



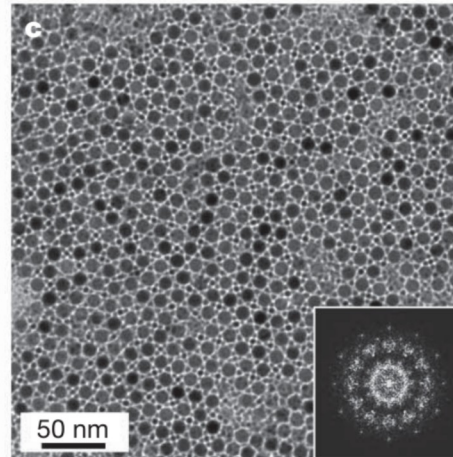
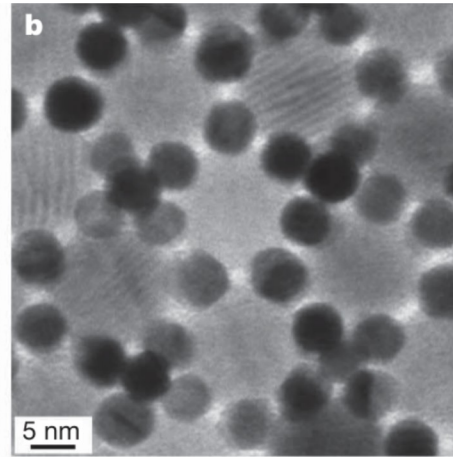
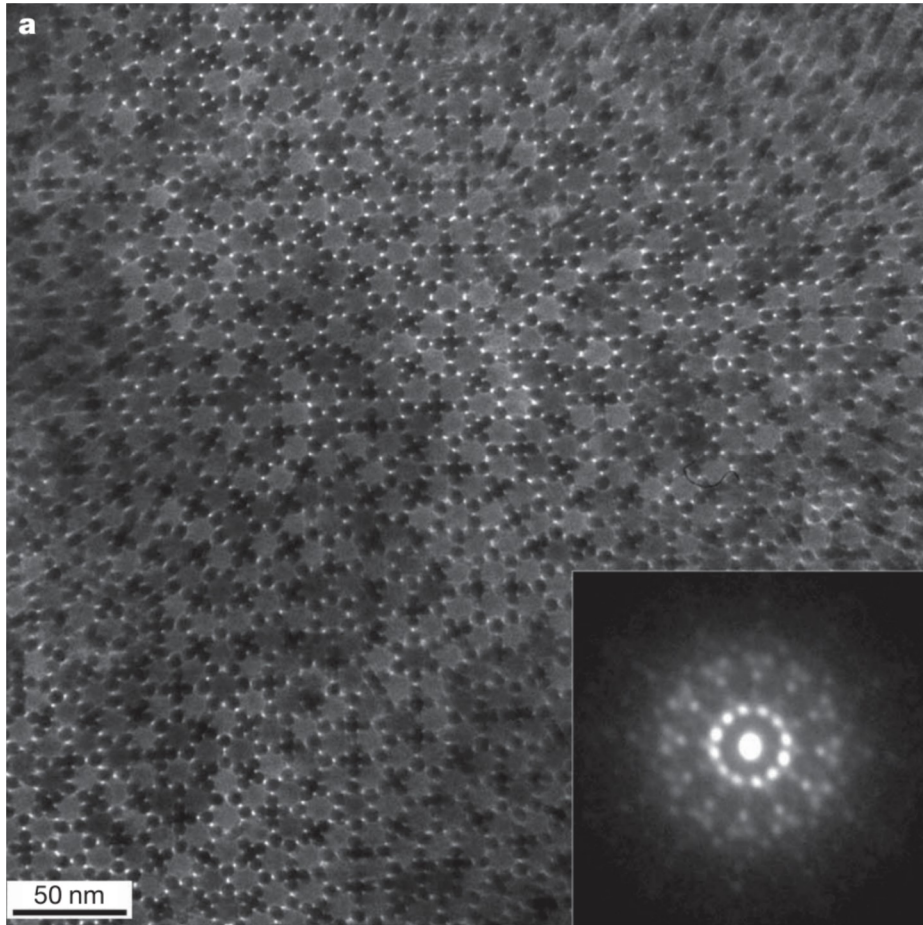
# Self assembly of binary hard discs : towards quasicrystals

Etienne FAYEN, PhD student at Laboratoire de Physique des Solides

# Outline

- Our system : binary hard discs mixtures
- How to build the phase diagram ?
- Floppy Box Monte Carlo simulations
- Relevance to quasicrystals
- Extension to non-additive hard discs

# Why are there stable QC ?



Nanoparticles  
self-assembly

Talapin et al.,  
NatLett, **461**,  
964-967, 2009

# Why are there stable QC ?

# The SoftQC ANR team



Anuradha Jagannathan  
(Theory)



Benjamin Abécassis  
(NP synthesis/assembly)



Brigitte Pansu  
(NP synthesis/assembly)



Chiara Moretti  
(NP synthesis/assembly)



Claire Goldmann  
(NP synthesis)



Giuseppe Foffi  
(Theory/simulations)



Jean-François Sadoc  
(Theory)



Marianne Impérator  
(X-rays)



Frank Smallenburg  
(Theory/simulations)



Etienne Fayen  
(Theory/simulations)

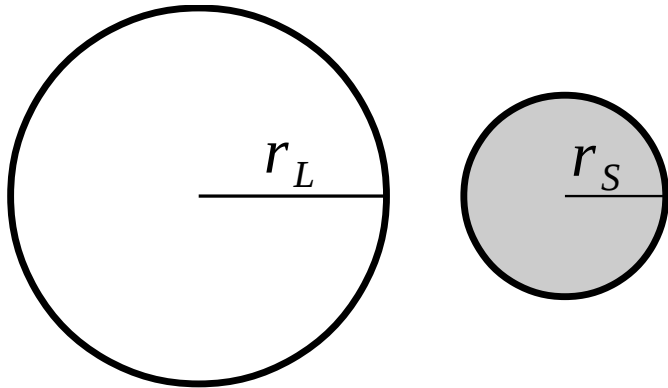
# Why are there stable QC ?

Quasiperiodicity seems favored by **interplay of length scales**

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Binary hard-disc mixtures



Size ratio :  $q = \frac{r_S}{r_L}$

Concentration of small discs :  $p = \frac{n_S}{n_S + n_L}$

Why are there **stable** QC ?

The diagram illustrates the Helmholtz free energy equation,  $F = U - T \times S + P \times V$ . Each variable in the equation is linked to a physical quantity by an arrow:

- Free energy** points to  $F$ .
- Energy** points to  $U$ .
- Temperature** points to  $T$ .
- Entropy** points to  $S$ .
- Pressure** points to  $P$ .
- Volume** points to  $V$ .

The equation is written as  $F = U - T \times S + P \times V$ .

Why are there **stable** QC ?

The diagram shows the equation  $F = U - T \times S + P \times V$ . Arrows point from labels to terms: 'Free energy' to  $F$ , 'Energy' to  $U$ , 'Temperature' to  $T$ , 'Entropy' to  $S$ , 'Pressure' to  $P$ , and 'Volume' to  $V$ . A red diagonal line is drawn through the  $U$  term.

$$F = U - T \times S + P \times V$$

Hard interactions



Why are there **stable** QC ?

Free energy

Energy

Temperature

Entropy

Pressure

Volume

$$F = U - T \times S + P \times V$$

Hard interactions

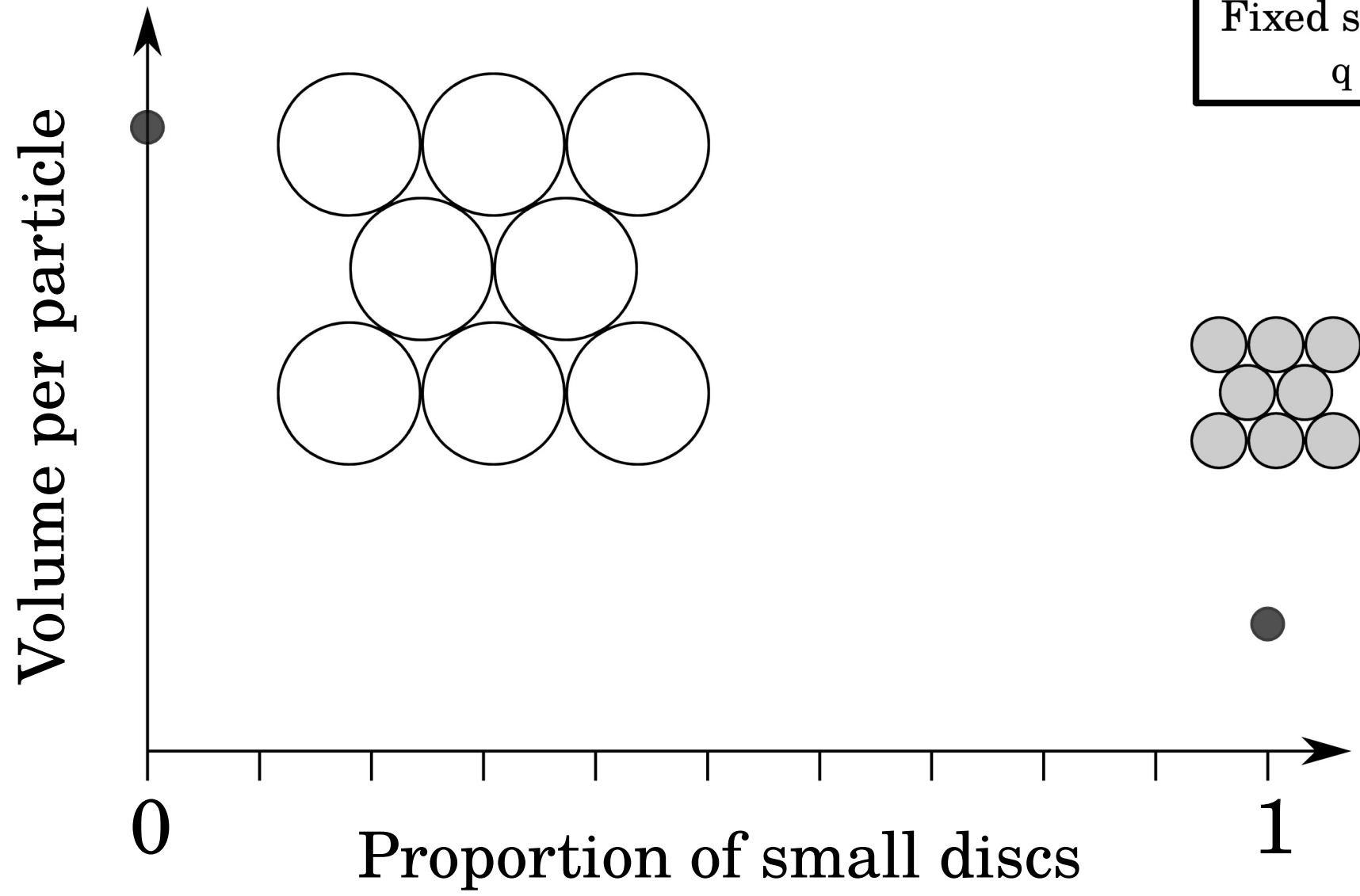


Infinite pressure limit

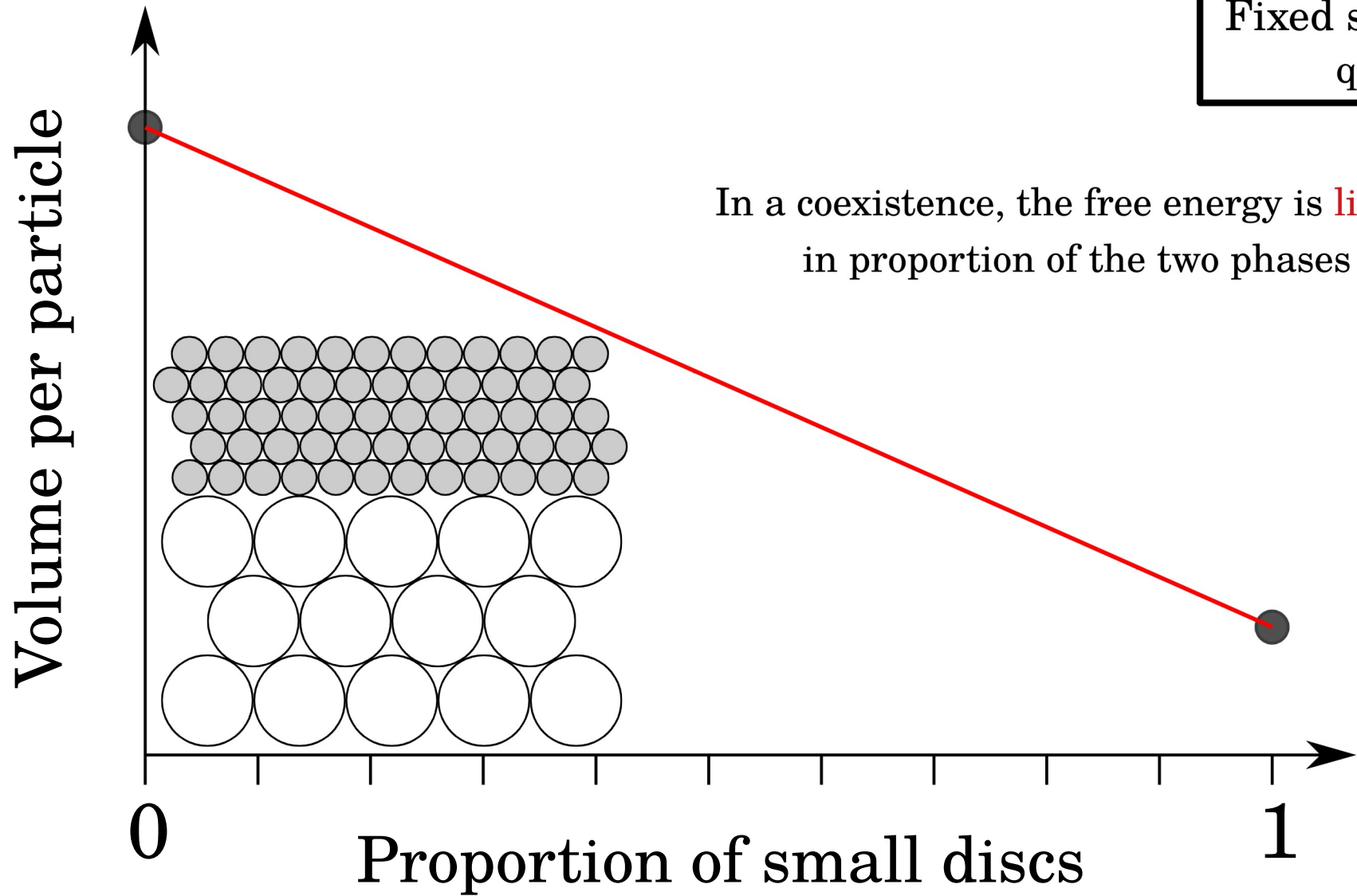
$$\frac{F}{N} = P \times \frac{V}{N}$$

Stable means optimal packing

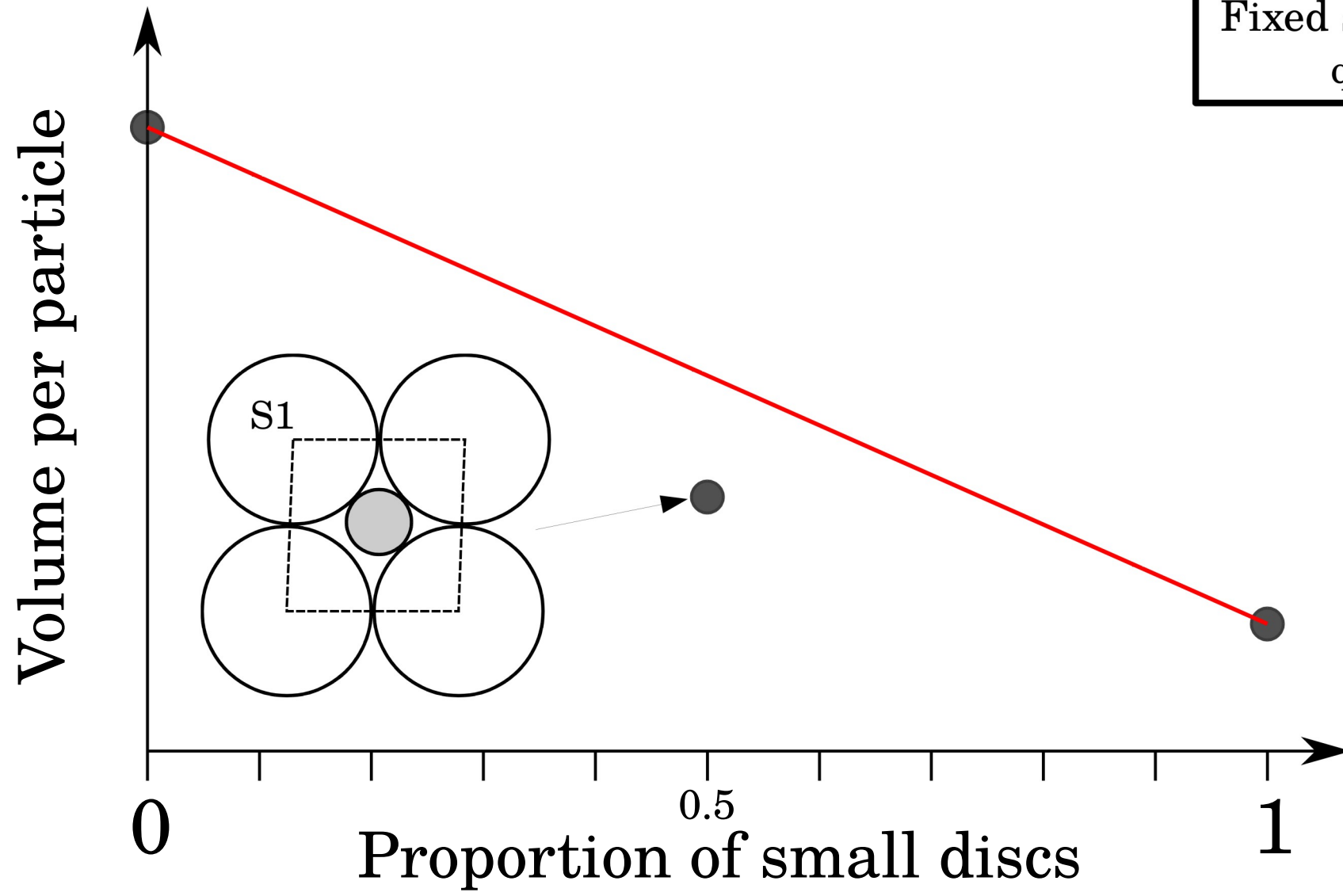
Fixed size ratio  
 $q = 0.4$



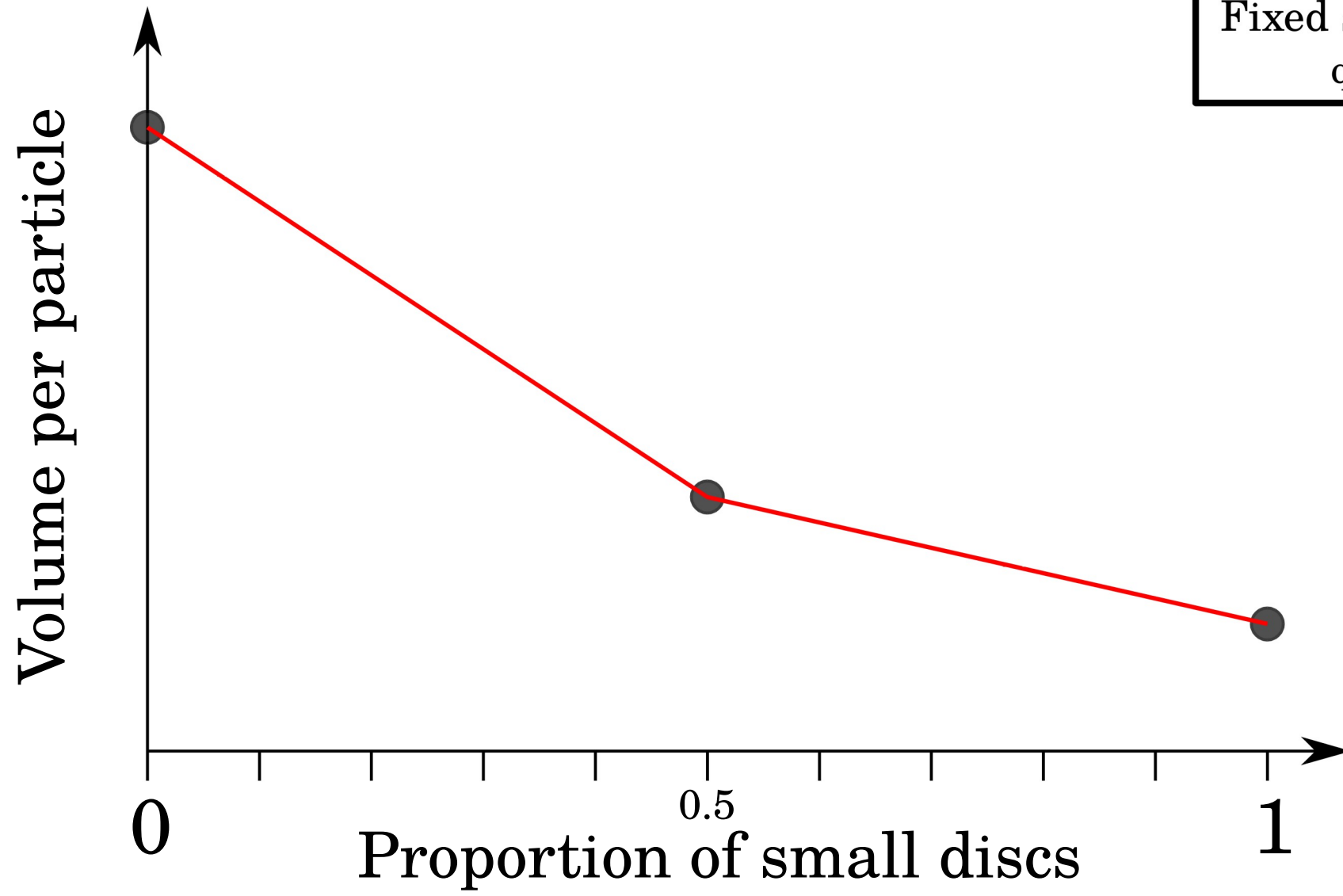
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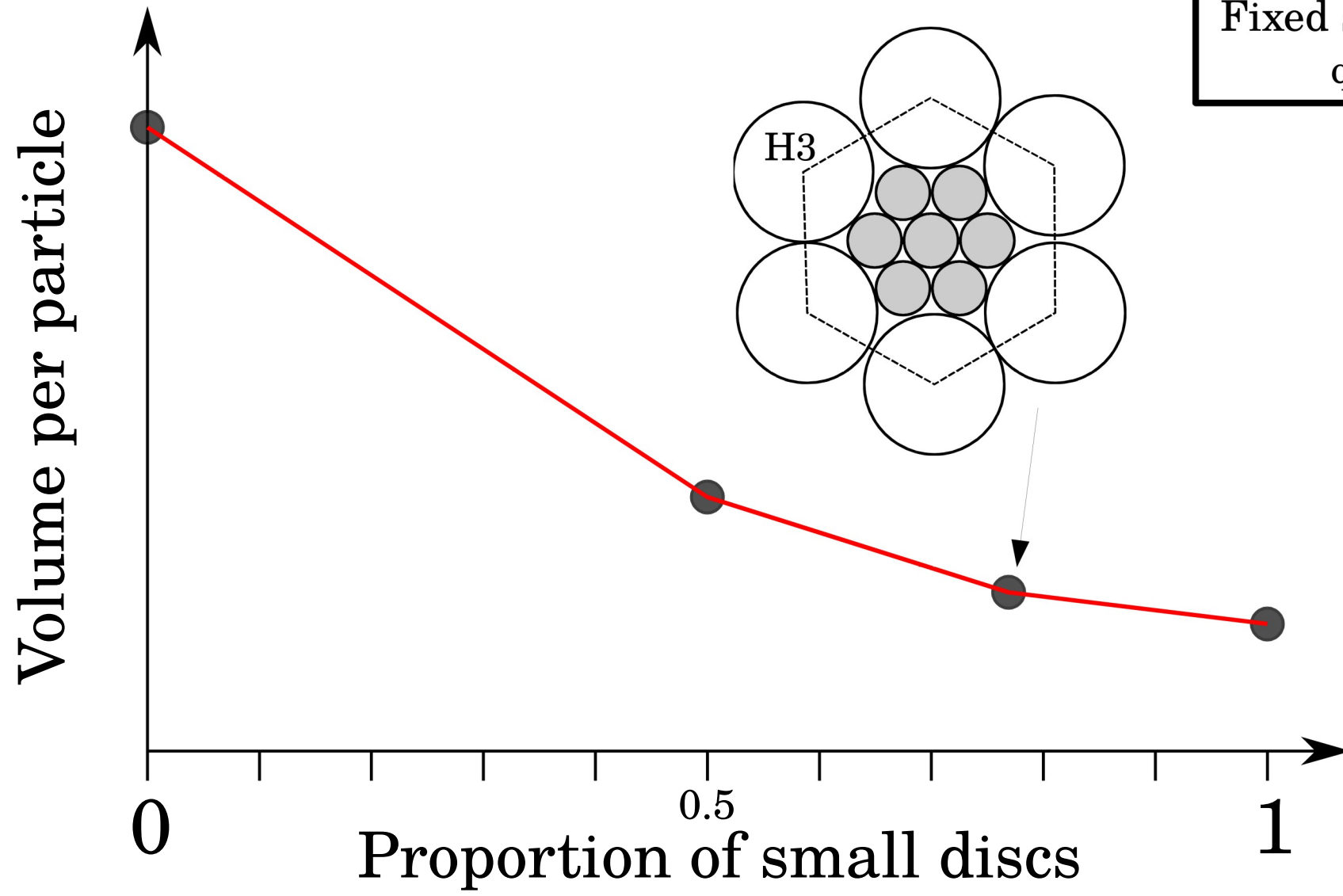
Fixed size ratio  
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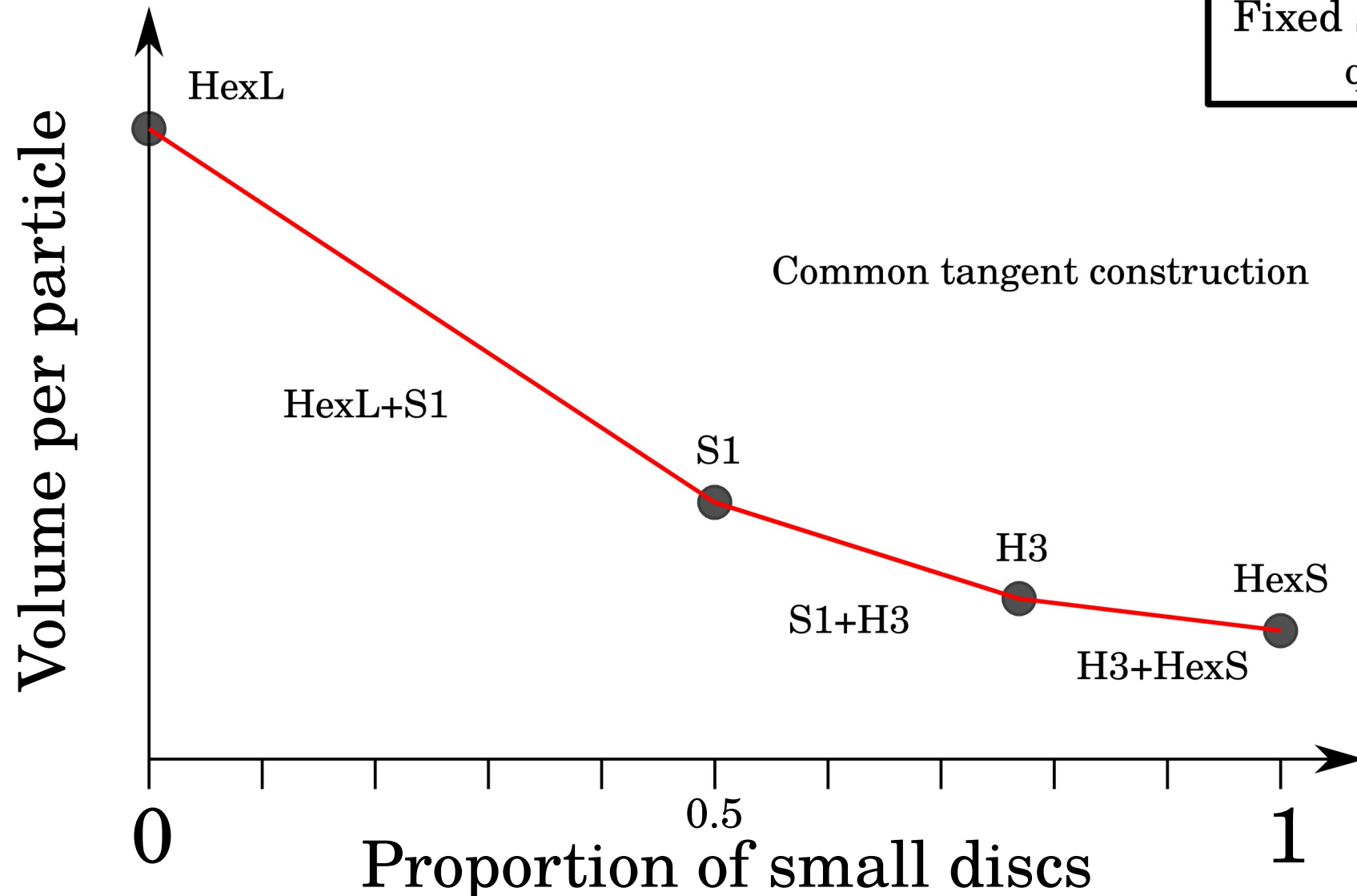
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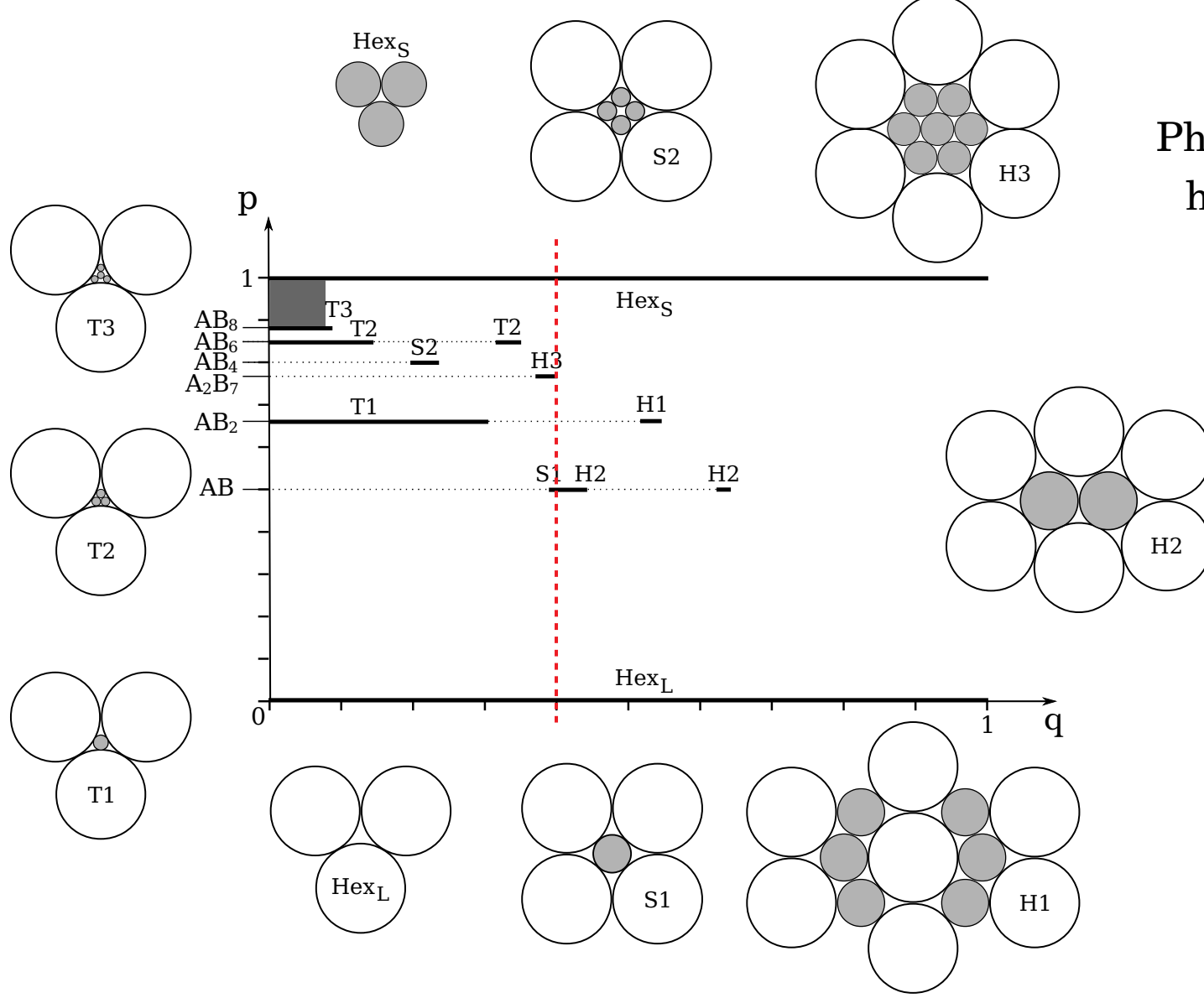


Fixed size ratio  
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Phase diagram for binary  
hard discs mixtures at  
infinite pressure

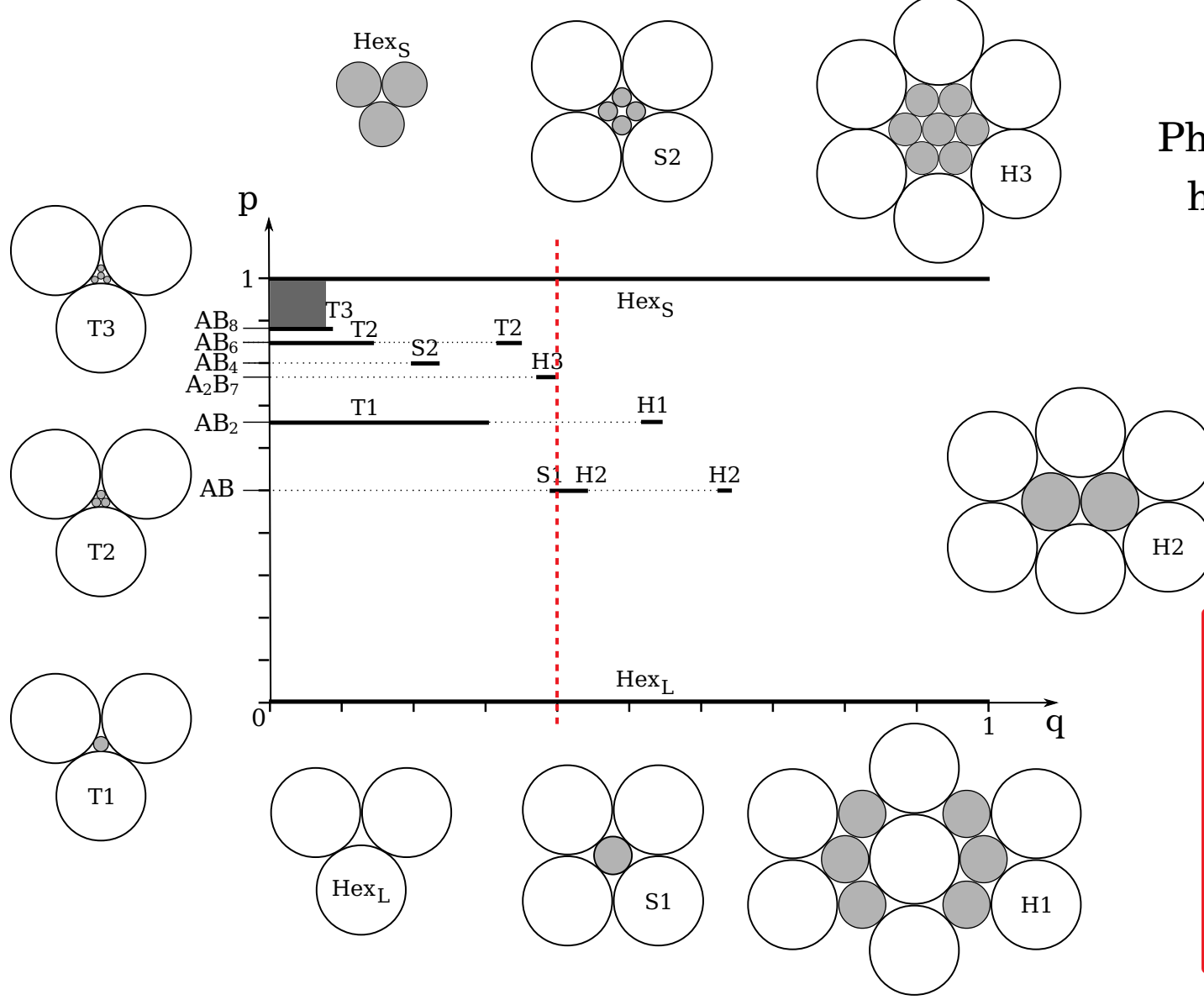
C.N. Likos, C.L. Henley,  
PhiloMagB, **68**, 85-113, 1993





Phase diagram for binary  
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C.N. Likos, C.L. Henley,  
PhiloMagB, **68**, 85-113, 1993



How can we  
sample candidate  
structures in a  
systematic way ?

# Floppy Box Monte Carlo (FBMC) simulations

Simulate only the unit cell of a crystal (periodic BC)

Let particles position and box shape fluctuate

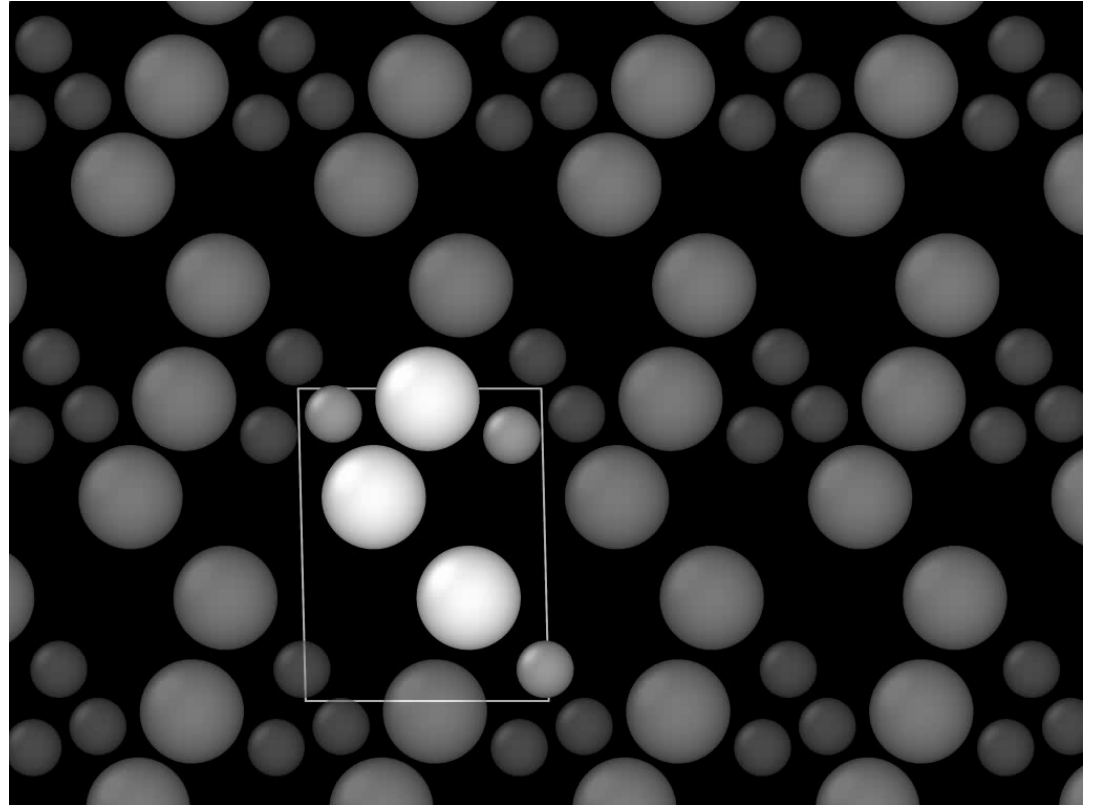
- Start from a dilute system
- Compress slowly
- Quench to infinite pressure
- Look at the resulting structure

# Floppy Box Monte Carlo (FBMC) simulations

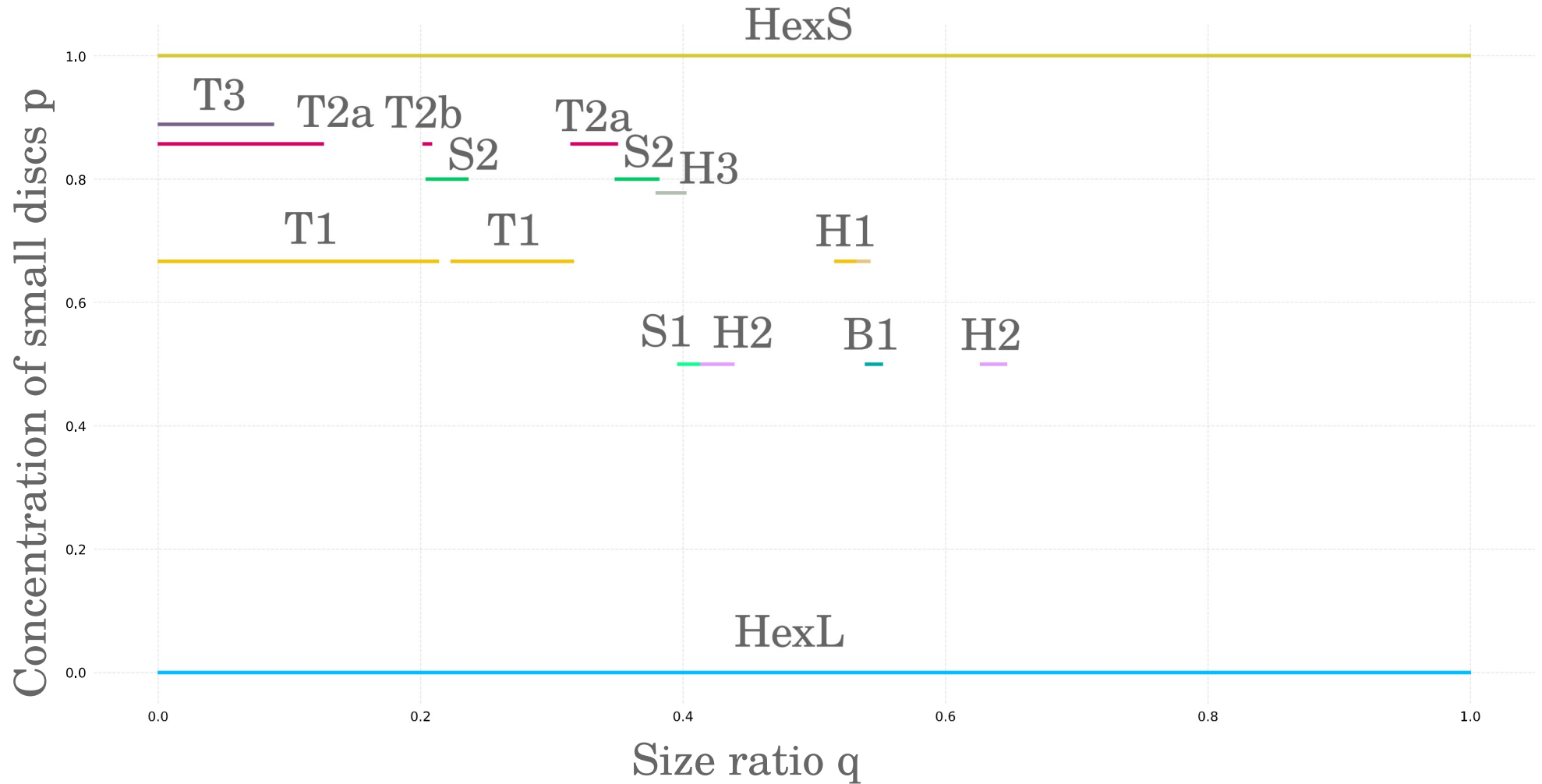
Simulate only the unit cell of a crystal (periodic BC)

Let particles position and box shape fluctuate

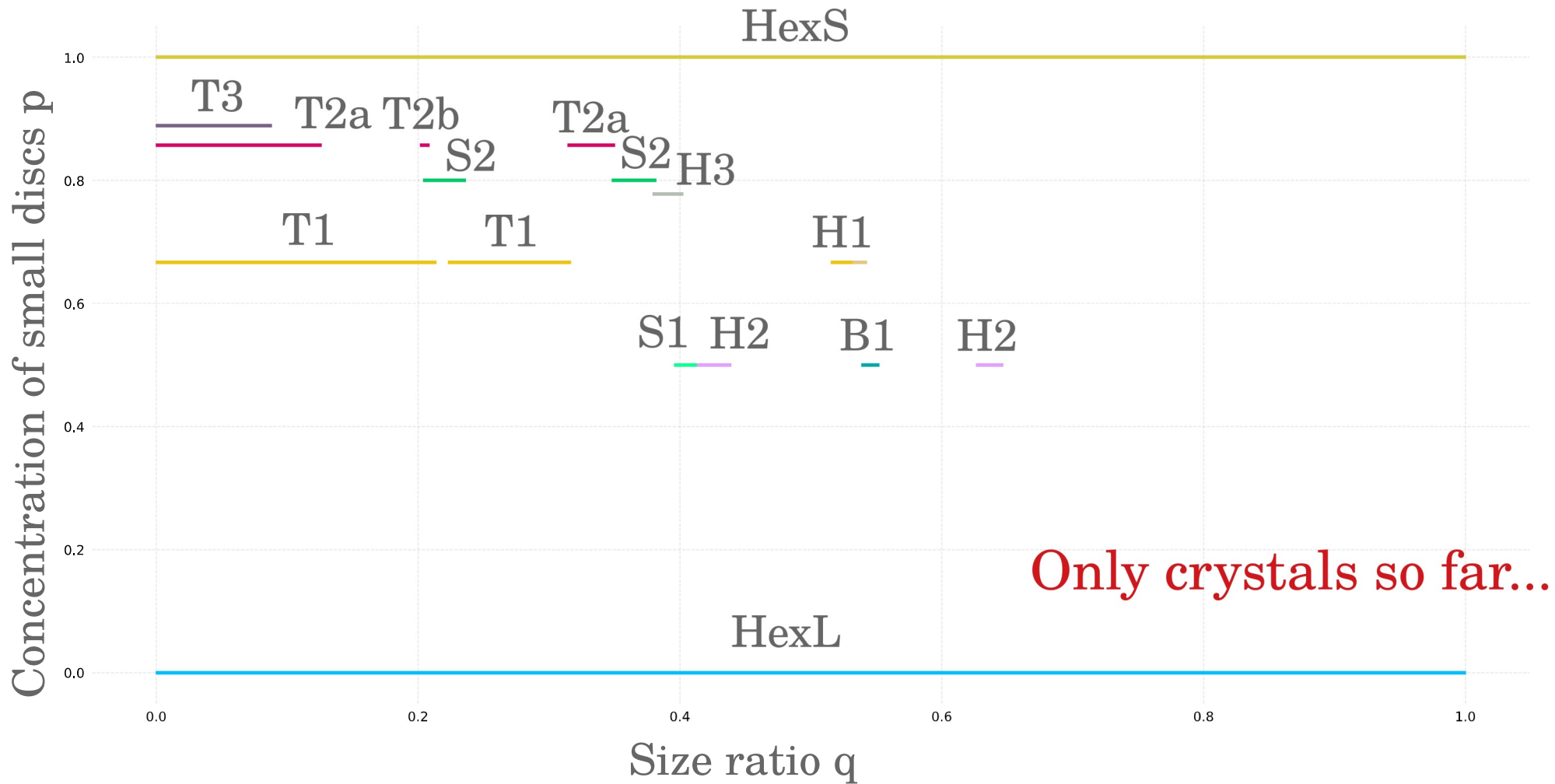
- Start from a dilute system
- Compress slowly
- Quench to infinite pressure
- Look at the resulting structure
- Start again !



# Updated phase diagram of binary hard discs



# Updated phase diagram of binary hard discs

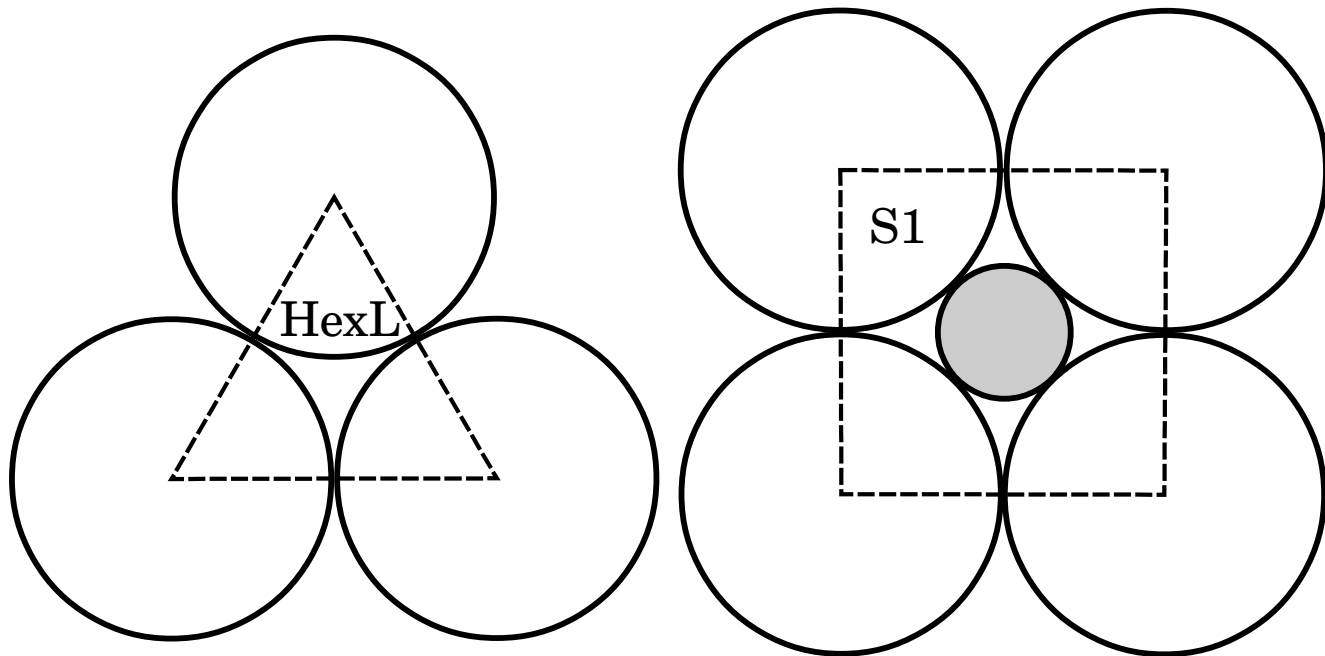


# Relevance to quasicrystals

FBMC can only sample periodic structures

Rely on **random tilings**, ie. coexisting phases that :

- Have no boundary cost

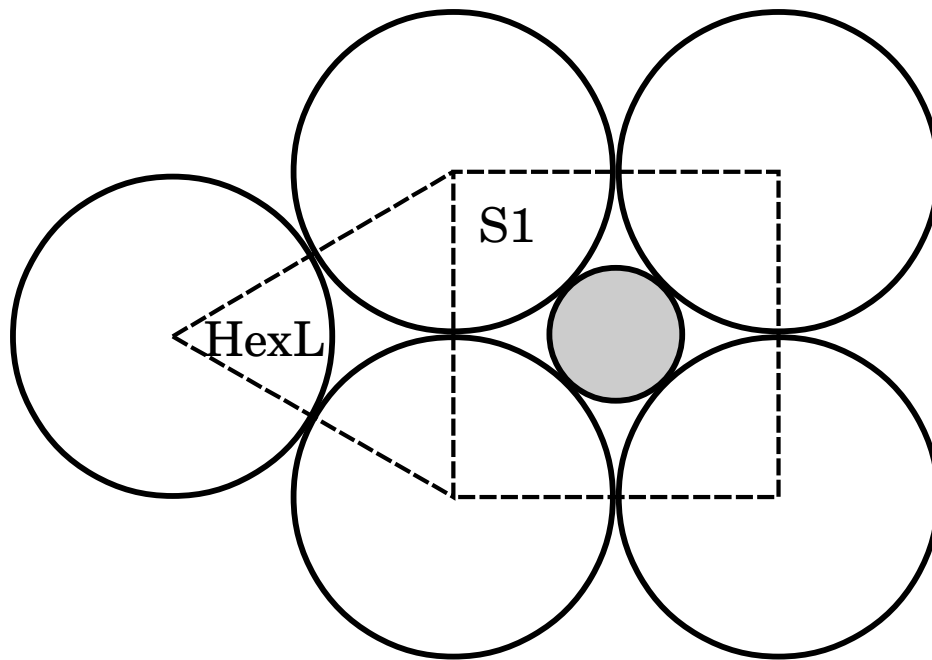


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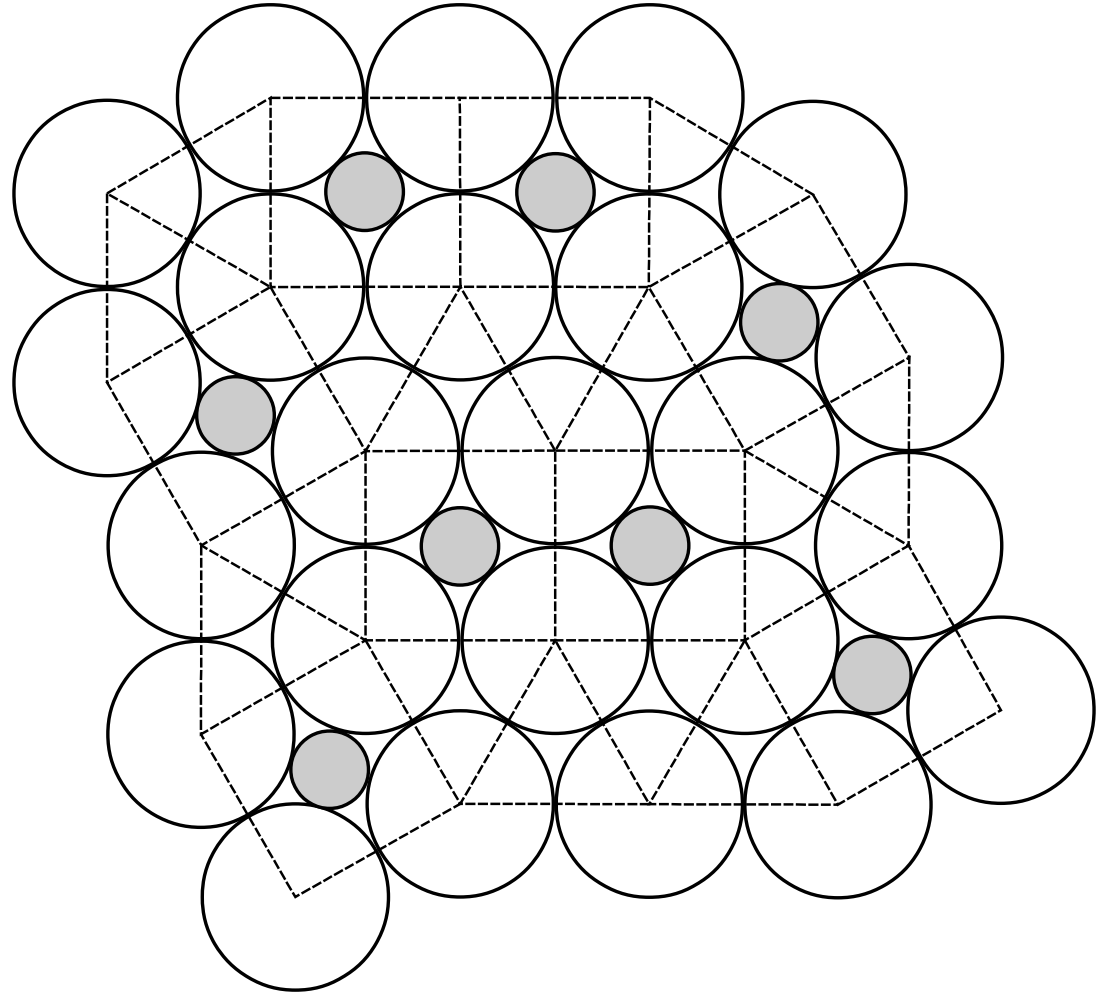


# Relevance to quasicrystals

FBMC can only sample  
periodic structures

Rely on **random tilings**, ie.  
coexisting phases that :

- Have no boundary cost
- Tile the plane





# Relevance to quasicrystals

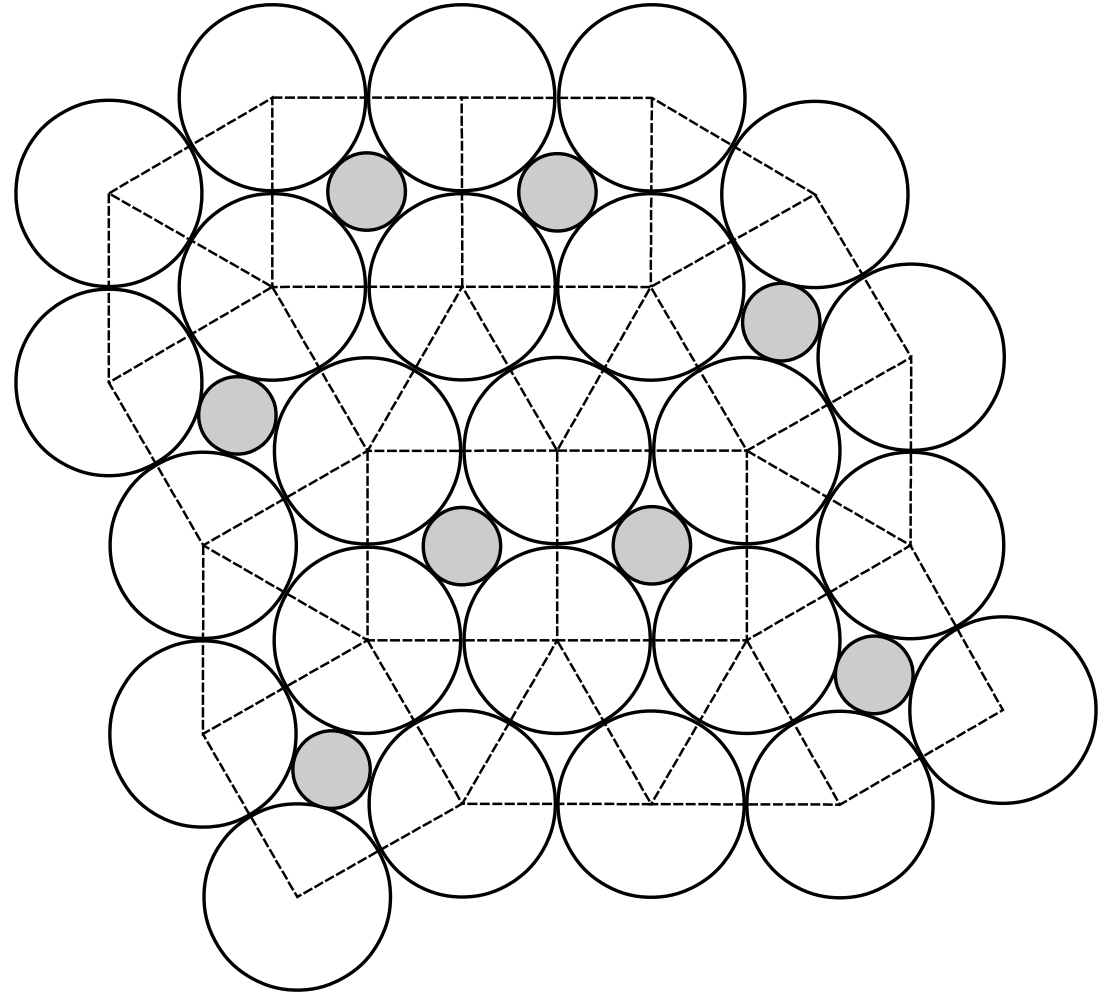
When  $\frac{N_{square}}{N_{triangle}} = \frac{\sqrt{3}}{4}$

Random tiling quasicrystal

H. Kawamura, ProgThPhys, **70**, 1983

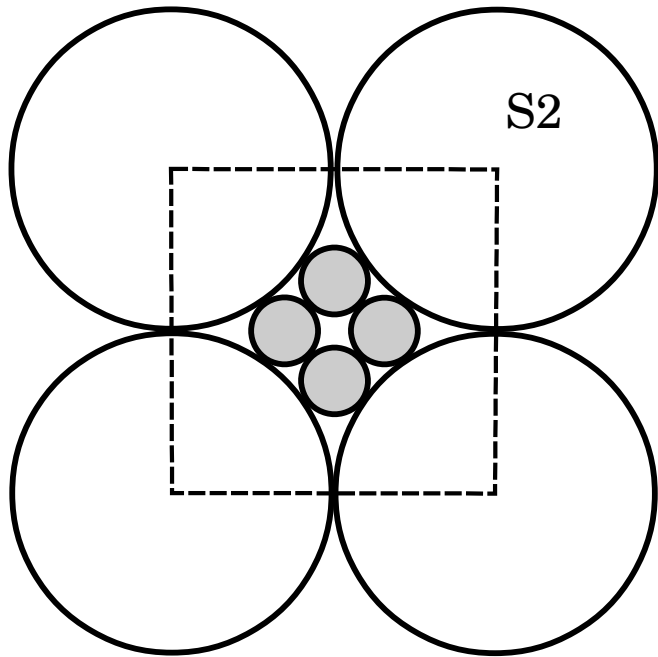
M. Oxborrow, C.L. Henley, PRB, **48**, 1993

M. Widom, PRL, **70**, 1993

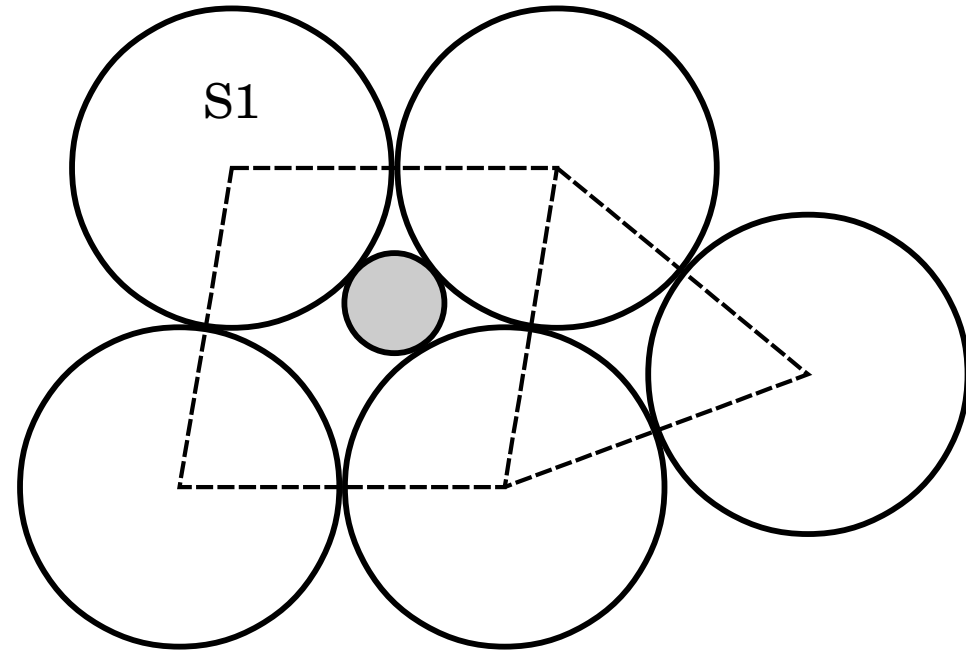


# Relevance to quasicrystals

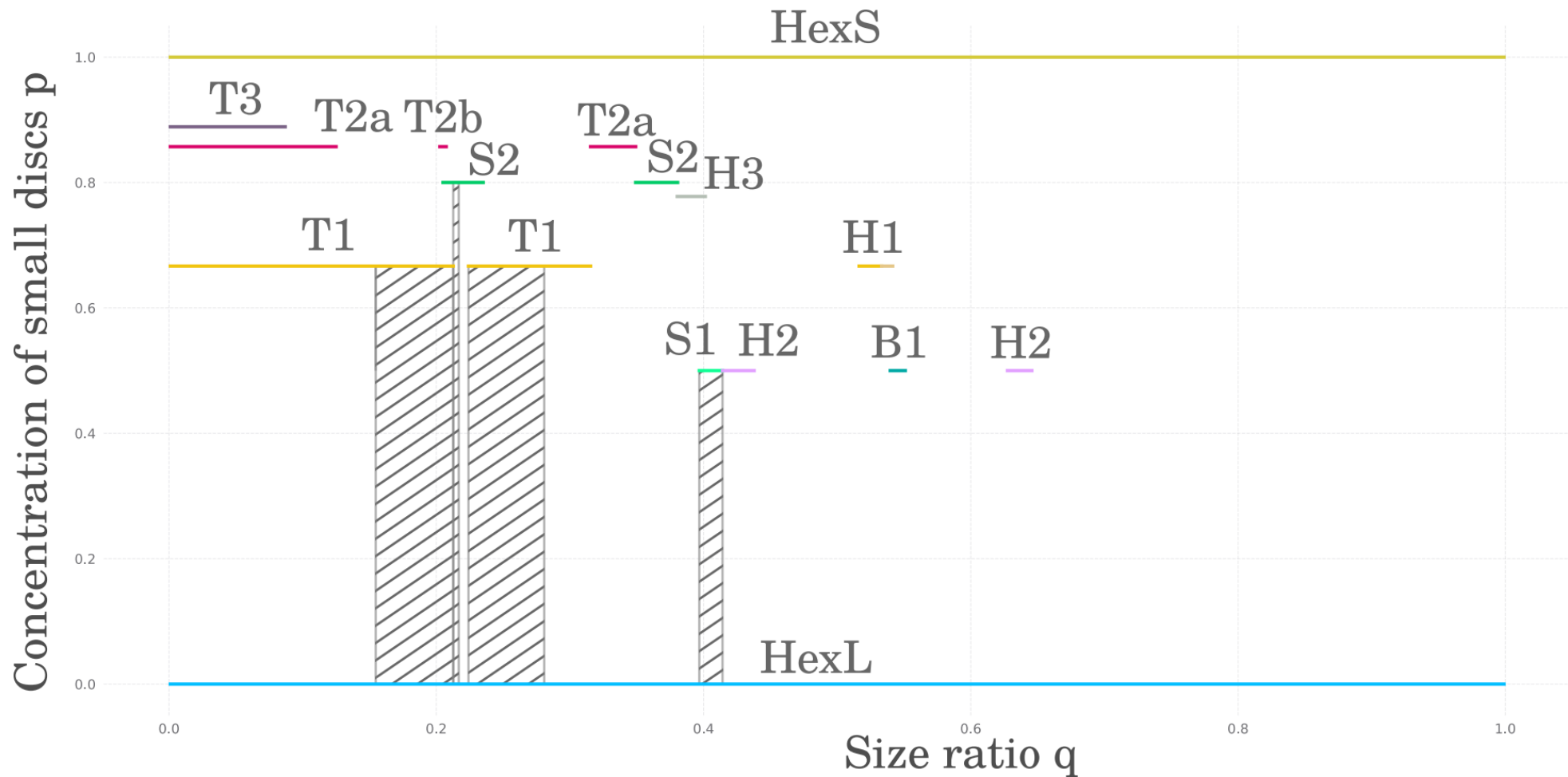
Works with other square cells:



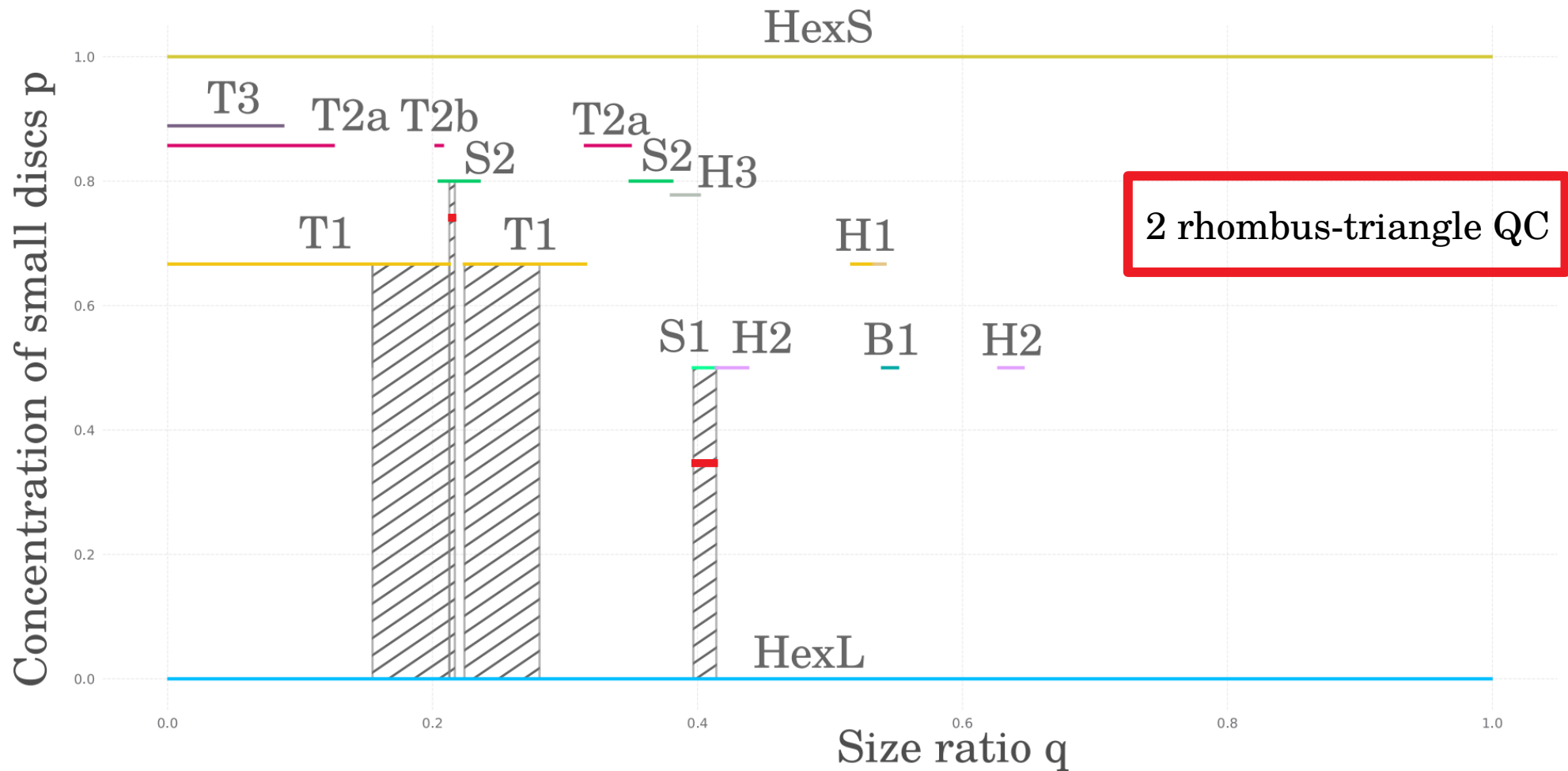
And also with rhombic cells:



# Relevance to quasicrystals



# Relevance to quasicrystals



## Extension to non-additive hard discs

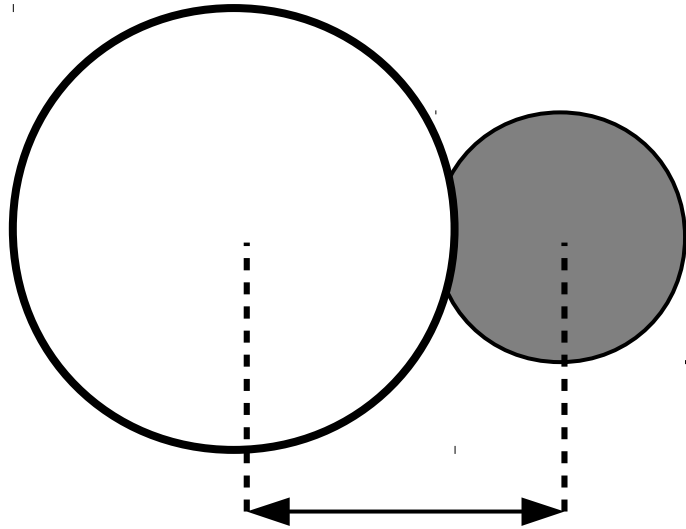
More interplaying length scales to favor quasiperiodic order ?

# Extension to non-additive hard discs

More interplaying length scales to favor quasiperiodic order ?

(top view)

Consider **non-additive hard discs**



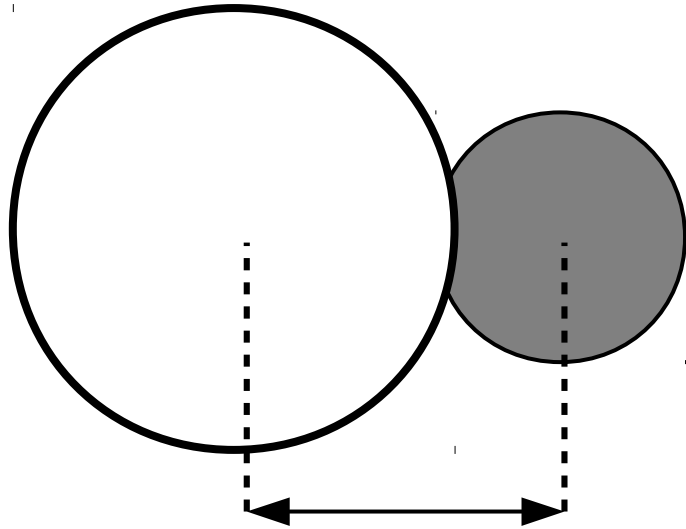
$$d_{LS} = (1+q) \times \text{delta}$$

# Extension to non-additive hard discs

More interplaying length scales to favor quasiperiodic order ?

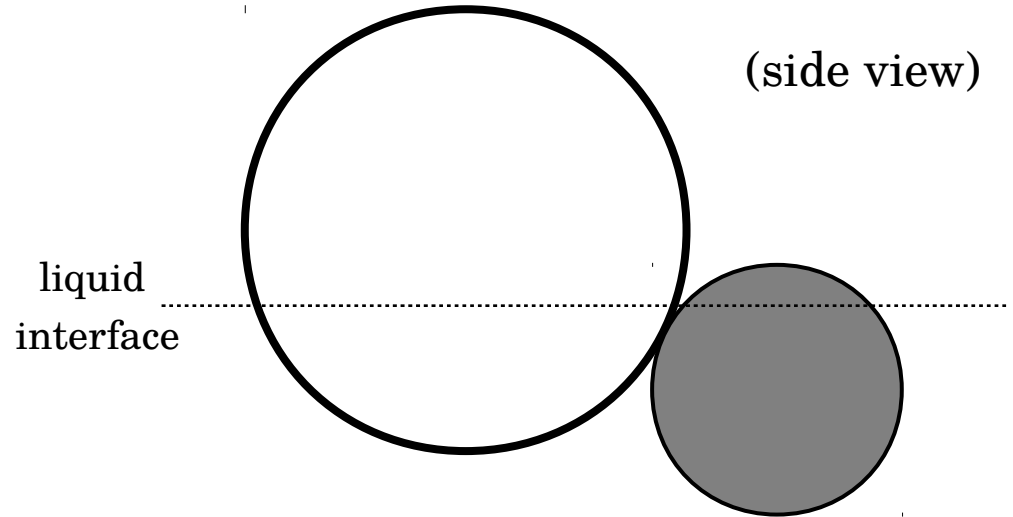
(top view)

Consider **non-additive hard discs**



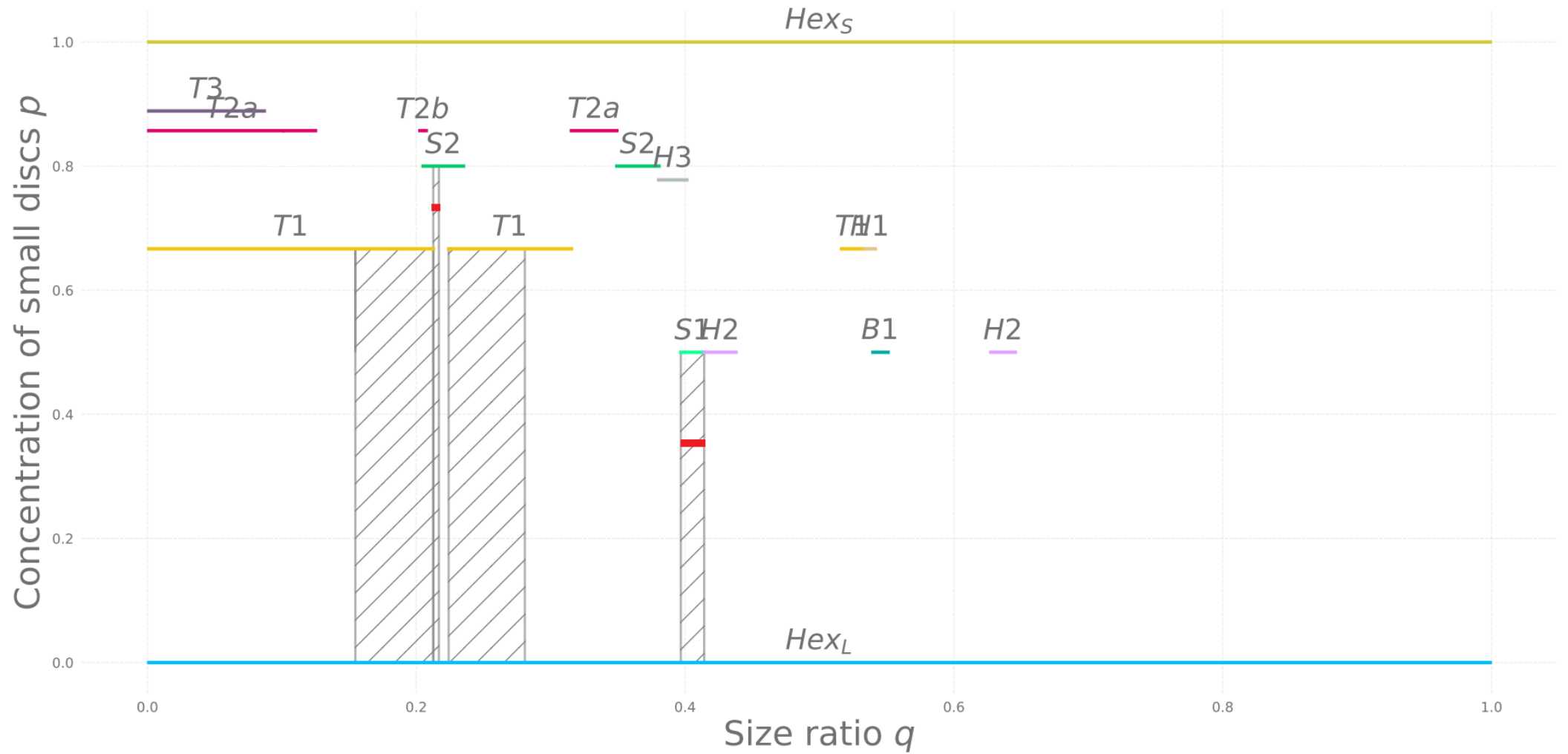
$$d_{LS} = (1+q) \times \text{delta}$$

(side view)



Delta = 1.00

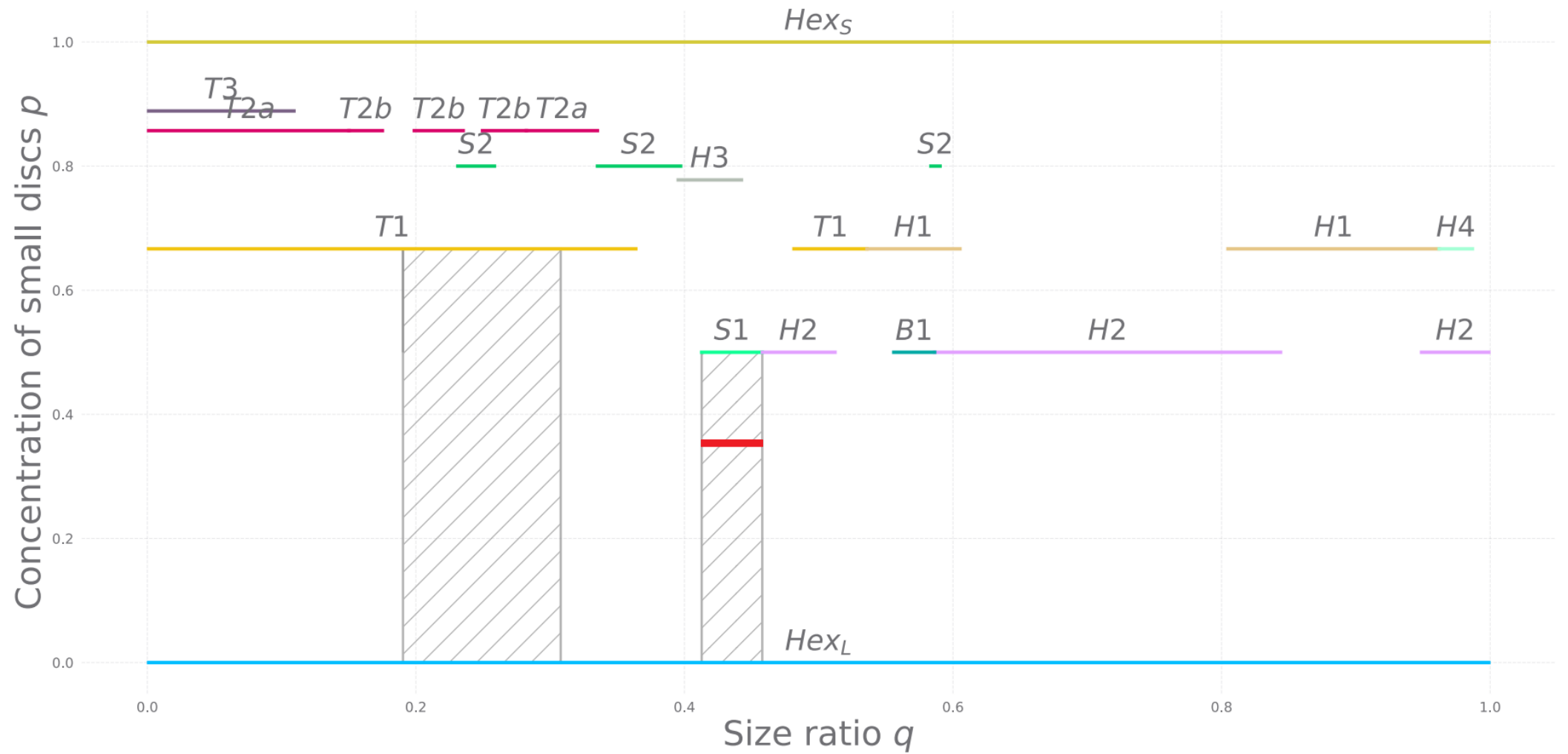
# Extension to non-additive hard discs





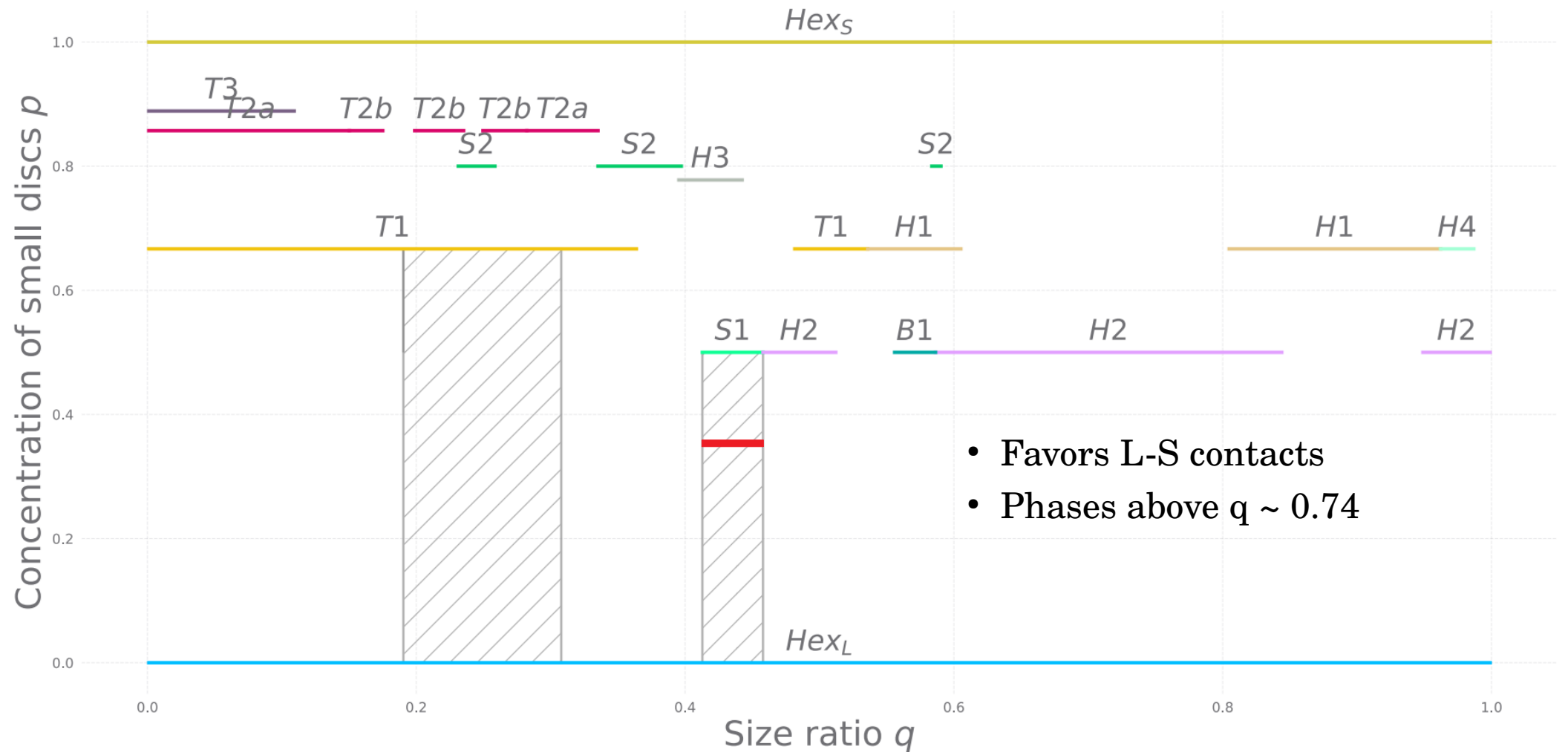
Delta = 0.97

# Extension to non-additive hard discs

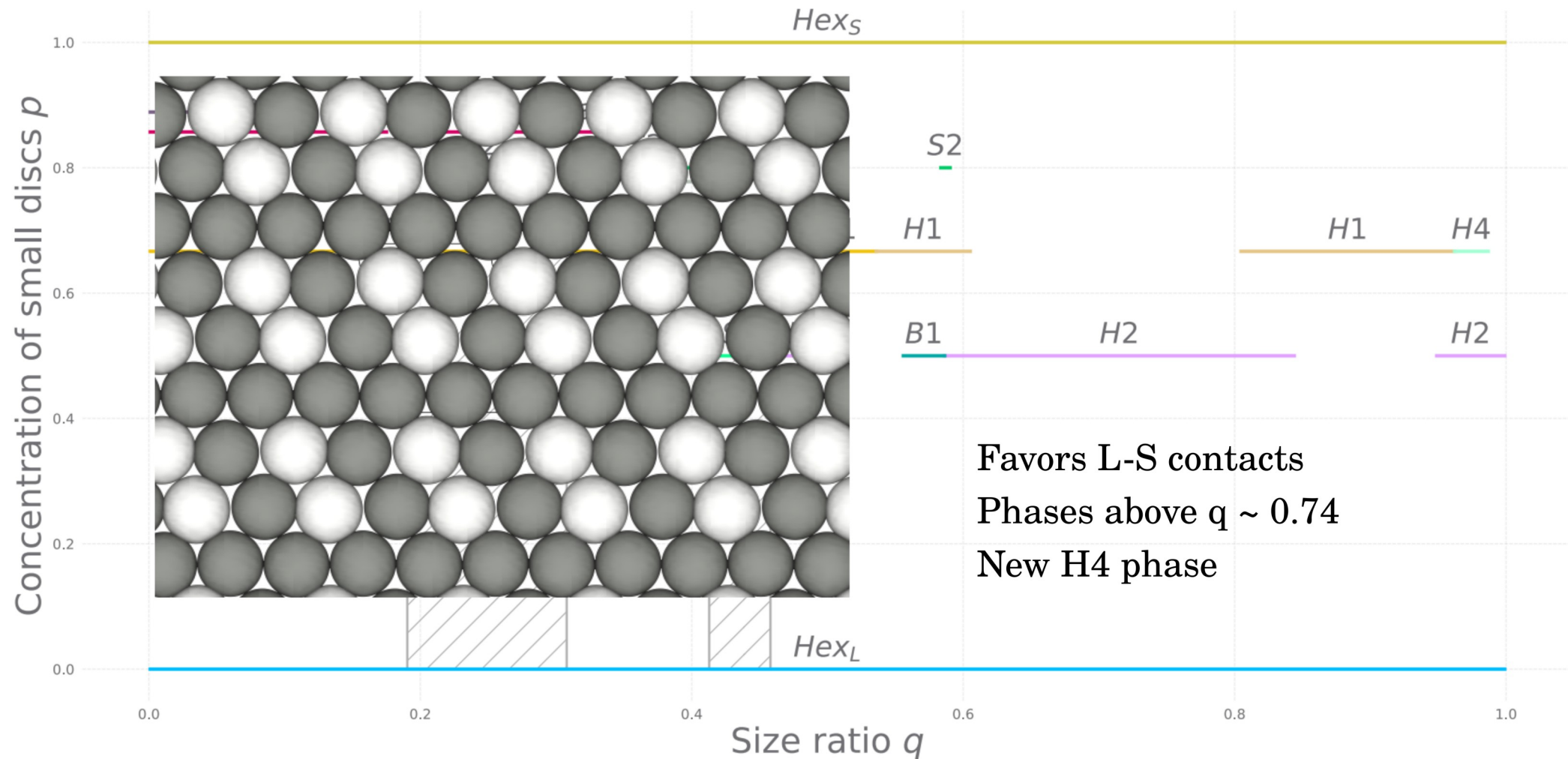


Delta = 0.97

# Extension to non-additive hard discs

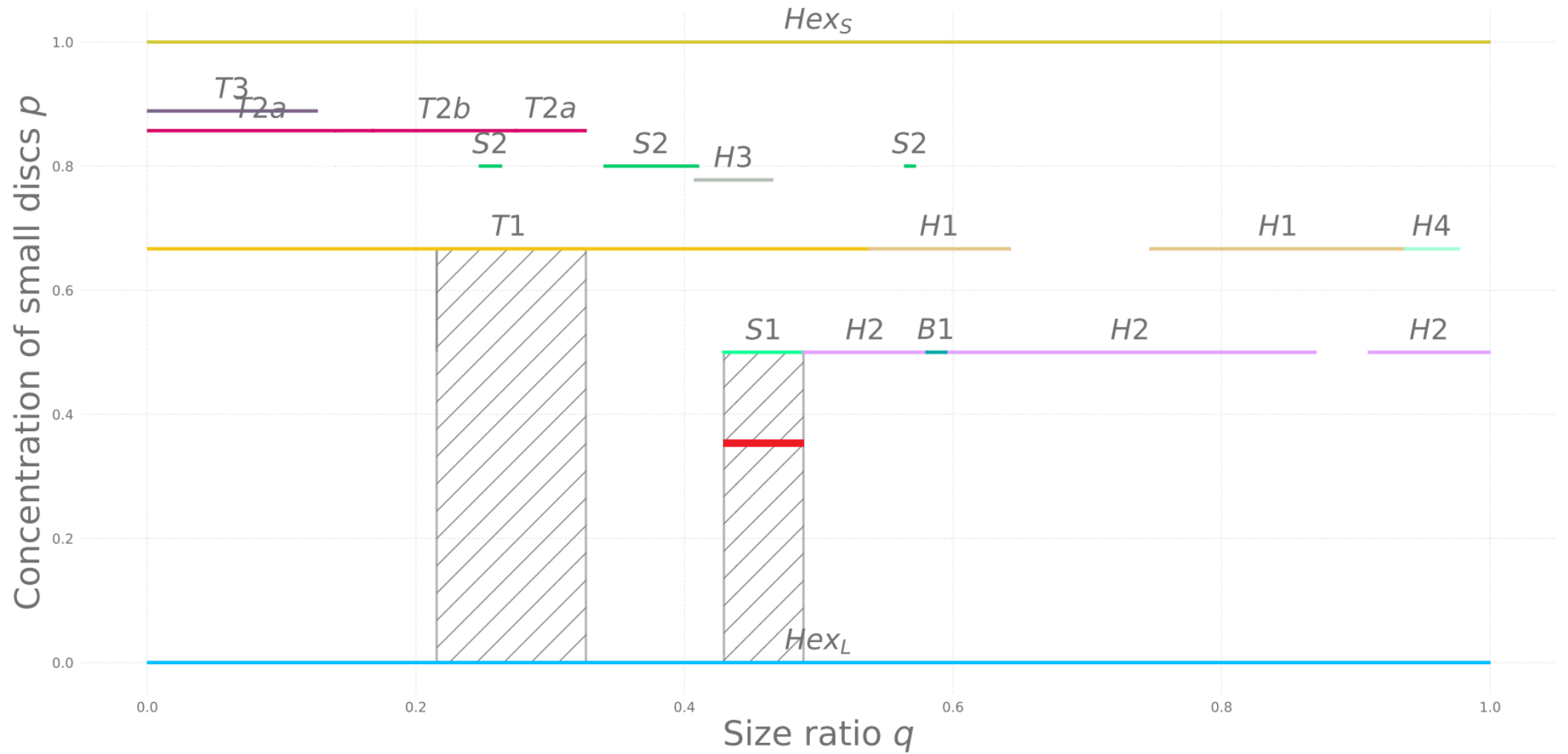


## Extension to non-additive hard discs



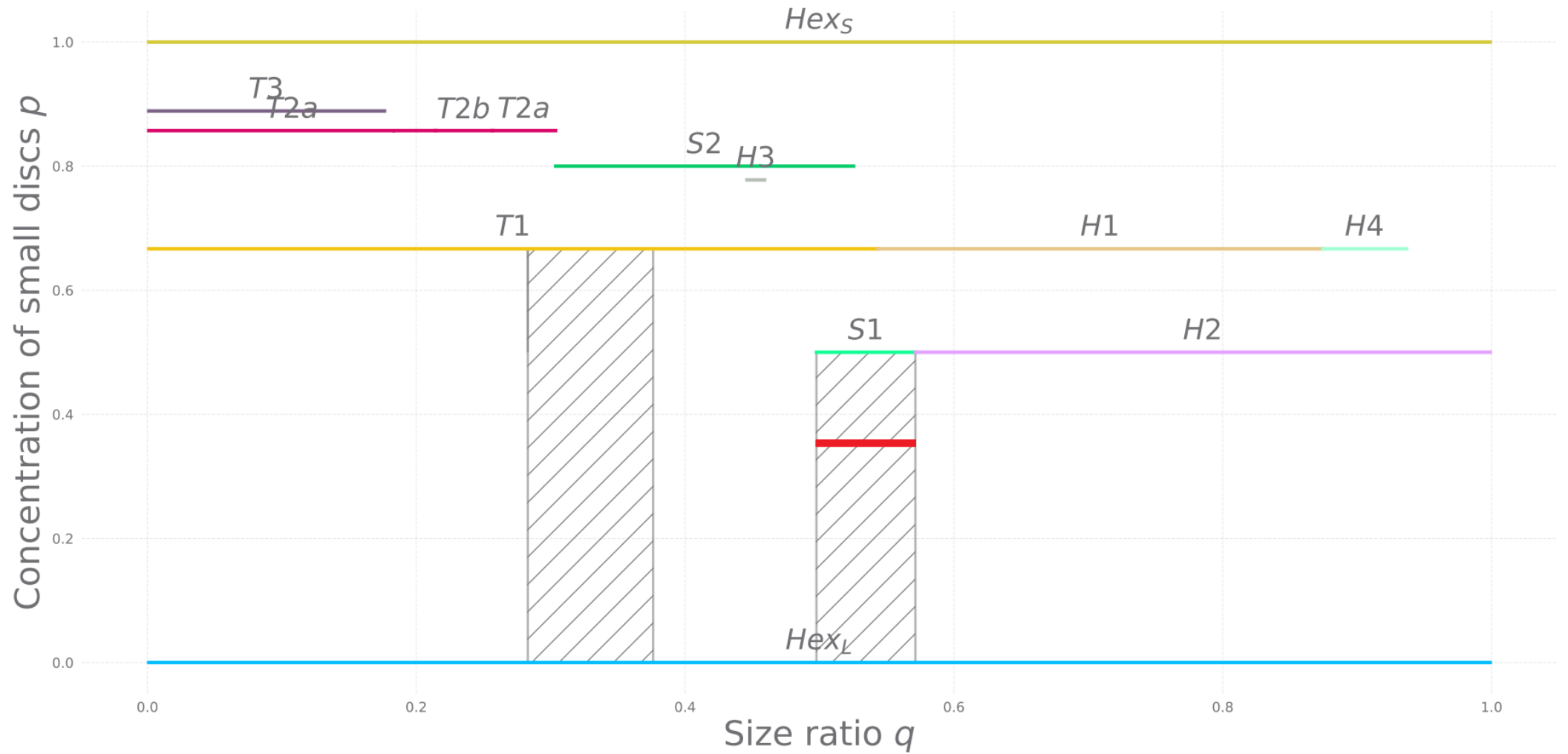
Delta = 0.95

# Extension to non-additive hard discs



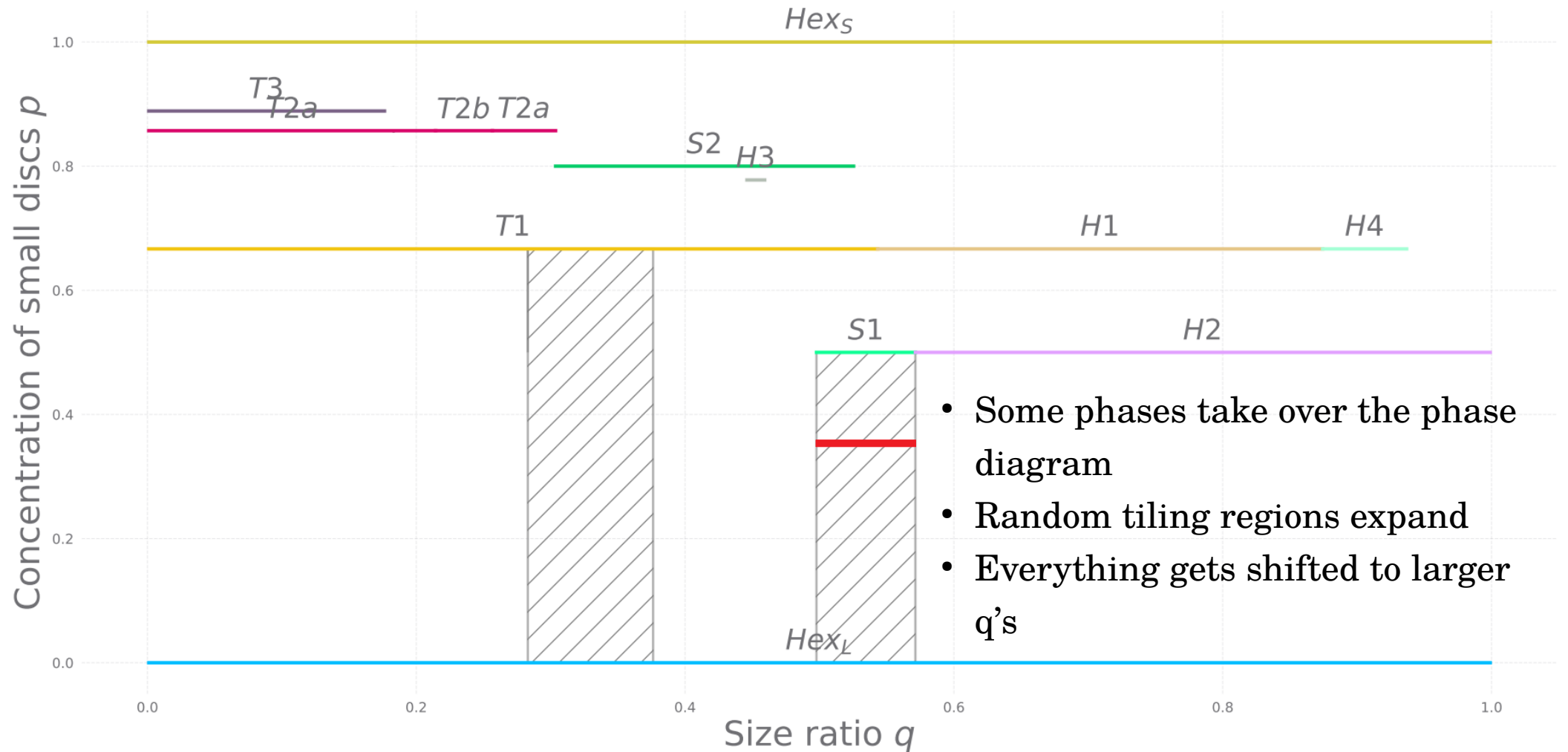
Delta = 0.90

# Extension to non-additive hard discs



Delta = 0.90

# Extension to non-additive hard discs



# Conclusion

FBMC: reliable method to generate candidate structures for the phase diagram  
=> updated phase diagram of binary mixtures of hard discs at infinite pressure  
=> **2 random tiling QC regions**

Non-additivity (extra length scale) **increases the range of stability of the QC**

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FBMC: reliable method to generate candidate structures for the phase diagram  
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Non-additivity (extra length scale) **increases the range of stability of the QC**

# Perspectives

Try and equilibrate some of those random tiling QCs using large scale molecular dynamics simulations.



# Acknowledgments

Frank SMALLENBURG

Giuseppe FOFFI

The SoftQC team

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Thank you for your attention

