On approximation of nonlinear generating series by rational series

Mikhail V. Foursov
IRISA/Université de Rennes–I
Campus Universitaire de Beaulieu
35042 Rennes Cedex
France

Christiane Hespel
IRISA/INSA de Rennes
20, avenue des Buttes de Coësmes
35043 Rennes Cedex
France

The causal input/output functionals can be described by a certain noncommutative formal power series: the generating (or Fliess) series. The generating series is a canonical representation of the causal functional, in the sense that different functionals have different generating series. The functional corresponding to a generating series is obtained as a product with another noncommutative power series depending on the input: the Chen series.

If the system of equations defining a causal functional is not known, we may consider it as a black box [3] and identify the coefficients of the generating series from the input/output behavior. It was shown by Hespel and Jacob that it is possible to identify the coefficients of the generating series G using a sufficient number of appropriate correlated input/output sets and their derivatives, up to an arbitrary order k [2].

Once a generating series is identified up to order k, it is possible to construct a rational series of minimal rank that coincides with it up to order k [1]. A rational series corresponds to a bilinear dynamical system that can be constructed using the dependencies between the columns of its Hankel matrix. As a result, the method of Hespel–Jacob allows one to construct a bilinear system that approximates an unknown system with an error of $O(t^k)$.

In general, there is more than one rational series of minimal rank that coincides with a given nonlinear generating series up to order k. In this article, we propose to reduce as much as possible the problem of choosing one of those rational series, for the case of systems with a single input. The main idea is to use the partial information about the coefficients of orders greater then k that was obtained during the identification. Indeed, during the modeling step, one uses only the values of the coefficients of orders up to k. However, some of linear combinations of the coefficients of higher order were also identified at the identification step. We propose thus an algorithm that uses this additional information in order to give the rational series that fits best to the known data. In the cases when the series is rational of rank r such that the output derivatives of orders up to $2r-1$ were used during the identification, we show that this rational series can be uniquely determined.

References

[1] C. Hespel and G. Jacob, *Approximation of nonlinear dynamic systems by rational series*, Theor. Comp. Sci. **79** (1991), 151–162.

[2] C. Hespel and G. Jacob, *On algebraic identification of causal functionals*, Discrete Math. **225** (2000), 173–191.

[3] A. Juditsky, H. Hjalmarsson, A. Benveniste, B. Delyon, L. Ljung, J. Sjöberg, and Q. Zhang, *Nonlinear black-box models in system dentification: mathematical foundations*, Automatica **31** (1995), 1725–1750.