Generating series : a combinatorial computation

F.Benmakrouha, C Hespel

We study, in this paper, the reliability and the quality of a model for nonlinear black-box identification. This modelling is of an unknown dynamical system (Σ) by a family (B_k) of bilinear systems. Two formal power series in noncommutative variables are used for describing (Σ) : the generating series for the system's behavior (G) and the Chen series for the system's input. The family (B_k) of bilinear systems is described by its rational generating series (G_k) such that the coefficients of (G) and (G_k) coincide up to order k. We investigate the quality of the model throughout two criteria : a convergence's measure, an amplitude's overestimation of the outputs ($\bar{y}_k$) of systems (B_k). We provide, by a symbolic computation, a valuation relating to the convergence of the family (B_k). This computation is a sum of differential monomials in the input functions and behavior system. We identify each differential monomial with its colored multiplicity and analyse our computation in the light of the free differential calculus.
We propose also a combinatorial interpretation of coefficients of (G_k), according to [13]. These coefficients are powers of an operator Θ which is in the monoid generated by two linear differential operators Δ and Γ.
The n-th power of Θ is equal to the sum of the labels of all forests of colored increasing trees.
Then we propose an overestimation of the output's amplitude of bilinear systems, for a bounded input.
More than a symbolic validation, these computing tools are parameterized by the input and the system's behavior. They can particularly provide a valuation process for rough and oscillating inputs as well as for smooth inputs.

References

[1] F. Benmakrouha, C. Hespel, G. Jacob, E. Monnier *Algebraic Identification algorithm and application to dynamical systems* CASC'2001,The 4th International Workshop on Computer Algebra in Scientific Computing

[2] B.Ninness, G C.Goodwin *Estimation of Model Quality*10th IFAC Symposium on System Identification, Copenhagen July 1994.

[3] J.Sjoberg,Q.Zhang, L.Ljung, A.Benveniste, B.Delyon, P.Y.Glorennec, H.Hjalmarsson, A.Juditsky,

[4] A.Juditsky, H.Hjalmarsson, A.Benveniste, B.Delyon, L.Ljung, J.Sjoberg, Q.Zhang, *Nonlinear black-box modeling in system identification:mathematical foundations*, Automatica, 31, 1995.

[5] Benmakrouha F., Hespel C., Jacob G., Monnier E., *A formal validation of Algebraic Identification algorithm: example of Duffing equation*, IMACS ACA'2000, Saint Petersburg, june 25-28, 2000.

[6] Fliess M., *Fonctionnelles causales non linaires et indtermines non commutatives*, Bull. Soc. Math. France 109, pp. 3-40, 1981.

[7] Fliess M., *Sur certaines familles de séries formelles*, Thèse d'état, Université de Paris 7, 1972.

[8] Hespel C., *Une étude des séries formelles non commutatives pour l'Approximation et l'Identification des systèmes dynamiques*, Thèse d'état, Université de Lille 1, 1998.

[9] Benmakrouha F., Hespel C., Monnier E. "Comparison of Identification Methods based on Fuzzy Systems and on Algebraic Model" *2001 WSES International Conference on Fuzzy sets and Fuzzy Systems*, Tenerife, Feb. 2001.

[10] M. Fliess, M.Lamnabhi, F. Lamnahbi-Lagarrigue *An Algebraic approach to nonlinear functional expansions* IEEE Trans. Circuits and Systems, vol. CAS-30, n^0 8,1983, . 554-570.

[11] I.G. Macdonald, *Symmetric Functions and Hall Polynomials*, 2d ed., Oxford Science Publications, 1995.

[12] G.E. Andrews *The theory of Partitions* Encyclopedia of Mathematics and its applications, Addison-Wesleys, 1984

[13] F. Bergeron,C Reutenauer,*Combinatorial interpretation of the powers of a linear differentiel operator* Rapport de recherche Université du Québec Montréal. Mars 1986.