Drawing approximated solution curve of differential equation

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Abstract

We develop a method for drawing approximated solution curves of differential equations. This method is based on the juxtaposition of local approximating curves on successive intervals $[t_i, t_{i+1}]_{0 \leq i \leq n-1}$.
The differential equation, considered as a dynamical system, is described by its state equations and its initial value at $t = t_0$.
A generic expression of its generating series G_t truncated at any order k, of the output and its derivatives $y^{(j)}(t)$ expanded at any order k, can be calculated. These expressions are obtained from the vector fields, from the observation of the state at time t, in the state equations [2, 3].
More precisely, the coefficients of G_t are expressed in terms of the vector fields and the observation. The output and its derivatives $y^{(j)}(t)$ are expressed in terms of the coefficients of the series G_t and of the Chen series [1].
At every initial point of the present interval, we specify the previous expressions of G_t and $y^{(j)}(t)$ for $t = t_i$. Then we obtain an approximated output $y(t)$ at order k in every interval $[t_i, t_{i+1}]_{0 \leq i \leq n-1}$.
By using Maple system, we have developed a package corresponding to the creation of the generic expression of G_t and $y^{(j)}(t)$ at order k and to the drawing of the local curves on every interval $[t_i, t_{i+1}]_{0 \leq i \leq n-1}$, by iterations on the initial points $t = (t_i)_{0 \leq i \leq n-1}$.

References

[1] Chen K.T., *Iterated path integrals*, Bull. Amer. Math. Soc. 83, pp. 831-879, 1977.

[2] Fliess M., *Fonctionnelles causales non linéaires et indéterminés non commutatives*, Bull. Soc. Math. France 109, pp. 3-40, 1981.

[3] Hespel C., *Une étude des séries formelles non commutatives pour l'Approximation et l'Identification des systèmes dynamiques*, Thèse d'état, Université de Lille 1, 1998.