Errata Corrige for "An Infinitary Affine Lambda-Calculus Isomorphic to the Full Lambda-Calculus"

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The paper mentioned in the title contains one false statement (Lemma 7 in Sect. 2.3) which compromises the validity of a couple of other statements and proofs in the rest of the paper. Fortunately, these are all accessory properties playing no role in the main result of the paper (the isomorphism theorem). Moreover, the isomorphism theorem may be restated using a different uniform structure for which Lemma 7 holds: it is the uniformity of uniform convergence on finitely branching trees considered in [Maz13].[1]

Corrections for Sect. 2.3

The main result of this section, namely that reduction is Cauchy-continuous (Proposition 8), is false. This is because, as mentioned above, Lemma 7 (Cauchy-continuity of substitution) is false. Indeed, Lemma 6 does not suffice to prove Lemma 7, as wrongly stated in the paper.

A counterexample to Proposition 8 is the following. Let, for all $n \in \mathbb{N}$,

$$t_n := (\lambda x_0 x_1.z\langle \overbrace{\bot, \dots, \bot}^{n}, x_0\langle y\langle x_1\rangle\rangle\rangle)\langle u\rangle,$$
$$t'_n := (\lambda x_0 x_1.z\langle \overbrace{\bot, \dots, \bot}^{n}, x_1\langle y\langle x_0\rangle\rangle\rangle)\langle u\rangle.$$

with u of height $h \geq 2$. It is easy to see that, with respect to the topology induced by the metric d used in the paper, the sequence $t_0, t'_0, t_1, t'_1, \dots, t_n, t'_n, \dots$ converges to $(\lambda x_0 x_1.z\langle\rangle)\langle u\rangle$ and is therefore Cauchy. However, if we consider the one-step head-reducts of t_n and t'_n, we obtain

$$H(t_n) = z\langle \overbrace{\bot, \dots, \bot}^{n}, u\langle y\langle\rangle\rangle\rangle,$$
$$H(t'_n) = z\langle \overbrace{\bot, \dots, \bot, \bot}^{n+1}\langle y\langle u\rangle\rangle\rangle,$$

[1] Unfortunately (and embarrassingly), that paper also contains a mistake, which is addressed in its own errata corrige.

whose heights are different: the height of $H(t_n)$ is $h+2$, the height of $H(t'_n)$ is $h+3$. Hence, the sequence $H(t_0), H(t'_0), H(t_1), H(t'_1), \ldots$ is not Cauchy, because all Cauchy sequences w.r.t. d must ultimately contain terms of equal height. This shows that reduction is not Cauchy-continuous.

The above may be easily reworked to give a counterexample to Lemma 7.

Corrections for Sect. 3.1

The proof of Proposition 10 uses the continuity of reduction (Proposition 8), so it is wrong. However, Proposition 10 itself holds (it may be proved by standard combinatorial arguments, the only point of giving it here was the topological proof).

Corrections for Sect. 4.2

The remark in the second paragraph after Definition 8 is false: given a λ-regular, height-bounded metric which is furthermore "syntactically discrete", Lemma 6 holds but, as observed above, Lemma 7 does not follow, and the head-reduction map H is not necessarily Cauchy-continuous.

Theorem 26 does hold if we replace the hypothesis "λ-regular, height-bounded" with "lambda-regular, height-bounded and such that H is Cauchy-continuous". However, it might be vacuous, since it may very well be that all uniformities satisfying these hypotheses are discrete (but I don't know this).

It would be more interesting if the result held with the weaker hypothesis "spinal-height-bounded". For that to be true, it is enough that point 3 of Lemma 27 holds with spinal height in place of height (I haven't checked if this is the case). The uniformity of uniform convergence on finitely-branching trees (see [Maz13]) is a non-trivial (*i.e.*, non-discrete) example of λ-regular, spinal-height-bounded uniformity w.r.t. which H is Cauchy-continuous. Interestingly, this uniformity is not metrizable (see the errata corrige of [Maz13]) and at this moment I don't know if there exists a metric which allows to prove the isomorphism theorem *and* at the same time makes reduction Cauchy-continuous.

References

[Maz13] Damiano Mazza. Non-linearity as the metric completion of linearity. In Masahito Hasegawa, editor, *Proceedings of TLCA*, volume 7941 of *Lecture Notes in Computer Science*, pages 3–14. Springer, 2013.