From neuronal oscillations to complexity

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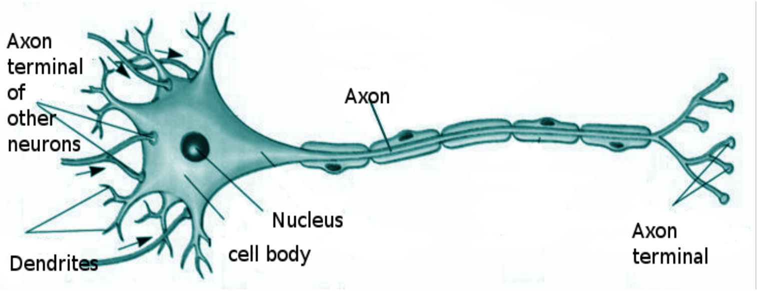
Neuron

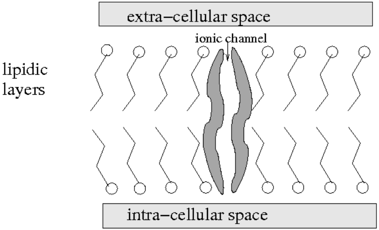
- *a cell body*, extended by :
- *dendrites*
- *an axon*
- *an axon terminal*



Ionic channels

Membrane is composed of :

- a double lipidic layer
- ionic channels



Ionic channels :

- voltage-dependant
- open / close / inactive
- conductance

Some important channels

- *Sodium channels* : 4 gates :
  - *3 to control the opening*
  - *1 to control the inactivity*

  → particularity : **inactivity period**

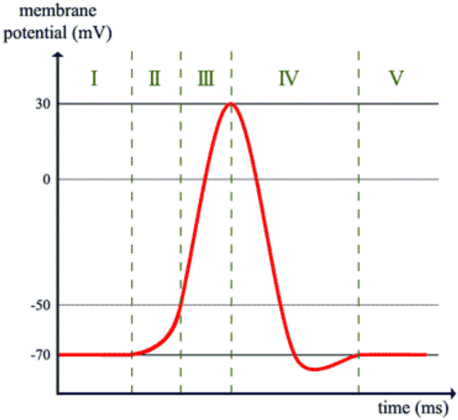
- *Potassium channels* : 4 gates
  all control the opening
  → particularity : **delay**

- *Leakage channels* :
  → particularity : **always opened**

Resting potential

- *Potential difference* between both sides of the membrane ;
  - more Na_+ outside
  - more K_+ inside
- Experimentally, $-90mV < ddp < -50mV$
  - intra-cellular space more negative
  - extra-cellular space more positive

Action potential



ACTION POTENTIAL :

- I : Resting state
- I-II : Stimulation
- II&III : Depolarization
- IV : Repolarization & hyperpolarization
- V : Resting state

IONIC SCALE :

- II : Sodium channels open
- III : More sodium channels open
- III-IV : Sodium channels close
- III-IV : Potassium channels open
- IV-V : Potassium channels close

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Hodgkin-Huxley model (1)

Alan Lloyd Hodgkin and *Andrew Fielding Huxley* :

- neurophysiologists
- experiments on squid giant neuron ;
- discovery of ionic channels ;
- description of electric activity of a neuron by mathematical equations ;
- Nobel price of physiology and medecine in 1952 ;

Hodgkin-Huxley model (2)

$$\begin{cases} -C\dfrac{dV}{dt} = I_K + I_{Na} + I_L - I \\ \dfrac{dm}{dt} = \alpha_m(1-m) - \beta_m m \\ \dfrac{dn}{dt} = \alpha_n(1-n) - \beta_n n \\ \dfrac{dh}{dt} = \alpha_h(1-h) - \beta_h h \end{cases}$$

Hodgkin-Huxley model (3)

$$I = C\frac{dV}{dt} + I_K + I_{Na} + I_L \Rightarrow -C\frac{dV}{dt} = I_K + I_{Na} + I_L - I$$

- $I_K = n^4 g_K(E - E_K)$;
- $n =$ one of the 4 gates of a potassium channel is open ;
- $g_K =$ conductance of a potassium channel $\times$ number of opened channels ;
- $E_K =$ equilibrium potential of potassium (= Nernst potential) ;

Hodgkin-Huxley model (4)

Similarly, we have :

- $I_{Na} = m^3 h g_{Na}(E - E_{Na})$;
- $I_L = g_L(E - E_L)$;

- α_i = opening coefficient, $i = m, n, h$;
- β_i = closing coefficient, $i = m, n, h$;

Hodgkin-Huxley model (5)

$$\begin{cases} -C\dfrac{dV}{dt} = I_{Na} + I_K + I_L - I \\ \dfrac{dm}{dt} = \alpha_m(1-m) - \beta_m m \\ \dfrac{dn}{dt} = \alpha_n(1-n) - \beta_n n \\ \dfrac{dh}{dt} = \alpha_h(1-h) - \beta_h h \end{cases}$$

Hindmarsh-Rose 1982 model (1)

From Hodgkin-Huxley model to Hindmarsh-Rose 1982 model ;

$\Rightarrow$ *Mathematical simplification* thanks to *biological observations* :

- m is substituted by a constant because of its fast activation ;
- $h + n = 0.8$ (experimentally) ;

$$\begin{cases} -C\dfrac{dV}{dt} = I_K + I_{Na} + I_L - I \\ \dfrac{dm}{dt} = \alpha_m(1-m) - \beta_m m \\ \dfrac{dn}{dt} = \alpha_n(1-n) - \beta_n n \\ \dfrac{dh}{dt} = \alpha_h(1-h) - \beta_h h \end{cases}$$

Hindmarsh-Rose 1982 model (2)

A Fitzhugh-Nagumo type model :

$$\left\{ \begin{array}{rcl} \frac{dx}{dt} & = & y - x^3 + ax^2 + I \\ \frac{dy}{dt} & = & 1 - dx^2 - y \end{array} \right.$$

a, d : parameters determined experimentally .
I : applied current.

Hindmarsh-Rose 1984 model

A third equation to be closer to reality : an *adaptation equation*

$$\begin{cases} \dfrac{dx}{dt} & = & y - x^3 + ax^2 - z + I \\ \dfrac{dy}{dt} & = & 1 - dx^2 - y \\ \dfrac{dz}{dt} & = & \epsilon(b(x - x_c) - z), \quad \epsilon << 1 \end{cases}$$

$\rightarrow$ *slow-fast system*

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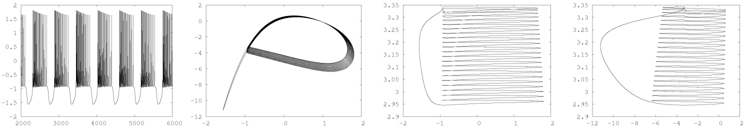
Numerical analysis (1)

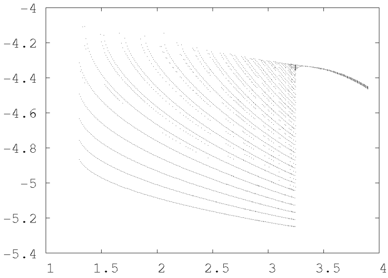


Time series (t, x) : bursting, and projections of the phase portrait on the planes (x, y), (x, z) et (y, z)

Numerical analysis (2)

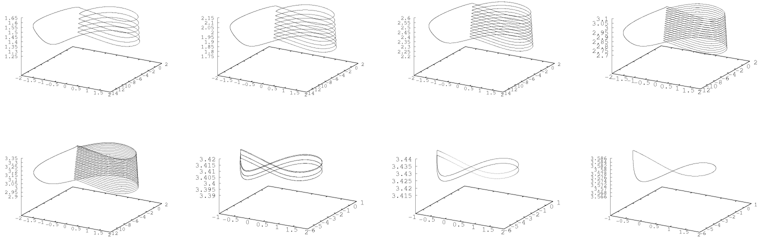
Bifurcation

Qualitative change of solutions due to parameters changes



- $I = 1.5$: 3-cycle
- $I = 2$: 5-cycle
- $I = 3$: 10-cycle
- $3.25 \lesssim I \lesssim 3.295$: chaos
- $I \gtrsim 3.295$: stop generating bursts

Numerical analysis (3)



$$I = 1.5,\ \ I = 2,\ \ I = 2.5,\ \ I = 3$$
$$I = 3.25, I = 3.33\ I = 3.35\ I = 3.50$$

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Synchronization-Generalities

- characteristic of many processes in natural systems and non linear science
- *syn* : common, *chronos* : time
- to share the same motion at the same time
- there exist many kind of synchronization (*identical, generalized, phase...*)
- neurons : optimal transmission of information

Coupling systems (1)

Coupling

Making different entities be dependant one from the others.

Linear coupling function

Mutual coupling of 2 neurons by a linear function : electrical connection

$$\begin{cases} \dot{x}_1 = a x_1^2 - x_1^3 - y_1 - z_1 - k(x_2 - x_1) \\ \dot{y}_1 = (a+\alpha) x_1^2 - y_1 \\ \dot{z}_1 = \mu(b x_1 + c - z_1) \\ \dot{x}_2 = a x_2^2 - x_2^3 - y_2 - z_2 - k(x_1 - x_2) \\ \dot{y}_2 = (a+\alpha) x_2^2 - y_2 \\ \dot{z}_2 = \mu(b x_2 + c - z_2) \end{cases}$$

$k(x_1 - x_2)$: coupling function, k : coupling strength

Coupling systems (2)

Sigmoid coupling function

Mutual coupling of 2 neurons by a sigmoid function : chemical connection

$$\begin{cases} \dot{x}_1 = a x_1^2 - x_1^3 - y_1 - z_1 - (x_1 - V_s) k_2 \dfrac{1}{1 + exp(-\lambda(x_2 - \Theta_s))} \\ \dot{y}_1 = (a + \alpha) x_1^2 - y_1 \\ \dot{z}_1 = \mu(b x_1 + c - z_1) \\ \\ \dot{x}_2 = a x_2^2 - x_2^3 - y_2 - z_2 - (x_2 - V_s) k_2 \dfrac{1}{1 + exp(-\lambda(x_1 - \Theta_s))} \\ \dot{y}_2 = (a + \alpha) x_2^2 - y_2 \\ \dot{z}_2 = \mu(b x_2 + c - z_2) \end{cases}$$

Θ_s : threshold, k_2 : coupling strength.

Coupling 2 neurons

Two neurons identically synchronize if :

$$\left\{\begin{array}{l} x_2 \rightarrow x_1 \\ y_2 \rightarrow y_1 \\ z_2 \rightarrow z_1 \end{array}\right. \Leftrightarrow \left\{\begin{array}{l} x_\perp = x_1 - x_2 \rightarrow 0 \\ y_\perp = y_1 - y_2 \rightarrow 0 \\ z_\perp = z_1 - z_2 \rightarrow 0 \end{array}\right.$$

$\Rightarrow$ Stability study of the transverse system :

$$\left\{\begin{array}{l} \dot{x_\perp} = \dot{x_1} - \dot{x_2} \\ \dot{y_\perp} = \dot{y_1} - \dot{y_2} \\ \dot{z_\perp} = \dot{z_1} - \dot{z_2} \end{array}\right.$$

$\Rightarrow$ Example of theoretical tool to study this system : *Lyapunov function*.

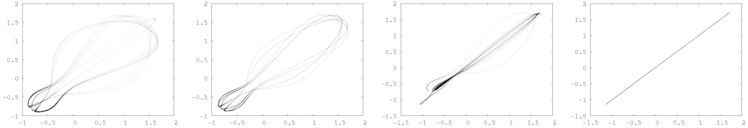
HR neurons mutually coupled by a sigmoid function

$$\begin{cases} \dot{x}_1 = a x_1^2 - x_1^3 - y_1 - z_1 - (x_1 - V_s) k_2 \dfrac{1}{1 + exp(-\lambda(x_2 - \Theta_s))} \\ \dot{y}_1 = (a + \alpha) x_1^2 - y_1 \\ \dot{z}_1 = \mu(b x_1 + c - z_1) \\ \\ \dot{x}_2 = a x_2^2 - x_2^3 - y_2 - z_2 - (x_2 - V_s) k_2 \dfrac{1}{1 + exp(-\lambda(x_1 - \Theta_s))} \\ \dot{y}_2 = (a + \alpha) x_2^2 - y_2 \\ \dot{z}_2 = \mu(b x_2 + c - z_2) \end{cases}$$

Numerical results

Remark

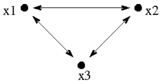
Diagonal $\Leftrightarrow x_1 = x_2 \Leftrightarrow$ synchronization

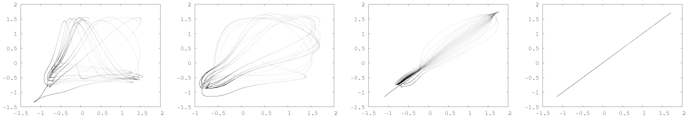


x_1 vs x_2 for $k_2 = 0.8, k_2 = 1, k_2 = 1.22, k_2 = 1.26$.

Same minimal coupling strength to obtain synchronization between y_1 and y_2 and between z_1 and z_2.

Coupling 3 neurons

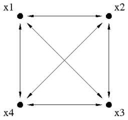


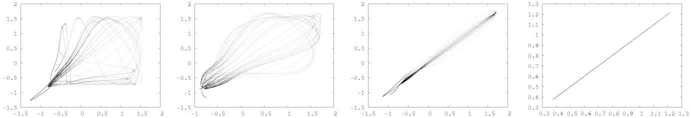


x_1 vs x_2 for $k_3 = 0.2, k_3 = 0.4, k_3 = 0.6, k_3 = 0.63$.

$\rightarrow$ same resuts for x_i vs x_j, y_i vs y_j, z_i vs z_j ($\forall i, j = 1, 2, 3$)

Coupling 4 neurons

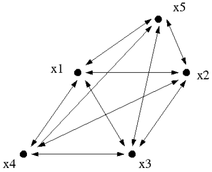


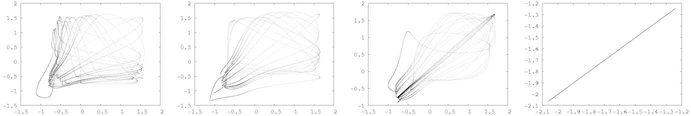


x_1 vs x_2 for $k_4 = 0.1, k_4 = 0.3, k_4 = 0.42, k_4 = 0.5$.

$\rightarrow$ same results for x_i vs x_j, y_i vs y_j, z_i vs z_j ($\forall i, j = 1, ..., 4$)

Coupling 5 neurons

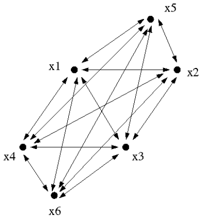


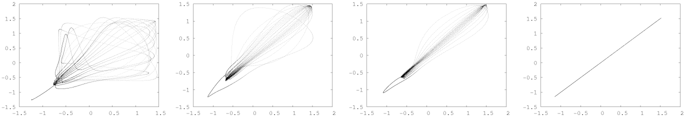


x_1 vs x_2 for $k_5 = 0.1$,$k_5 = 0.2$,$k_5 = 0.29$,$k_5 = 0.3$.

$\rightarrow$ same results for x_i vs x_j, y_i vs y_j, z_i vs z_j ($\forall i, j = 1, ..., 5$)

Coupling 6 neurons

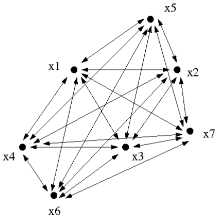


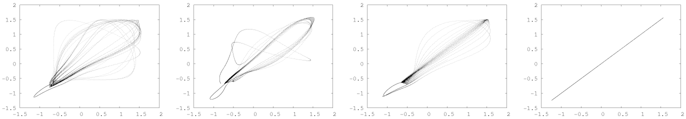


x_1 vs x_2 for $k_6 = 0.1, k_6 = 0.2, k_6 = 0.23, k_6 = 0.24$.

$\rightarrow$ same results for x_i vs x_j, y_i vs y_j, z_i vs z_j ($\forall i, j = 1, ..., 6$)

Coupling 7 neurons

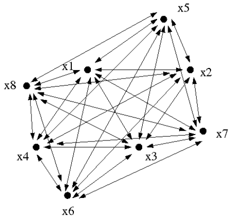


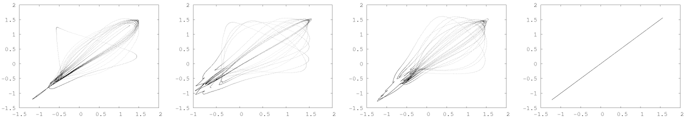


x_1 vs x_2 for $k_7 = 0.1, k_7 = 0.17, k_7 = 0.2, k_7 = 0.21$.

$\rightarrow$ same results for x_i vs x_j, y_i vs y_j, z_i vs z_j $(\forall i, j = 1, ..., 7)$

Coupling 8 neurons



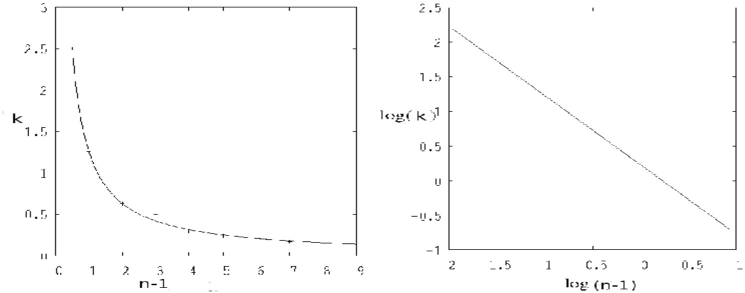


x_1 vs x_2 for $k_8 = 0.1, k_8 = 0.15, k_8 = 0.16, k_8 = 0.17$.

$\rightarrow$ same results for x_i vs x_j, y_i vs y_j, z_i vs z_j ($\forall i, j = 1, ..., 8$)

Coupling n neurons

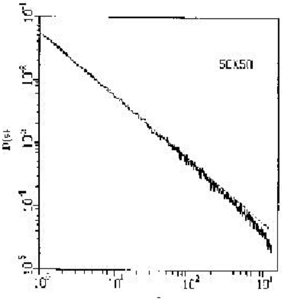
- each neuron is connected to all the others $\Leftrightarrow$ in a network of n neurons each neuron is connected to $n-1$ neurons ($n-1$ inputs)
- $$k_n = \frac{k_2}{n-1}$$
  - k_2 : 2 neurons mutually coupled synchronization threshold
  - $n-1$: number of inputs for each neuron



⇒ classical law which is found in many self-organized complex systems

Examples

- earthquakes : 1000 of magnitude 4 for 100 of magnitude 5 and 10 of magnitude 6
- linguistic : for 1000 occurrences of ’the’ in text, 100 occurrences of ’I’ and 10 of ’say’
- urban systems : big cities are rare and small ones are frequent in an exponential way.



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Conclusion and perspectives

- intersection between synchronization in coupled nonlinear systems and complex networks
- study conditions under which a complex network of dynamical systems synchronizes
- study of the different topological structures of networks and their consequences on the synchronization phenomenon
- synchronization and complex networks theoretical study

THANK YOU !