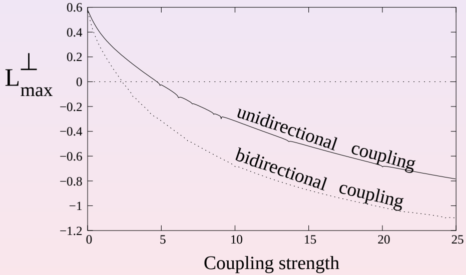


Figure: The largest transverse Lyapunov exponents $L^{\perp}_{max}$ as a function of coupling strength k in the 6-dimensional uni-directional (solid) and the bi-directional (dotted) coupled systems.

Regular behavior emerges from chaotic behavior

This result shows that simple bi-directional coupling of two (3-Dim) chaotic systems ”does not increase the chaoticity” of the (6-Dim) new system, unlike what one might expect.

Thus, in some sense, **regular behavior emerges from chaotic behavior** (i.e. the motion is confined in the ‘synchronization’ manifold).

THUS : Regular emergent properties are unanticipated.

Desynchronization motion

Synchronization depends on the coupling strength, but also on the vector field and the coupling function. For some choice of these quantities, synchronization may occur only within a finite range $[k_1, k_2]$ of coupling strength, in such a case a **desynchronization** phenomenon occurs.

Thus, increasing k beyond the critical value k_2 yields loss of the synchronized motion ($L_{max}^{\perp}$ becomes positive).

DeSynchronization and chaotic emergent properties

or chaos from order

Consider a continuous time dynamical system, modelling a tritrophic food chain (predator-prey) :

$$\begin{cases} \frac{dX}{dT} &= a_o X - b_o X^2 - \frac{v_o XY}{d_o + X} \\ \frac{dY}{dT} &= -a_1 Y + \frac{v_1 XY}{d_1 + X} - \frac{v_2 YZ}{d_2 + Y} \\ \frac{dZ}{dT} &= c_3 Z - \frac{v_3 Z^2}{d_3 + Y}, \end{cases} \tag{6}$$

a_o is the growth rate of prey X ,b_o measures the strength of competition among X, v_o is the maximum value which *per capita* reduction rate of X can attain, ... etc, ...

Regular behavior for the predator-prey system

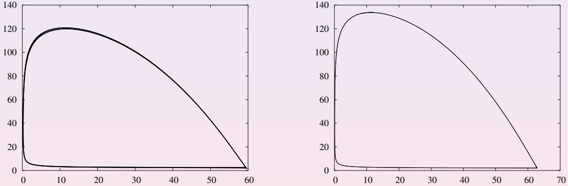


Figure: Period 1 and 2 limit cycles, for $a_0 = 3.6$ and $a_0 = 3.8$ respectively, found for system (6).

... or chaotic ...

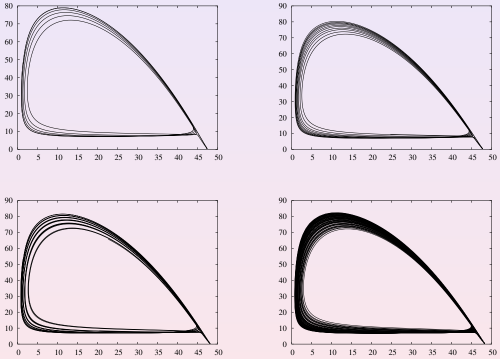


Figure: Transition to chaotic (or quasi-periodic) behavior is established via period doubling bifurcation, for respectively $a_0 = 2.85$, $a_0 = 2.87$, $a_0 = 2.89$ and $a_0 = 2.90$, found for system (6).

Uni-directional desynchronization : Predator-Prey system

$$\left\{\begin{array}{lcl}
\dot{X}_1 & = & a_o X_1 - b_o X_1^2 - \dfrac{v_o X_1 Y_1}{d_o + X_1} \\
\dot{Y}_1 & = & -a_1 Y_1 + \dfrac{v_1 X_1 Y_1}{d_1 + X_1} - \dfrac{v_2 Y_1 Z_1}{d_2 + Y_1} \\
\dot{Z}_1 & = & c_3 Z_1 - \dfrac{v_3 Z_1^2}{d_3 + Y_1} \\
\dot{X}_2 & = & a_o X_2 - b_o X_2^2 - \dfrac{v_o X_2 Y_2}{d_o + X_2} - k(X_2 - X_1) \\
\dot{Y}_2 & = & -a_1 Y_2 + \dfrac{v_1 X_2 Y_2}{d_1 + X_2} - \dfrac{v_2 Y_2 Z_2}{d_2 + Y_2} \\
\dot{Z}_2 & = & c_3 Z_2 - \dfrac{v_3 Z_2^2}{d_3 + Y_2}
\end{array}\right. \qquad (7)$$

Uni-directional desynchronization

We have choose, for the coupled system, a range of parameters for which both subsystems constituents parts evolve periodically, as the previous figure shows.

BUT : we have no synchronization, even for strong coupling !

Emergence of chaotic properties

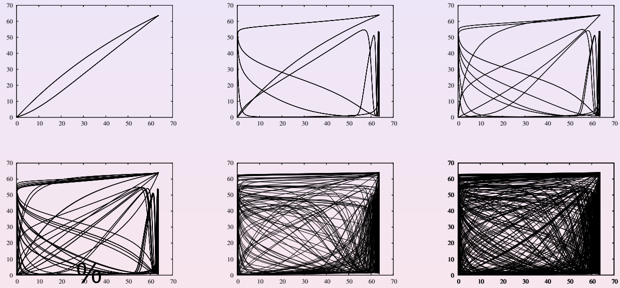


Figure: Illustration of the onset of desynchronization of the unidirectionel coupled system (7) the system synchronizes (in the generalized sense) for very very small values of k. But a desynchronization processus fastly arrises by increasing k, the larger is the coupling coefficient the weaker is the synchronization-.

Emergence of chaotic properties

Hence, we have emergence of chaotic properties :
The coupled system displays behavioral chaotic phenomena which is not exhibited by systems' constituent parts, that are the two predator-prey systems before coupling, which exhibit the limit-cycle of the previous figure, and for the same parameters, same initial conditions.

This phenomenon is robust with respect to small parameters variations.

Emergent chaotic properties are typically novel and unanticipated, for this example.

Bidirectional desynchronization : Predator-Prey system

Many biological or physical systems consist of bi-dir- ectionally interacting elements or components, let us use a bi-directionally (*mutual*) coupling, in order that both drive and response subsystems are connected in such a way that they mutually influence each other's behavior:

Bidirectional desynchronization

$$
\left\{
\begin{array}{rcl}
\dot{X}_1 & = & a_o X_1 - b_o X_1^2 - \dfrac{v_o X_1 Y_1}{d_o + X_1} - k(X_1 - X_2) \\
\dot{Y}_1 & = & -a_1 Y_1 + \dfrac{v_1 X_1 Y_1}{d_1 + X_1} - \dfrac{v_2 Y_1 Z_1}{d_2 + Y_1} \\
\dot{Z}_1 & = & c_3 Z_1 - \dfrac{v_3 Z_1^2}{d_3 + Y_1} \\
\dot{X}_2 & = & a_o X_2 - b_o X_2^2 - \dfrac{v_o X_2 Y_2}{d_o + X_2} - k(X_2 - X_1) \\
\dot{Y}_2 & = & -a_1 Y_2 + \dfrac{v_1 X_2 Y_2}{d_1 + X_2} - \dfrac{v_2 Y_2 Z_2}{d_2 + Y_2} \\
\dot{Z}_2 & = & c_3 Z_2 - \dfrac{v_3 Z_2^2}{d_3 + Y_2}
\end{array}
\right. \tag{8}
$$

Emergence of chaotic properties

We have also choose, for this bi-directionally coupled system, the same range of parameters for which the subsystems constituents parts evolve periodically, as the previous figure shows.

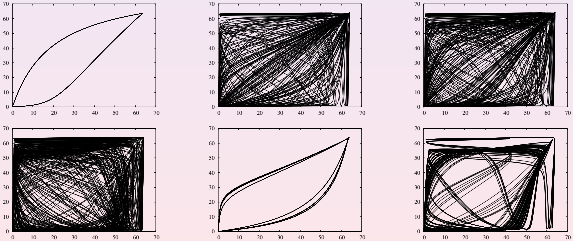


Figure: Figures plot amplitudes x_1 against x_2. The system synchronizes (in the generalized sense) for $k \leq 0.01$. But the desynchronization processus arrises by increasing k, fastly in comparison with the unidirectional case.

Emergence of chaotic properties

Thes figures show that the larger is this coupling coefficient the weaker is the synchronization.

Furthermore, the bidirectional case enhances the desynchronization processus that is the occurence of new complex phenomenon, and makes it occuring fastly in comparison to the unidirectinal case.

Conclusion : Chaotic emergent properties

All our results show that the whole predator-prey food chain in 6-dimensional space, exhibits behavioral phenomena which are unexplainable by any conventional analysis of the 3-dimensional systems' constituent parts (which have for the same ranges of parameters 1-periodic solutions).
New Emergent properties of the "whole" 6-dimensional system that are not possessed by any of the individual parts (that are the two 3-dimensional subsystems).

Conclusion : Chaotic emergent properties

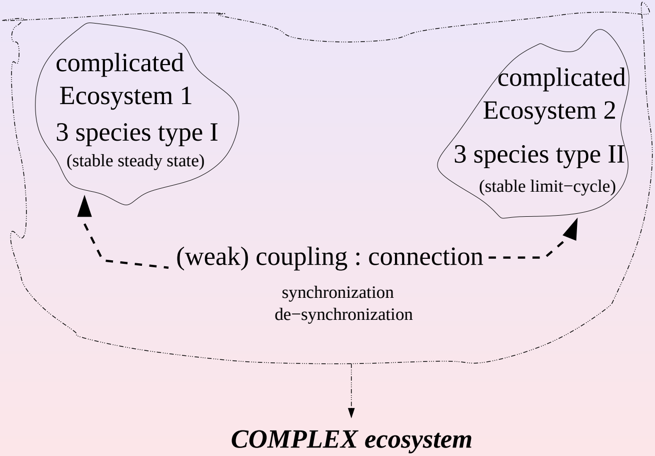


Figure:

Conclusion

In this presentation I did not mention a number of interesting topics on complexity ! The selected topics that I tried to develop concerns Complexity from THE EMERGENCE point of view.

References :

References

- Just google-it !!!
- But you can also see the book: Aziz-Alaoui M.A. and Bertelle C., Eds, (2006) *Emergent properties in Natural and Artificial Dynamical Systems*, Springer, Berlin Heidelberg New york, "Understanding Complex Systems" series.
- L'Enigme de l'Emergence, *Sciences et Avenir*, 143, Juil-Août, 2005.
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- ...

Remark : To measure complexity

While several measures of complexity have been proposed in the research literature, they all fall into two general classes:

1. **Static Complexity** -which addresses the question of how an object or system is put together (i.e. only pure structural informational aspects of an object), and is independent of the processes by which information is encoded and decoded.
2. **Dynamic Complexity** -which addresses the question of how much dynamical or computational effort is required to describe the informational content of an object or state of a system.

These two measures are clearly not equivalent.

Conclusion

Differences between equilibrium and complex systems.

Equilibrium systems are divisible and satisfy the ergodic theorem.

Complex systems are composed out of interdependent parts and violate the ergodic theorem. They have many degrees of freedom whose time dependence is very slow on a microscopic scale.

Master and Slave systems

Master system :

$$\frac{dX}{dt} = F(X), \quad X \in R^n \tag{9}$$

Slave system :

$$\frac{dY}{dt} = G(X, Y), \quad Y \in R^m, \tag{10}$$

Identical Synchronization

Definition

Identical synchronization System (10) synchronizes with system (9), if the set $M = \{(X, Y) \in R^n \times R^n, Y = X\}$ is an attracting set with a basin of attraction B ($M \subset B$) such that $\lim_{t \to \infty} ||X(t) - Y(t)|| = 0$, forall $(X(0), Y(0)) \in B$.

Generalized Synchronization

Definition

Generalized synchronization System (10) synchronizes with system (9), in the generalized sense, if there exists a transformation $\psi : R^n \longrightarrow R^m$, a manifold $M = \{(X, Y) \in R^{n+m}, Y = \psi(X)\}$ and a subset B ($M \subset B$), such that for all $(X_o, Y_o) \in B$, the trajectory based on the initial conditions (X_o, Y_o) approaches M as time goes to infinity.