

Triple Birth of the New Quantum Theory

Now a remarkable report on the end of 25 years of confusion. During the twelve month period from June 1925 to June 1926, not one, not two, but **three** distinct and independent developments of a complete quantum theory were published . . . and then shown to be equivalent.



**THE FIRST —
MATRIX MECHANICS —
BY WERNER HEISENBERG.**



**THE SECOND —
WAVE MECHANICS —
BY ERWIN SCHRÖDINGER**



**THE THIRD —
QUANTUM ALGEBRA —
BY PAUL DIRAC.**

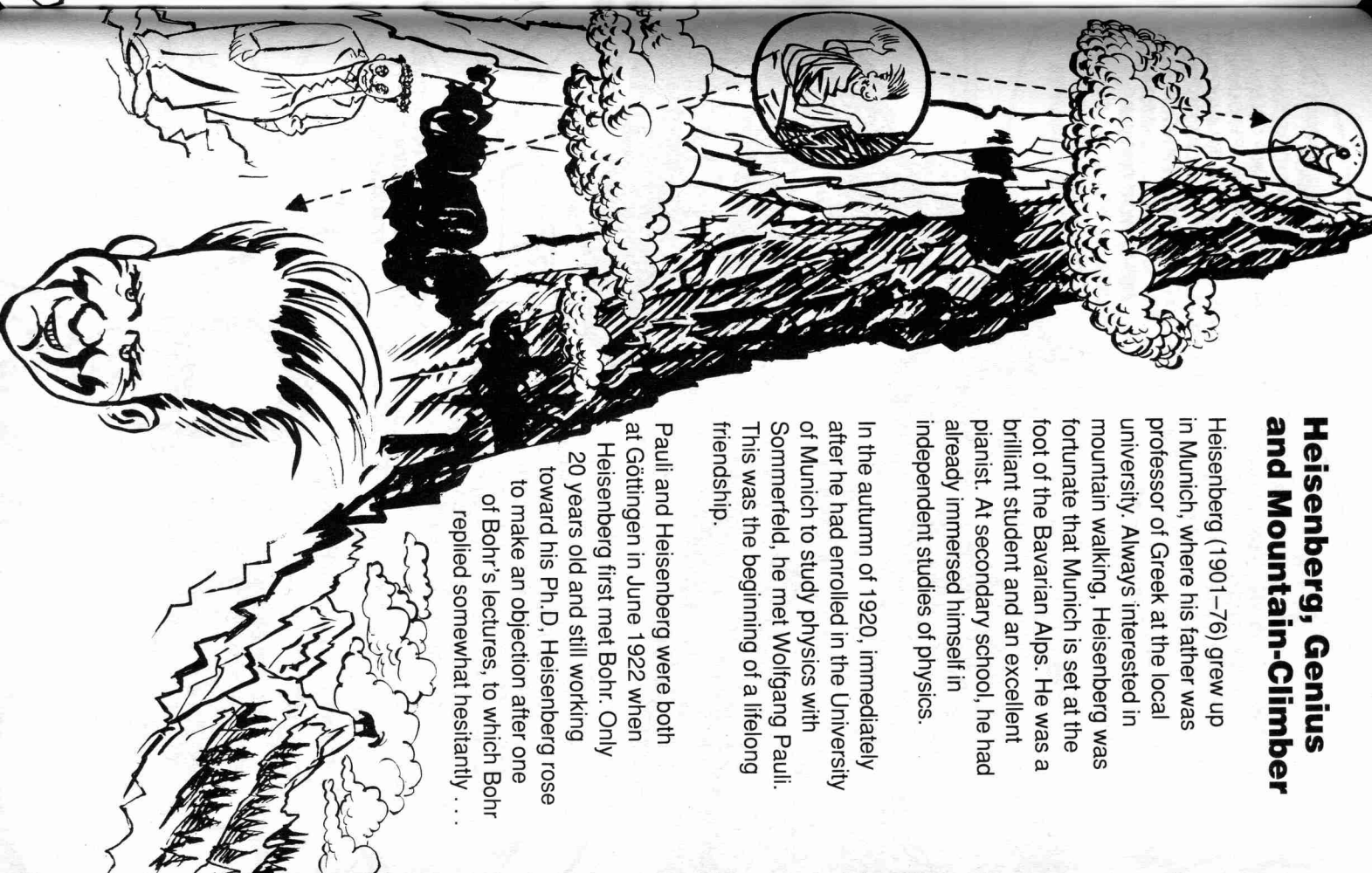
The following pages will outline how these discoveries were made and the context which made them possible. The story begins with Bohr and his new protégé, Werner Heisenberg.

Heisenberg, Genius and Mountain-Climber

Heisenberg (1901–76) grew up in Munich, where his father was professor of Greek at the local university. Always interested in mountain walking, Heisenberg was fortunate that Munich is set at the foot of the Bavarian Alps. He was a brilliant student and an excellent pianist. At secondary school, he had already immersed himself in independent studies of physics.

In the autumn of 1920, immediately after he had enrolled in the University of Munich to study physics with Sommerfeld, he met Wolfgang Pauli. This was the beginning of a lifelong friendship.

Pauli and Heisenberg were both at Göttingen in June 1922 when Heisenberg first met Bohr. Only 20 years old and still working toward his Ph.D., Heisenberg rose to make an objection after one of Bohr's lectures, to which Bohr replied somewhat hesitantly . . .



AT THE
END OF THE LECTURE, BOHR
CAME OVER AND ASKED ME TO
JOIN HIM THAT AFTERNOON
FOR A WALK OVER THE
HAINBERG MOUNTAIN.

THIS WALK
WAS TO HAVE PROFOUND
REPERCUSSIONS ON MY
SCIENTIFIC CAREER. PERHAPS IT
IS MORE CORRECT TO SAY THAT
MY REAL SCIENTIFIC CAREER ONLY
STARTED THAT AFTERNOON WHEN
BOHR TOLD ME... ATOMS
WERE NOT THINGS!

WE TALKED
FOR ABOUT THREE HOURS.
AND FOR THE FIRST TIME I SAW THAT
ONE OF THE FOUNDERS OF QUANTUM
THEORY WAS DEEPLY WORRIED BY
ITS DIFFICULTIES. BOHR HAD IMMENSE
SIGHT, A RESULT NOT OF MATHEMATICAL
ANALYSIS BUT OF OBSERVATION OF
THE ACTUAL PHENOMENA.

HE
COULD SENSE
A RELATIONSHIP INTUITIVELY
RATHER THAN DERIVE IT
FORMALLY.

HEISENBERG
UNDERSTANDS EVERYTHING.
NOW THE SOLUTION IS IN HIS
HANDS. HE MUST FIND A WAY
OUT OF THE DIFFICULTY
OF THE QUANTUM
THEORY.

After returning from this walk,
Bohr told friends about Heisenberg...

Evidently, Bohr had quickly
recognized Heisenberg as
a young physicist of
exceptional gifts.

But Heisenberg had a surprise for Niels Bohr.
He hated the imaginary electron orbits in Bohr's
atomic model...

THEY
COULD NEVER BE
OBSERVED. WHAT GOOD IS
IT TO SPEAK OF INVISIBLE
ELECTRON PATHS INSIDE
INVISIBLE TINY ATOMS?

IF
AN ATOM CAN'T
BE SEEN, THEN IT IS
NOT A MEANINGFUL
CONCEPT.

In the spring of 1925, he left
Copenhagen and returned to
Göttingen where Max Born
(1882-1970) had made him a
privatdozent at the age of only 22!
In Germany he was bothered by two
major irritants: the pollen in the air
and the problem of the atomic orbits.

I HAD
THIS VERY BAD
ATTACK OF HAY FEVER. I
COULDN'T EVEN
SEE.

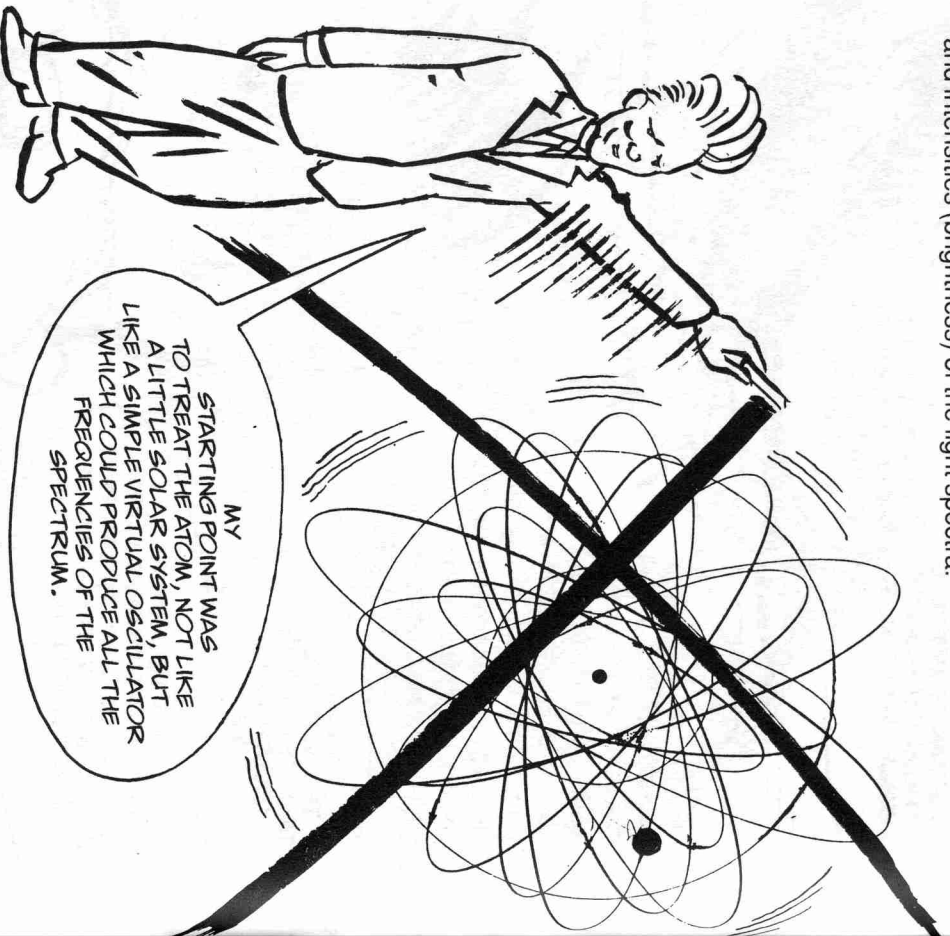
I WAS IN
A TERRIBLE STATE AND DECIDED
TO SEEK BETTER, I.E. POLLEN-FREE,
AIR. I LEFT FOR THE ISLAND OF
HELGOLAND IN THE NORTH
SEA.

I WAS
EXTREMELY TIRED WHEN I
ARRIVED AND MY WHOLE FACE
WAS SWOLLEN. THE LANDLADY
AT AN INN ASKED IF I HAD BEEN
BEATEN BY SOMEBODY.



Heisenberg's Picture of the Atom

Heisenberg hardly slept, dividing his time between inventing quantum mechanics, climbing rocks and memorizing poems by Goethe. He was attempting to work out a **code** for connecting the quantum numbers and energy states in an atom with the experimentally determined frequencies and intensities (brightness) of the light spectra.

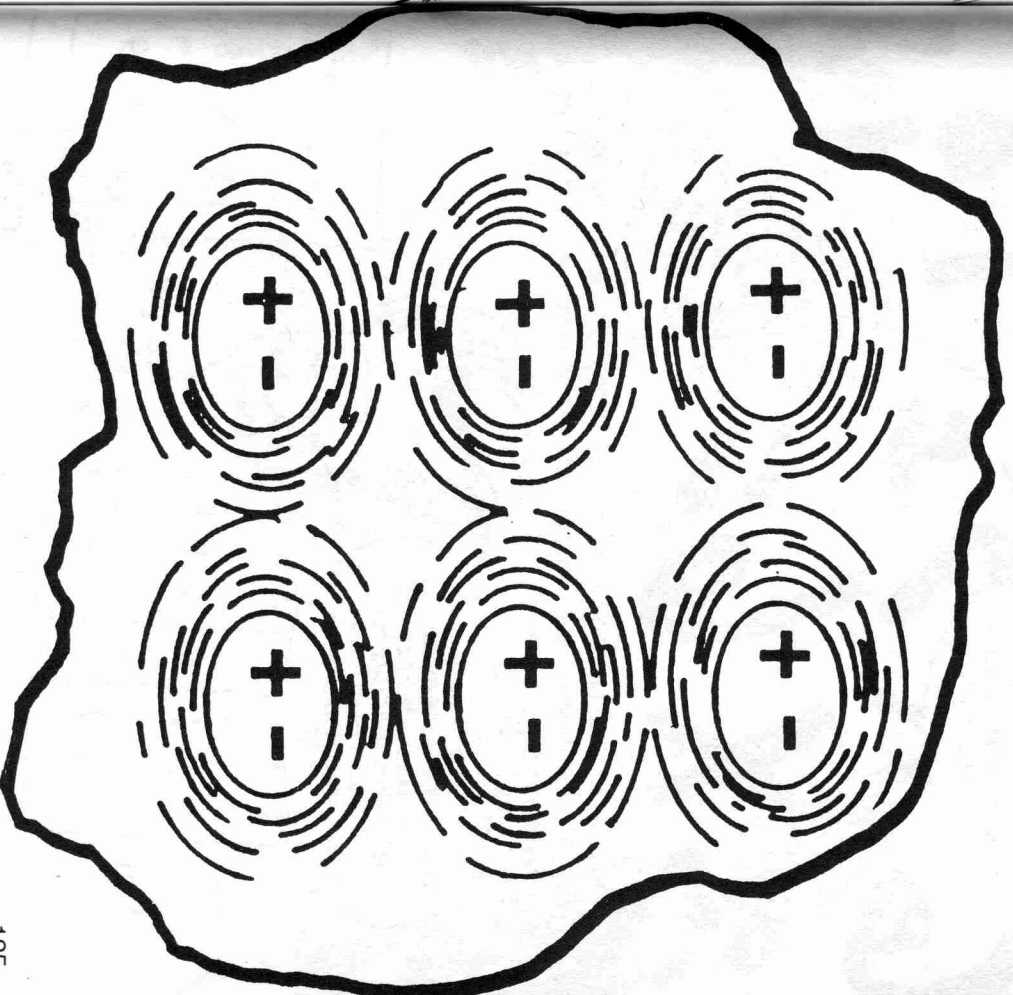


This was similar to what Planck had done on black-body radiation in 1900.

Using the concept which Bohr had called the *correspondence principle*, (where quantum and classical regions overlap), Heisenberg imagined the Bohr atom at very large orbits. There the orbital frequency would equal the radiation frequency and the atom would be like a simple linear oscillator.

He knew how to analyse this problem from classical physics. Familiar quantities like the linear momentum (p) and the displacement from equilibrium (q) could now be used. Classically, he could solve the equation of motion, then calculate the energy of the particle in the state n , the quantized values, E_n .

From the largest orbit – where he could get answers – he then tried to extrapolate *inside* the atom. Here his intuition, some would call it genius, led him to a formula for including all the possible states. **He had broken the spectral code.**



At this point, knowing he was close to something quite new, Heisenberg made a startling discovery.

IN CLASSICAL THEORY THE PRODUCT P TIMES Q ALWAYS EQUALS THE REVERSE, Q TIMES P . . .

BUT IN QUANTUM THEORY, THIS IS NOT NECESSARILY THE CASE.

He was much troubled by this property, which violates the fundamental commutative law for multiplication.

THIS IS A VERY DISAGREEABLE SITUATION AND I AM TERRIBLY WORRIED THAT P Q DOES NOT EQUAL Q P!

$$pq \neq qp$$

In order to obtain the correct frequencies and intensities of the spectral lines for his theory, Heisenberg somehow had to include the quantum postulate, as did Bohr.

I GUESSED THAT THE DIFFERENCE . . . Pq - qp WAS NOT ZERO BUT EQUAL TO $wz\pi i$, WHERE i IS $\sqrt{-1}$, AN IMAGINARY NUMBER.

| | | | | | |
|---|---|---|---|---|---|
| 1 | 3 | 5 | 3 | 5 | 4 |
| 2 | 5 | 1 | X | 1 | 1 |
| 4 | 3 | 2 | | 2 | 3 |

we get:

| | | |
|----|----|----|
| 16 | 23 | 32 |
| 13 | 18 | 18 |
| 19 | 29 | 29 |

we get:

| | | |
|---|---|---|
| 3 | 5 | 4 |
| 1 | 3 | 5 |

| | | |
|----|----|----|
| 29 | 46 | 28 |
|----|----|----|

That very night on Helgoland, he was able to show that the energy states were **quantized** and **time independent**, i.e. they were stationary as in the Bohr atom. He later called this . . .

. . . A GIFT FROM HEAVEN.

IT WAS ABOUT THREE O'CLOCK AT NIGHT WHEN THE FINAL RESULT OF THE CALCULATION LAY BEFORE ME. AT FIRST I WAS DEEPLY SHAKEN, SO EXCITED THAT I COULD NOT THINK OF SLEEP.

SO I LEFT THE HOUSE AND AWAITED THE SUNRISE ON TOP OF A ROCK. THAT WAS "THE NIGHT OF HELGOLAND".

On 19 June, Heisenberg returned to Göttingen and sent his results to Pauli, the invaluable critic. If his theory was correct, he had taken a first step towards killing the concept of orbits. He was now almost fully recovered from both his illnesses. . . . the hay fever and the electron orbits!

Max Born and Matrix Mechanics

Pauli's reaction was favourable. So before setting off for a visit to the Cavendish Laboratory in Cambridge and a walking holiday, Heisenberg set the paper before Max Born.



And with this, **matrix mechanics** was born – or perhaps one should write “Born”. Working with a talented student, **Pascual Jordan** (1902–80) – an expert on matrix methods – Born transposed Heisenberg's theory into a systematic *matrix language*.

Now the frequencies of the optical spectrum could be represented by an infinite matrix which looks like this . . .

| | | | | | | | |
|-----------|----------|----------|----------|----------|----------|----------|------|
| $f_{m,n}$ | f_{11} | f_{12} | f_{13} | f_{14} | f_{15} | f_{16} | etc. |
| | f_{21} | f_{22} | f_{23} | f_{24} | f_{25} | f_{26} | etc. |
| | f_{31} | f_{32} | f_{33} | f_{34} | f_{35} | f_{36} | etc. |
| | f_{41} | f_{42} | f_{43} | f_{44} | f_{45} | f_{46} | etc. |
| | etc. | etc. | etc. | etc. | etc. | etc. | etc. |

Since Heisenberg's idea was that the individual oscillators with momentum $p(t)$ and displacement $q(t)$ vibrate with these frequencies, they will also be infinite matrices.

| | | | | | | | |
|-------|----------|----------|----------|----------|------|----------|----------|
| $p =$ | p_{11} | p_{12} | p_{13} | p_{14} | etc. | | |
| | p_{21} | p_{22} | p_{23} | p_{24} | etc. | and | |
| | p_{31} | p_{32} | p_{33} | p_{34} | etc. | | |
| | etc. | etc. | etc. | etc. | etc. | | |
| | | | | | | q | |
| | | | | | | q_{11} | q_{12} |
| | | | | | | q_{21} | q_{22} |
| | | | | | | q_{31} | q_{32} |
| | | | | | | q_{41} | q_{42} |
| | | | | | | etc. | etc. |
| | | | | | | etc. | etc. |

Heisenberg's quantum postulate was introduced to obtain the correct frequencies and intensities, each represented by a set of two numbers in *matrix form*.

$pq - qp = (h/2\pi i) I$ (quantum condition)

I is the unit matrix which looks like this . . .

| | | | | |
|-------|------|------|------|------|
| $I =$ | 1 | 0 | 0 | etc. |
| | 0 | 1 | 0 | etc. |
| | 0 | 0 | 1 | etc. |
| | etc. | etc. | etc. | etc. |