Let $(B,\mu,1_B,\Delta,\varepsilon)$ be a bialgebra (the ground field will be taken as k), we define the convolution in $\mathsf{End}(B)$ for $f,g\in\mathsf{End}(B)$ and $x\in B$ as

$$f*g(x)=\mu(f\otimes g)\Delta(x) \tag{1}$$

one can check that (exercise 11.9 (2) of the tutorial) this is an algebra with the linear mapping

$$convunit\ :\ x\mapsto\varepsilon(x)1_B \tag{2}$$

as unit.
One can check easily that this mapping is the projector on the line $k.1_B$ and kernel

$$B^+=ker(\varepsilon)=\{x\in B\mid\varepsilon(x)=0\}\ . \tag{3}$$

We will denote $I^+=Id_B-convunit$ the complementary projector (in fact, in my previous letter $\Delta^+=(I^+\otimes I^+)\circ\Delta$ and it can be proved that Δ^+ is coassociative).

The antipode S, when it exists, is defined as the convolutional inverse of Id_B.
But $Id=convunit+I^+$ (of type $1+X$), and then if the sum

$$S=\sum_{n=0}^{\infty}(-1)^n(I^+)^{*n} \tag{4}$$

converges (in some sense), it is the antipode.
Here, in the two cases of $\mathcal{U}$ and $\mathcal{G}$, the sequence of endomorphisms $(I^+)^{*n}$ is locally finite which means that, for all $x\in B$ it exists a number $N\in\mathbb{N}$ from which $(I^+)^{*n}(x)=0$. Formally

$$(\forall x\in B)(\exists N\in\mathbb{N})(\forall n\geq N)((I^+)^{*n}(x)=0) \tag{5}$$

then, it is easy to check that the sum

$$S_n(x)=\sum_{m=0}^{n}(-1)^m(I^+)^{*m}(x) \tag{6}$$

is stationnary from N. Taking as $S(x)$ the common value of the $S_n(x)$, one has then the antipode.
It is obvious that, if $x\in B^+$, $S(x)=\sum_{n=1}^{\infty}(-1)^n(I^+)^{*n}(x)$ as $(I^+)^{*0}(x)=convunit(x)=0$ and the explains formula (33) of the paper.
Now, if you have two bialgebras $(B_i,\mu_i,1_{B_i},\Delta_i,\varepsilon_i),\ i=1,2$ with a morphism of bialgebras $\phi\ :\ B_1\to B_2$, this is a morphism for the convolutions in the sense that, if $f_i,g_i\in\mathsf{End}(B_i)$ with

$$f_2\circ\phi=\phi\circ f_1\ ;\ g_2\circ\phi=\phi\circ g_1$$

then $(f_2*_2g_2)\circ\phi=\phi\circ(f_1*_1g_1)$ (almost straigthforward). On the other hand,

$$I_2^+\phi=\phi\circ I_1^+$$

and then,

$$S_2\circ\phi=(\sum_{n=0}^{\infty}(-1)^n(I_2^+)^{*n})\circ\phi=(\sum_{n=0}^{\infty}(-1)^n\phi\circ(I_1^+)^{*n})=\phi\circ(\sum_{n=0}^{\infty}(-1)^n(I_1^+)^{*n})=\phi\circ S_1 \tag{7}$$