The Hopf Algebra of Heisenberg-Weyl Diagrams

G H E Duchamp[a], **P Blasiak**[b], **A Horzela**[b], **K A Penson**[c], **A I Solomon**[c,d],

[a] LIPN - UMR 7030
CNRS - Université Paris 13
F-93430 Villetaneuse, France

[b] H. Niewodniczański Institute of Nuclear Physics, Polish Academy of Sciences
ul. Eliasza-Radzikowskiego 152, PL 31342 Kraków, Poland

[c] Laboratoire de Physique Théorique de la Matière Condensée
Université Pierre et Marie Curie, CNRS UMR 7600
Tour 24 - 2ième ét., 4 pl. Jussieu, F 75252 Paris Cedex 05, France

[d] The Open University, Physics and Astronomy Department
Milton Keynes MK7 6AA, United Kingdom

E-mail:
`ghed@lipn-univ.paris13.fr`, `a.i.solomon@open.ac.uk`
`pawel.blasiak@ifj.edu.pl`, `andrzej.horzela@ifj.edu.pl`,
`penson@lptl.jussieu.fr`,

Abstract. We show here that, keeping track of the annihilations in the Wick formula allows to construct a Hopf algebra of diagrams which explains at the both the Hopf defect of the HW algebra and the graphical mechanism of coupling-separating the diagrams themselves.

Contents

1. Introduction

Since the advent of Hopf algebras in Combinatorial Physics, when facing an algebra, one always ask oneself whether its structure could be (more or less naturally) be enriched to a Hopf algebra one. For one of the simplest in Quantum Physics, the Heisenberg-Weyl (HW) algebra, the answer is "no" since the counit provides one with a one-dimesional representation HW and it is well-known (and easily checked) that this algebra has no non-zero finite dimensional representation.

References

[1] Bayen (F.), Flato (M.), Fronsdal (C.), Lichnerowicz (A.) and Sternheimer (D.), *Deformation and Quantization*, Ann. of Phys. 111 (1978), pp. 61-151.

[2] J. Berstel, C. Reutenauer, *Rational series and their languages EATCS Monographs on Theoretical Computer Science*, Springer (1988).

[3] C. M. Bender, D. C. Brody, and B. K. Meister, Quantum field theory of partitions, J. Math. Phys. Vol 40 (1999)

[4] P. Blasiak, A. Horzela, K. A. Penson, G. H. E. Duchamp, A.I. Solomon, *Boson normal ordering via substitutions and Sheffer-Type Polynomials*, Phys. Lett. A **338** (2005) 108

[5] P. Blasiak, K. A. Penson, A.I. Solomon, A. Horzela, G. H. E. Duchamp, *Some useful formula for bosonic operators*, Jour. Math. Phys. **46** 052110 (2005).

[6] Bourbaki N., *Algebra, chapter III*, Springer

[7] Bourbaki N., *Algebra, chapter VI*, Springer

[8] Bourbaki N., *Theory of sets*, Springer

[9] Cartier P., *Séminaire "Sophus Lie", 2ème année*, Faculté des Sciences de Paris (1955-56)

[10] P. Cartier, *Fonctions polylogarithmes, nombres polyzeta et groupes pro-unipotents, Séminaire Bourbaki, Asterisque n. 282 (2002)*

[11] P. Cartier, *A primer of Hopf algebras*, Septembre (2006), IHES preprint IHES/M/06/40.

[12] V. Chari, A. Pressley, A guide to quantum groups. Cambridge (1994).

[13] G. H. E. Duchamp, P. Blasiak, A. Horzela, K. A. Penson, A. I. Solomon, Feynman graphs and related Hopf algebras, Journal of Physics: Conference Series, SSPCM'05, Myczkowce, Poland. arXiv : cs.SC/0510041

[14] Duchamp G., Flouret M., Laugerotte E., Luque J-G., *Direct and dual laws for automata with multiplicities*
`arXiv` : math.CO0607412

[15] Duchamp G., Luque J-G., *Congruences Compatible with the Shuffle Product*
`arXiv` : math.CO0607419

[16] G. Duchamp, F. Hivert, J. Y. Thibon, *Non commutative functions VI: Free quasi-symmetric functions and related algebras*, International Journal of Algebra and Computation Vol 12, No 5 (2002).

[17] G. H. E. Duchamp, G. Koshevoy, K. A. Penson, C. Tollu, F. Toumazet, *Geometric combinatorial twisting and shifting*, Séminaire Lotharingien (in preparation).

[18] G. H. E. Duchamp, J. -G. Luque, J. -C. Novelli, C. Tollu, F. Toumazet, *Hopf algebras of diagrams*, FPSAC07.

[19] G. Duchamp, A.I. Solomon, K.A. Penson, A. Horzela and P. B lasiak, *One-parameter groups and combinatorial physics*, Proceedings of the Symposium Third International Workshop on Contemporary Problems in Mathematical Physics (COPROMAPH3) (Porto-Novo, Benin, Nov. 2003), J. Govaerts, M. N. Hounkonnou and A. Z. Msezane (eds.), p.436

(World Scientific Publishing 2004)
arXiv: `quant-ph/04011262`

[20] L. Foissy, *Isomorphisme entre l'algèbre des fonctions quasi-symétriques libres et une algèbre de Hopf des arbres enracinés décorés plans*, personal communication.

[21] L. Foissy, *Les algèbres de Hopf des arbres enracinés decorés*, PhD Memoir, Reims University (2002).

[22] L. Foissy, *Personal Communication.*

[23] M. Hazewinkel, Hopf algebras of endomorphisms of Hopf algebras, (Oct 2004) `ArXiv : math.QA/0410364`

[24] M. E. Hoffman, *Quasi-shuffle products*, J. Algebraic Combin. (2000), 49-68

[25] A. Horzela, P. Blasiak, G. Duchamp, K. A. Penson and A.I. Solomon, *A product formula and combinatorial field theory*, Proceedings of the XI International Conference on Symmetry Methods in Physics (SYMPHYS-11) (Prague, Czech Republic, June 2004), C. Burdik, O. Navratil, and S. Posta (eds.) (JINR Publishers, Dubna)
arXiv:`quant-ph/0409152`

[26] D. Kreimer, *Knots and Feynman Diagrams*, Cambridge Lecture Notes in Physics (2000).

[27] P. Ochsenschläger, Binomialkoeffitzenten und Shuffle-Zahlen, Technischer Bericht, Fachbereich Informatik, T. H. Darmstadt,1981.

[28] Reutenauer C., *Free Lie algebras*, Oxford University Press (1993).

[29] M. Rosso, *Quantum groups and quantum shuffles*, Inventiones Math. 133 (1998).

[30] A.I. Solomon, P. Blasiak, G. Duchamp, A. Horzela and K.A. Penson, *Combinatorial Physics, Normal Order and Model Feynman Graphs*, Proceedings of the Symposium 'Symmetries in Science XIII', Bregenz, Austria, 2003, B. Gruber, G. Marmo and N. Yoshinaga (eds.), p.527 (Kluwer Academic Publishers 2004)
arXiv : `quant-ph/0310174`

[31] A.I. Solomon, G. Duchamp, P. Blasiak, A. Horzela and K.A. Penson, *Normal Order: Combinatorial Graphs Quantum Theory and Symmetries*, Proceedings of the 3rd International Symposium P.C. Argyres, T.J. Hodges, F. Mansouri, J.J. Scanio, P. Suranyi, and L.C.R. Wijewardhana (eds.), p.398 (World Scientific Publishing 2004)
arXiv:`quant-ph/0402082`

[32] A.I. Solomon, G. Duchamp, P. Blasiak, A. Horzela and K. A. Penson, *Partition functions and graphs: A combinatorial approach*, Proceedings of the XI International Conference on Symmetry Methods in Physics (SYMPHYS-11) (Prague, Czech Republic, June 2004), C. Burdik, O. Navratil, and S. Posta (eds.) (JINR Publishers, Dubna, 2004)
arXiv:`quant-ph/0409082`

[33] A. I. Solomon, G. Duchamp, P. Blasiak, A. Horzela and K. A. Penson, *Hopf algebra structure of a model quantum field theory*, Proceedings of the 26th International Colloquium on Group Theoretical Methods in Physics, New York 2006, Editor: S. Catto (2007).

[34] G. Duchamp, A. I. Solomon, P. Blasiak, A. Horzela and K. A. Penson, *A multipurpose Hopf deformation of the algebra of Feynman-like diagrams*, Proceedings of the 26th International Colloquium on Group Theoretical Methods in Physics, New York 2006, Editor: S. Catto (2007).