Monotonic Subsequences

Three (Nice) Open Problems

Nabil H. Mustafa

Large monotone subsequence

Erdős–Szekeres Theorem

Given a sequence S of n reals,
there exists a monotonic subsequence of S of size at least $\sqrt{n}$.

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$\Longrightarrow$ leads to the proof

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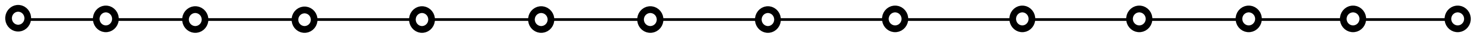
either $\exists\ i$ with $t_i \geq \sqrt{n}$ or the same integer appears $\frac{n}{\sqrt{n}}$ times

CONFLICT-FREE COLORINGS

Conflict-free colorings



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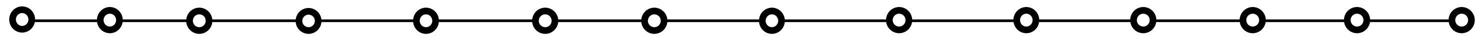
[Even, Lotker, Ron, Smorodinsky]



Goal : coloring such that each interval contains a unique color

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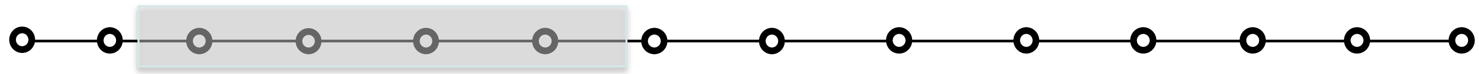

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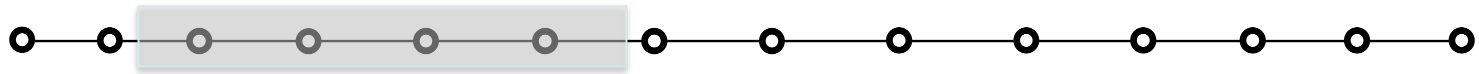

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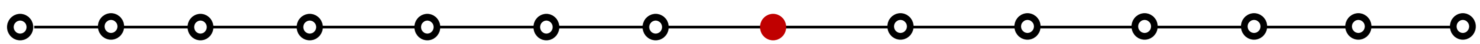

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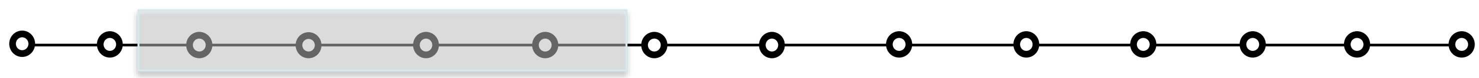

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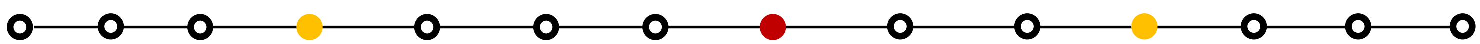

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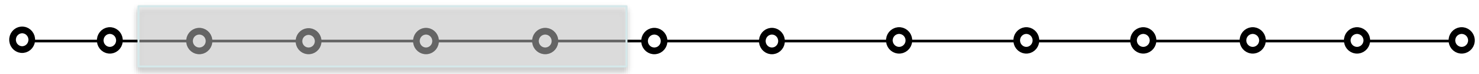

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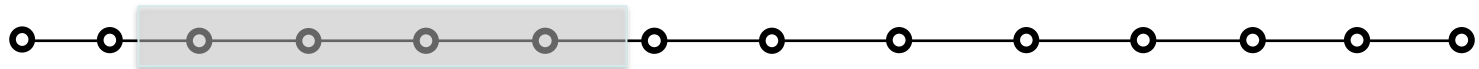

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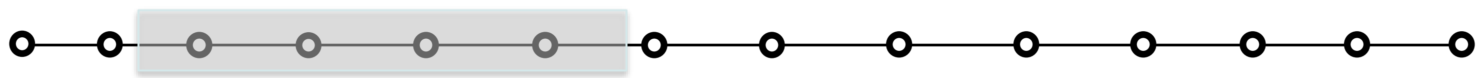

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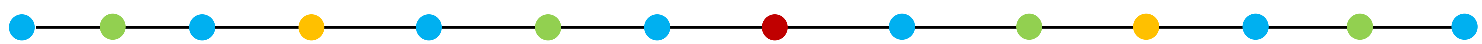

→ possible with $O(\log n)$ colors

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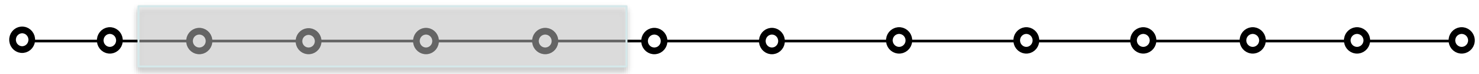

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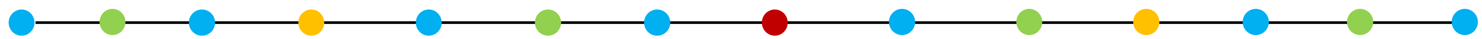

$\rightarrow$ possible with $O(\log n)$ colors

$\rightarrow$ need $\Omega(\log n)$ colors

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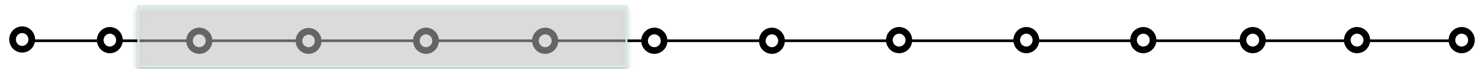

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$\rightarrow$ **Disks** in $\mathbb{R}^2$

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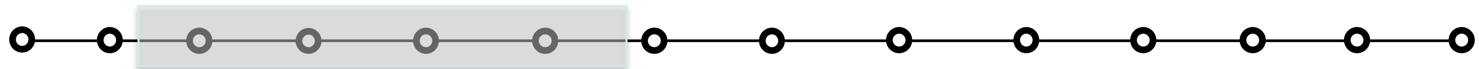

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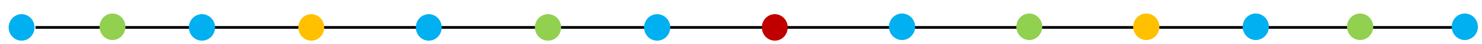

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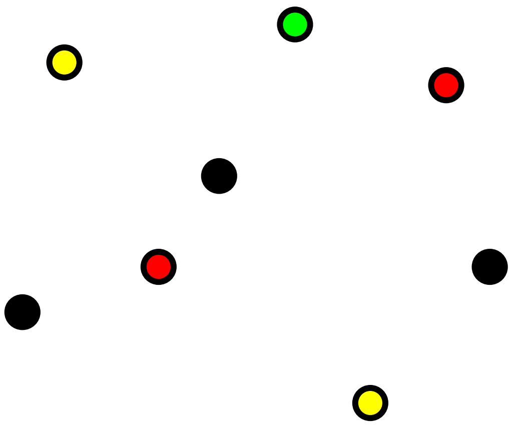

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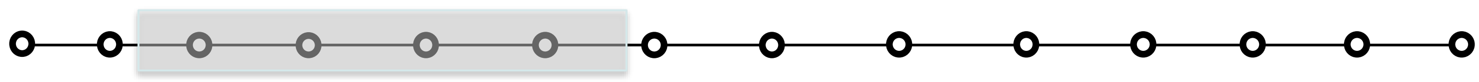

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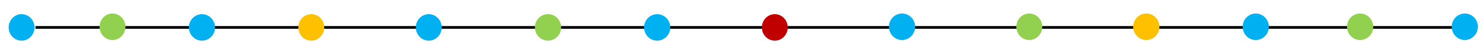

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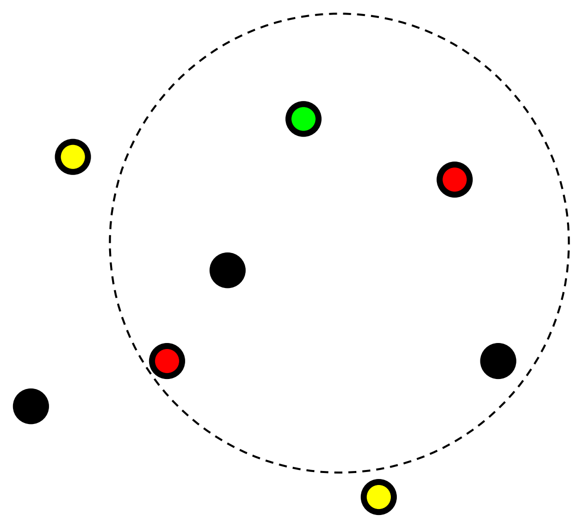

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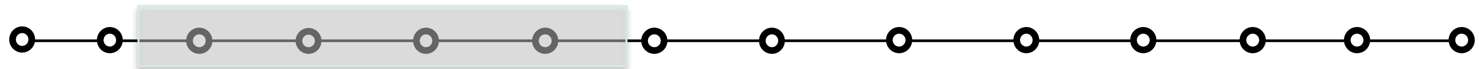

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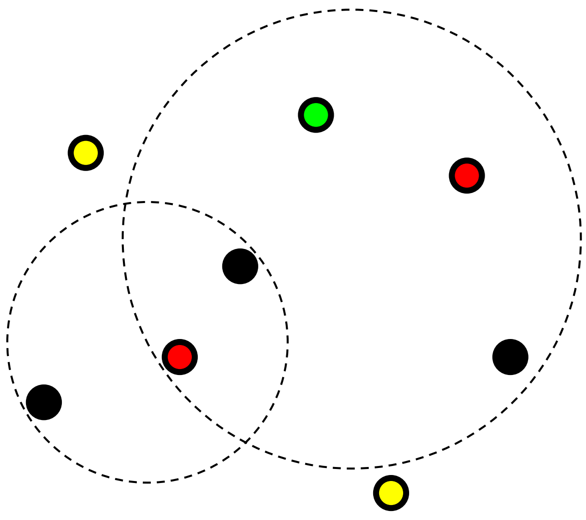

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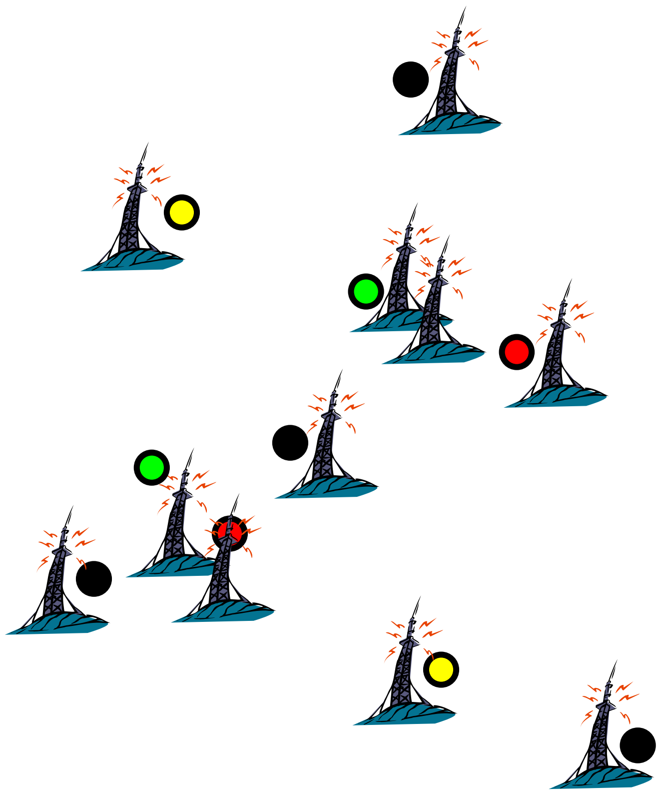

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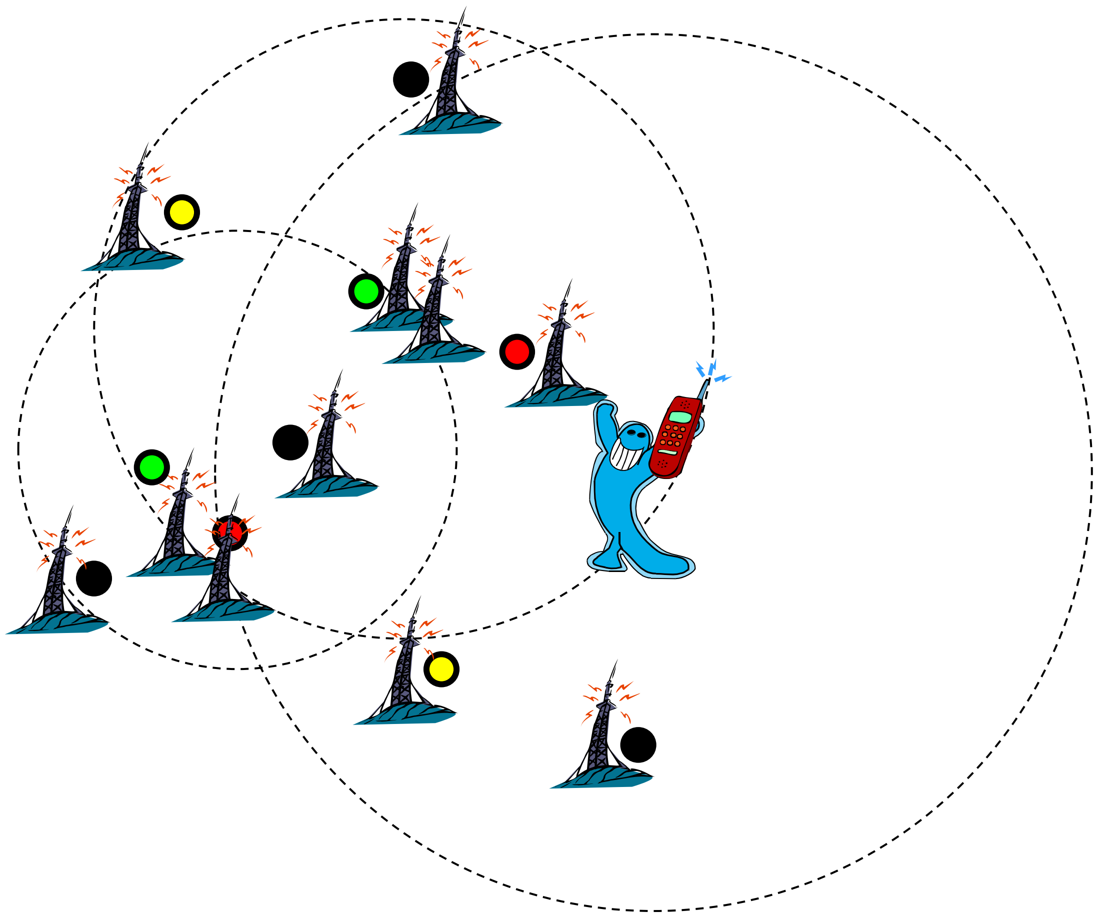

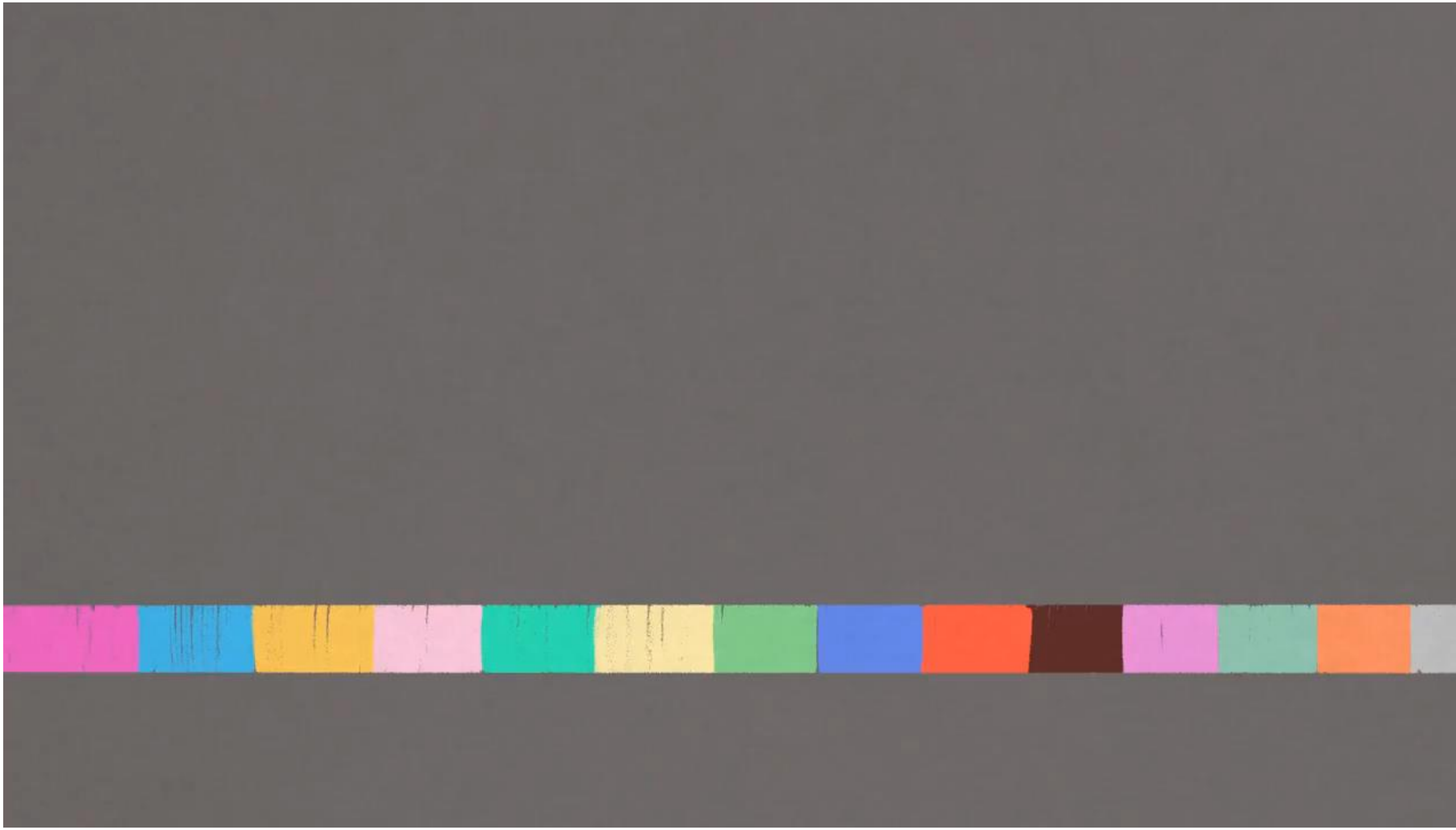

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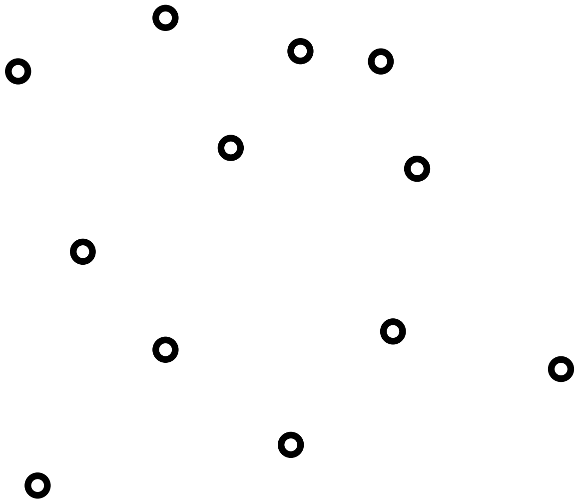

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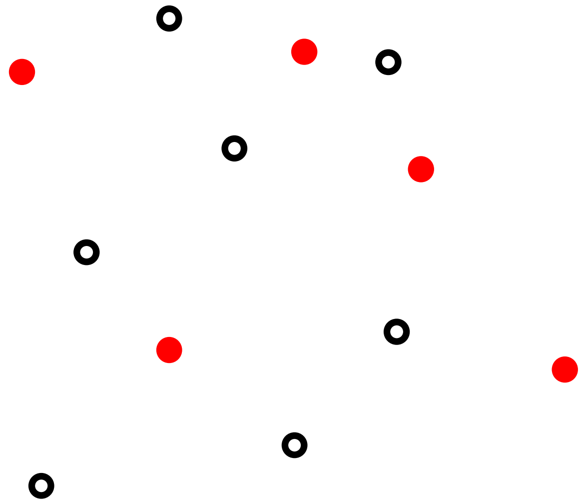

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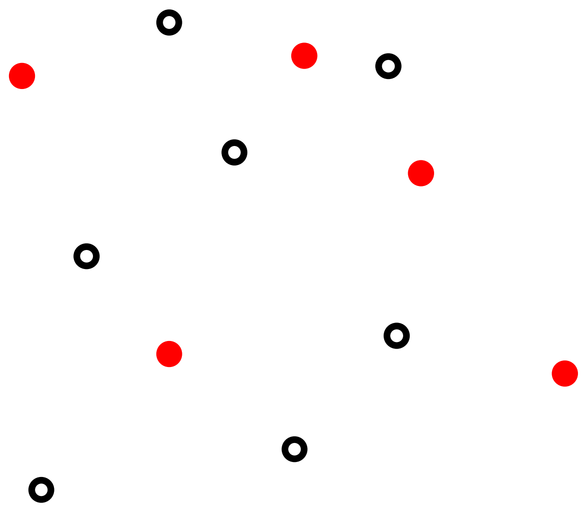
Coloring procedure:



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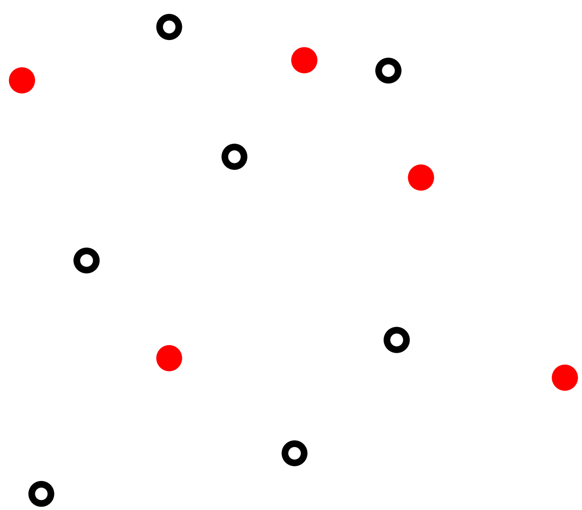

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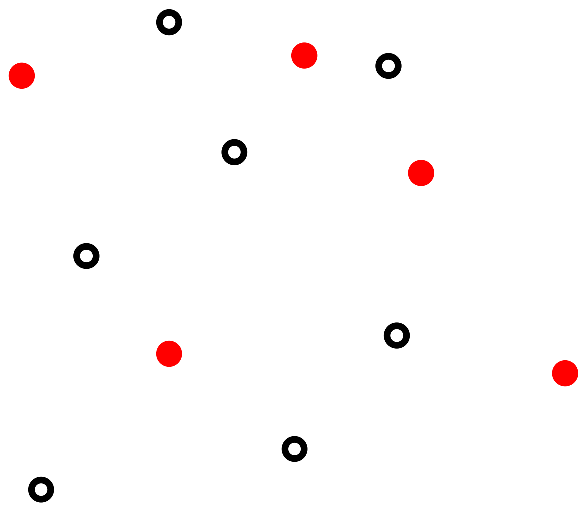

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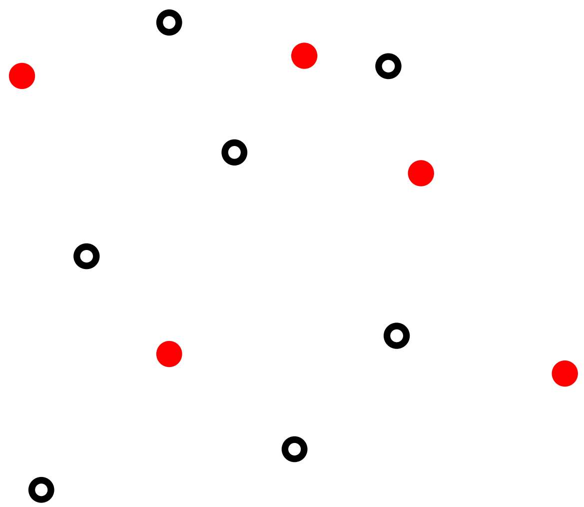
$P = P - Q$



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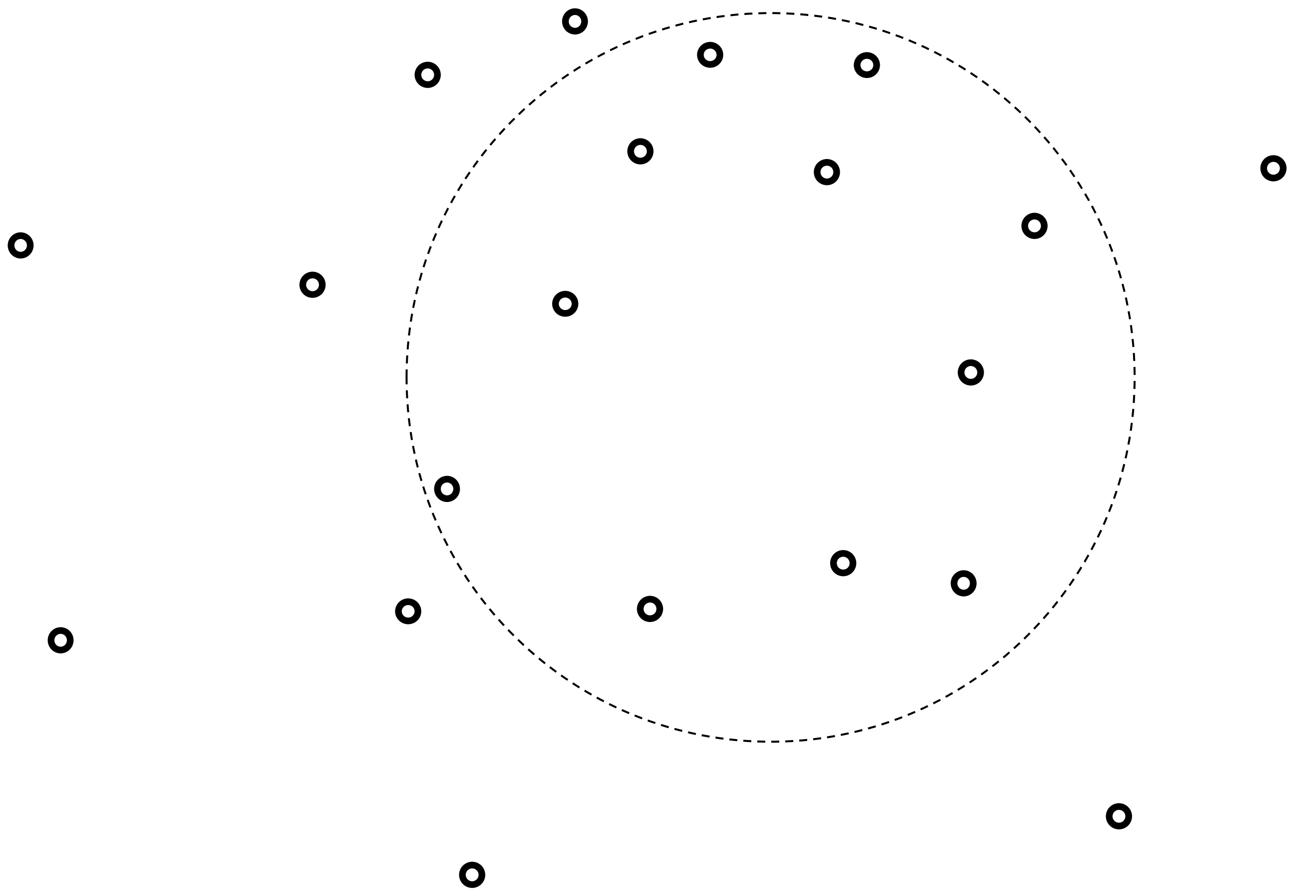

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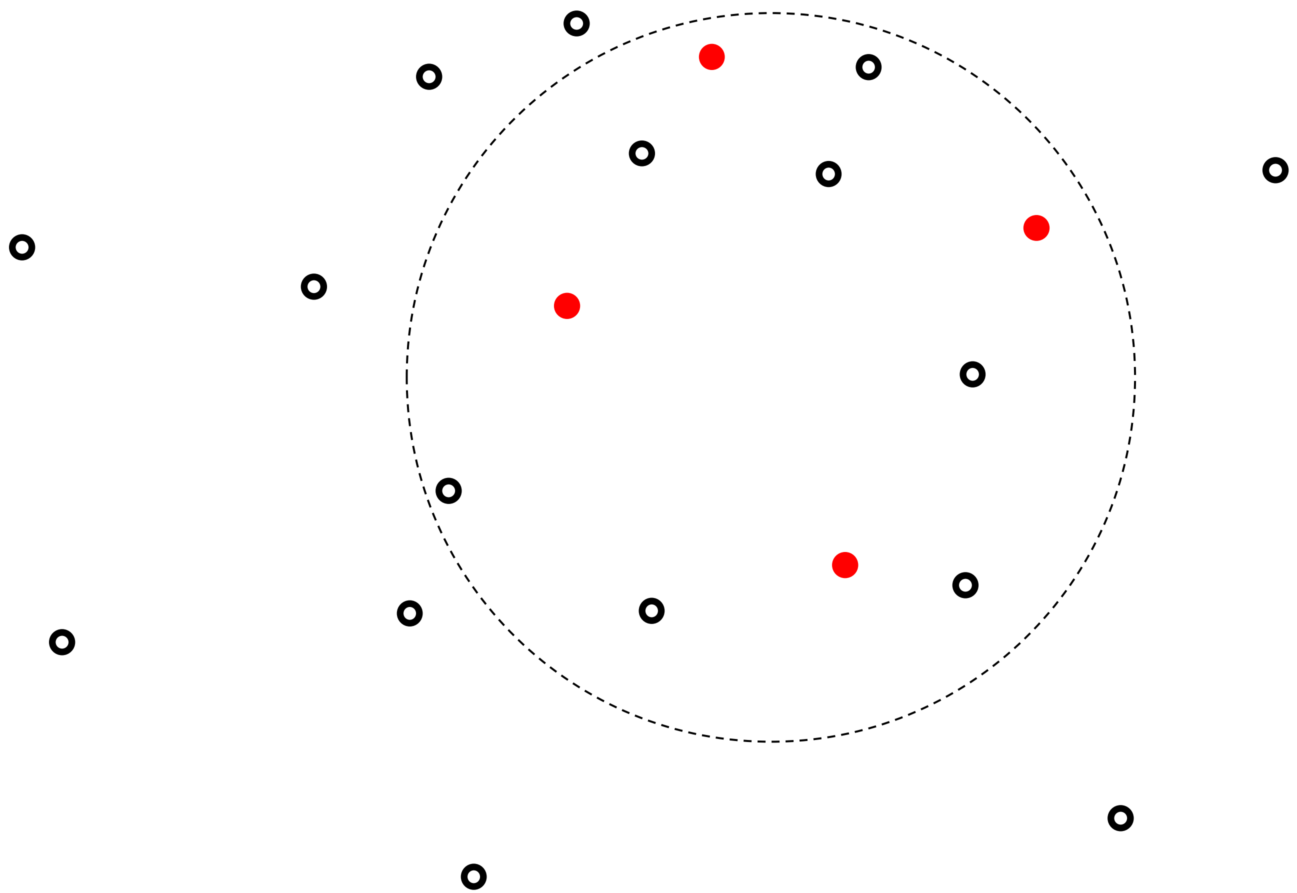

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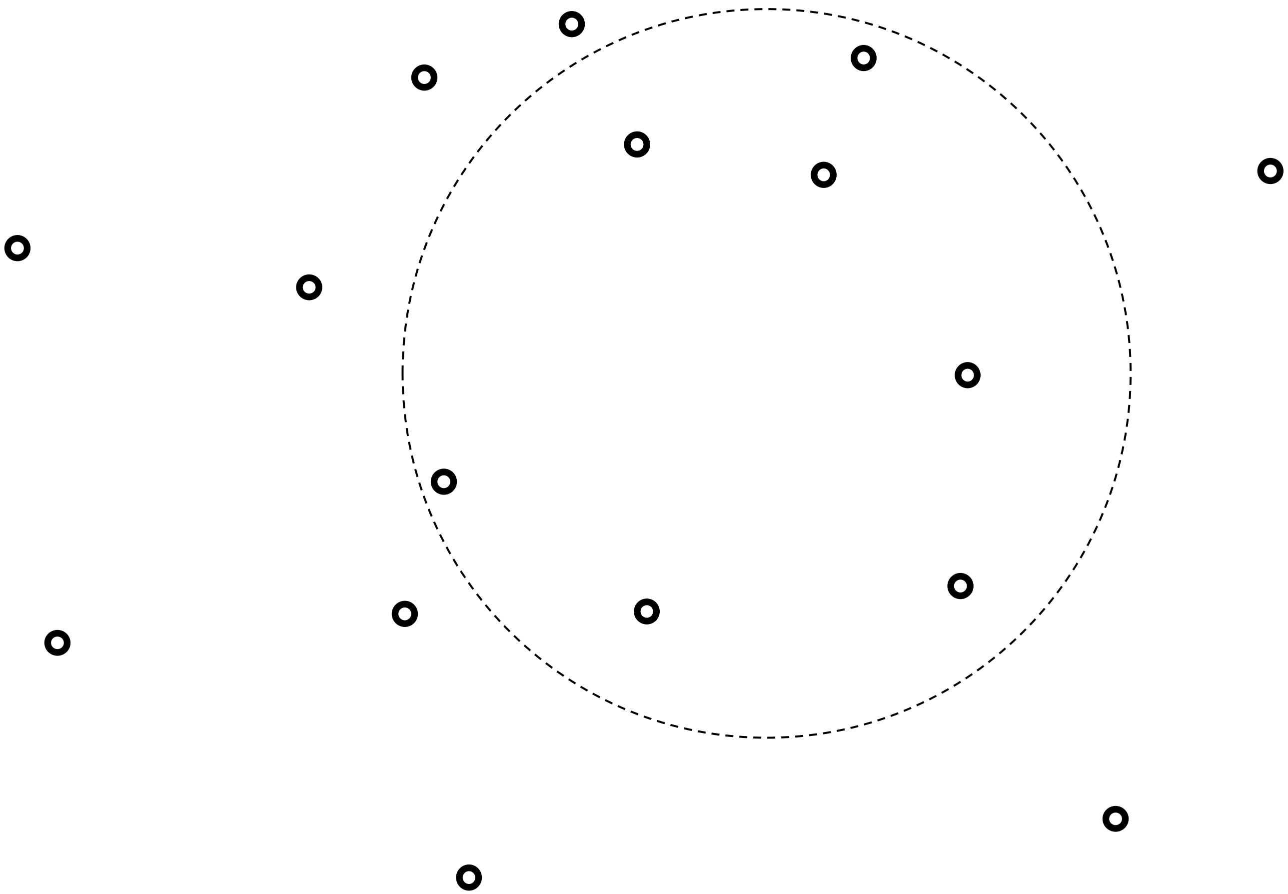

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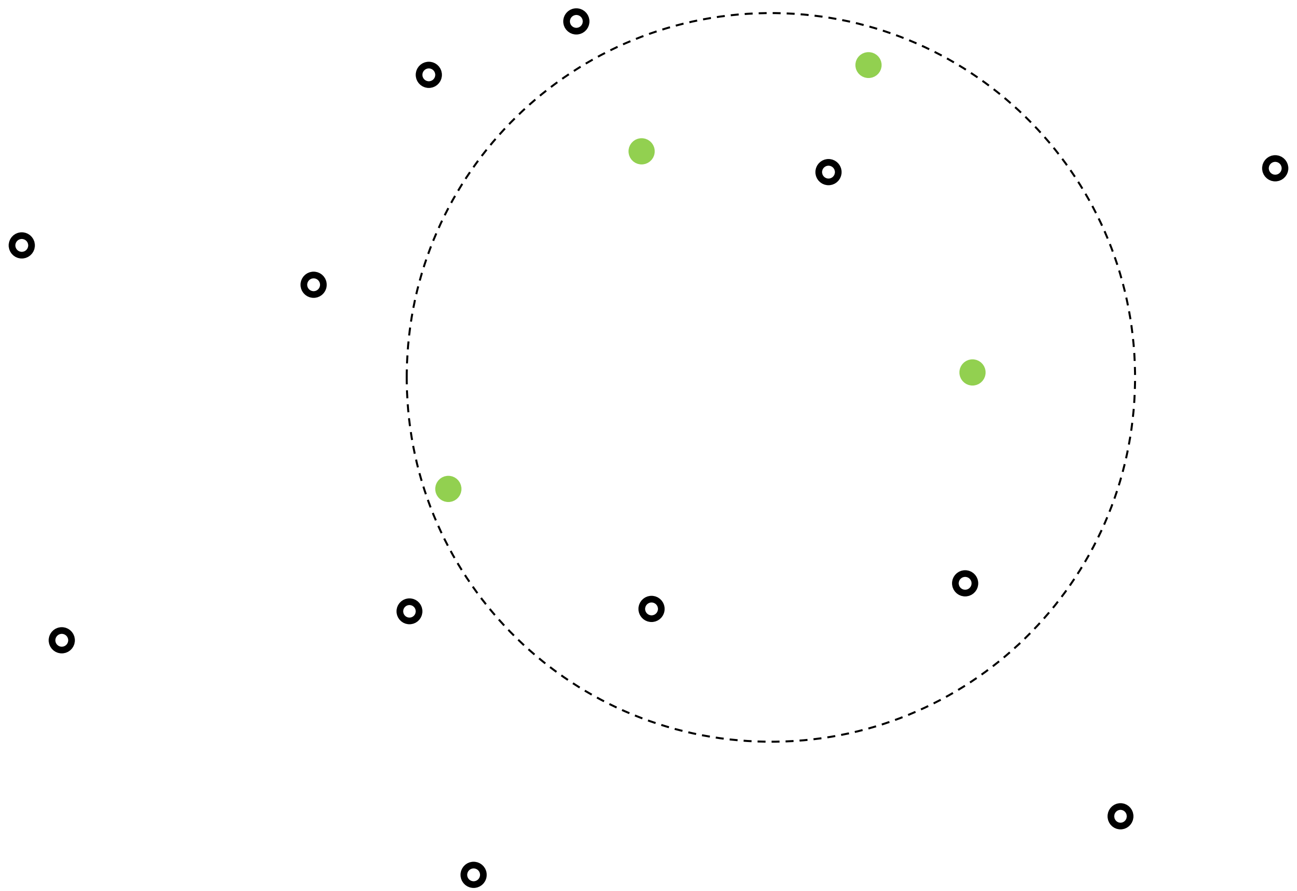

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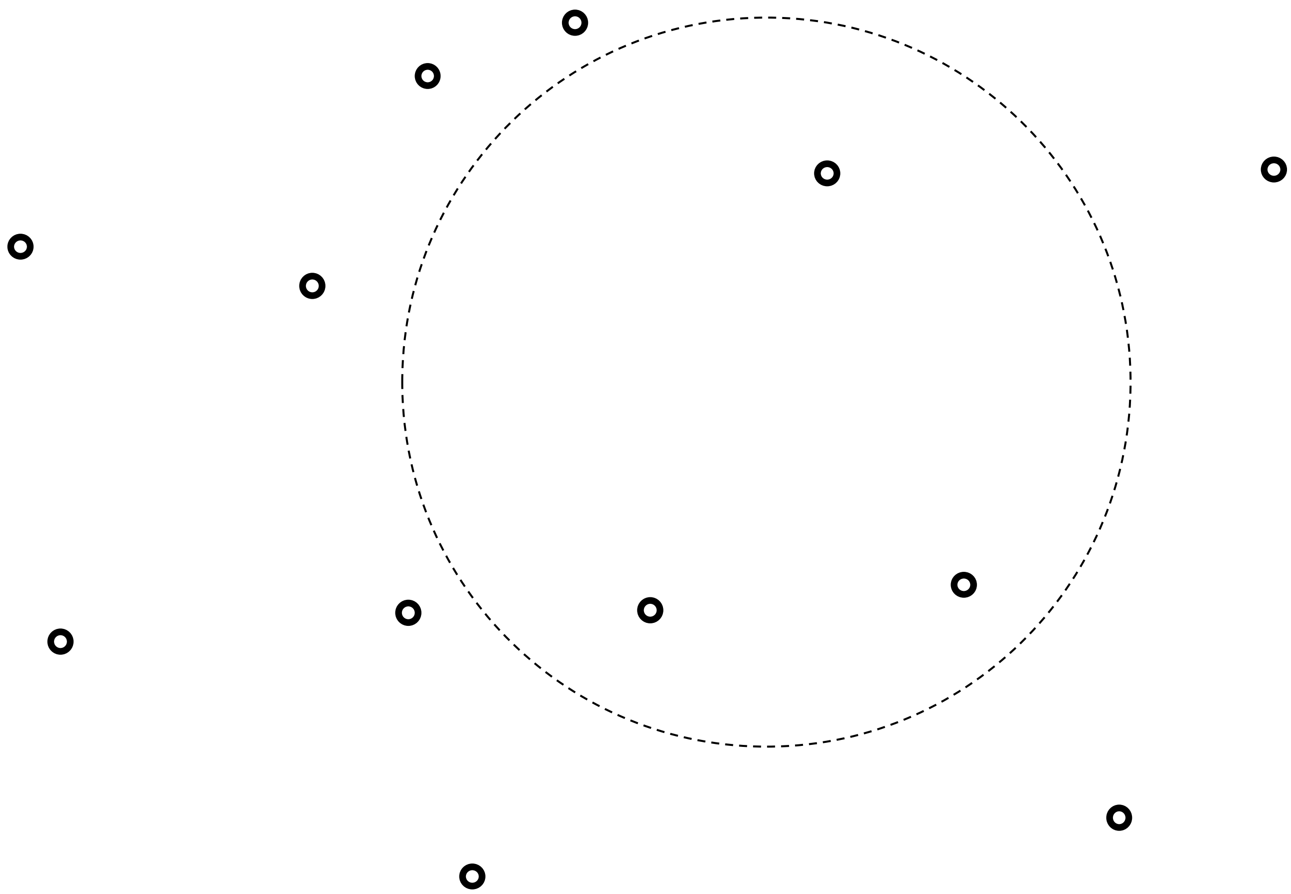

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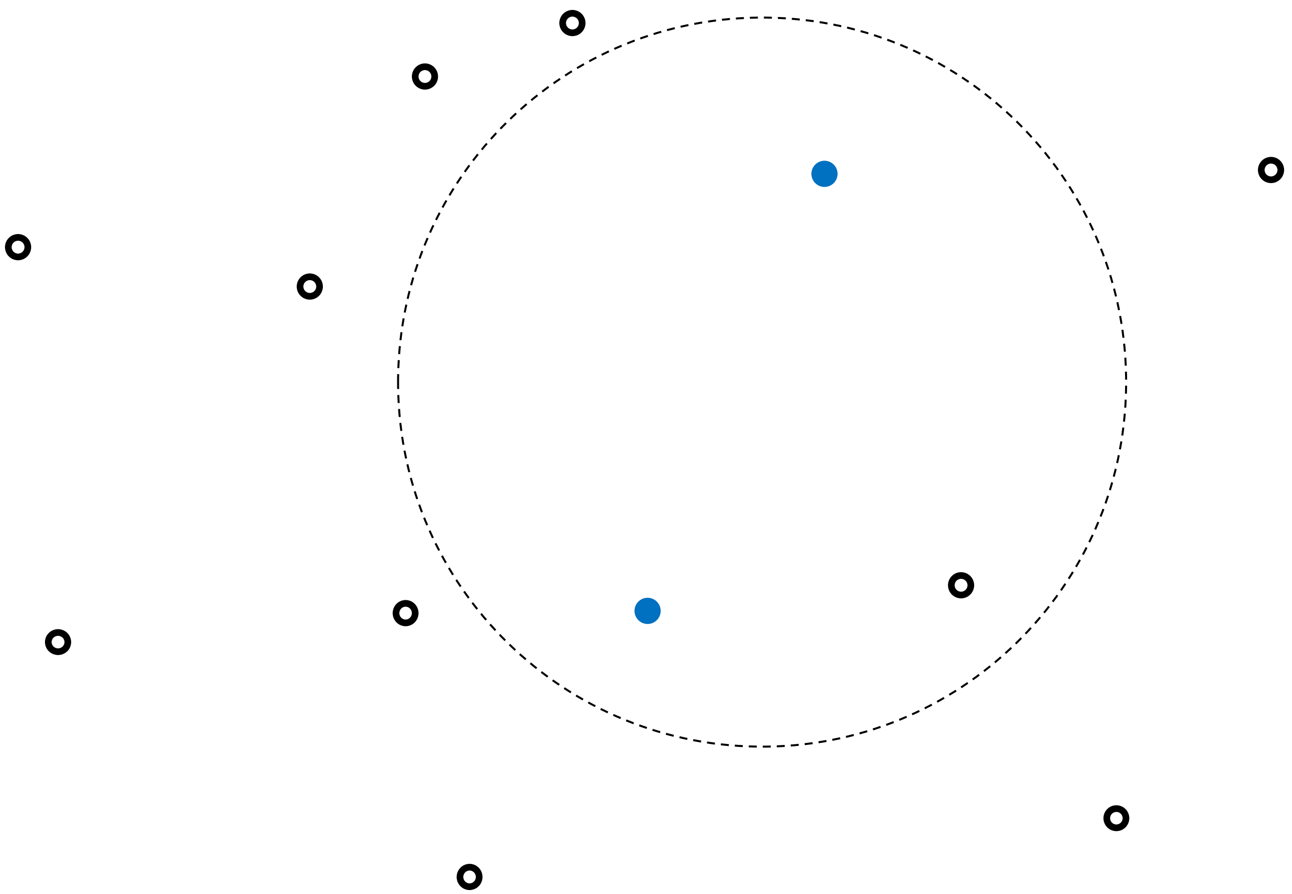

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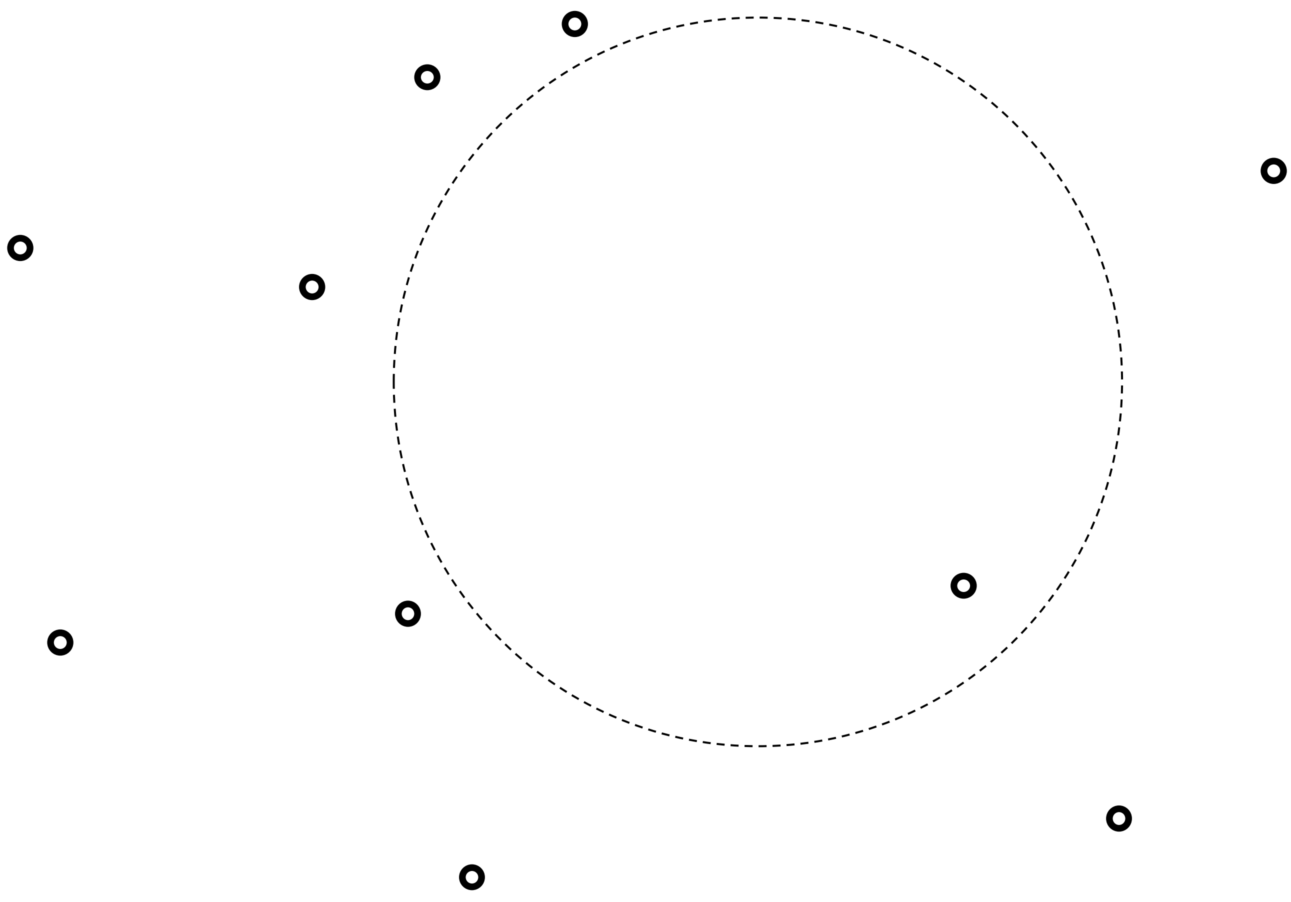

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(4-color theorem)

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Rectangles : ??

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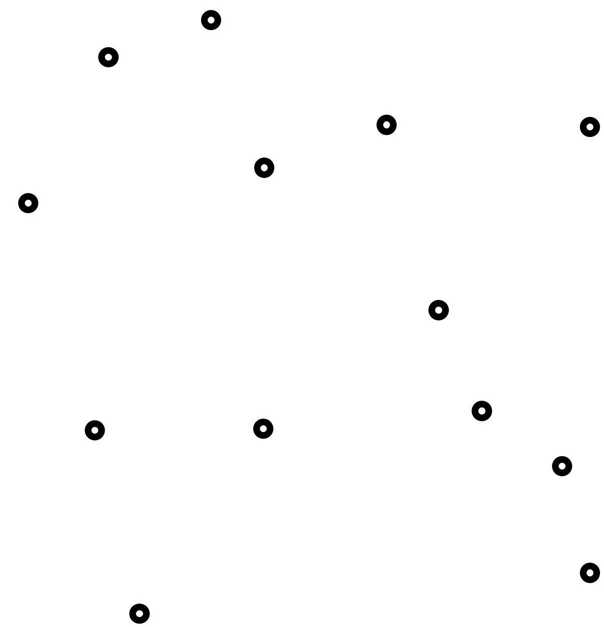

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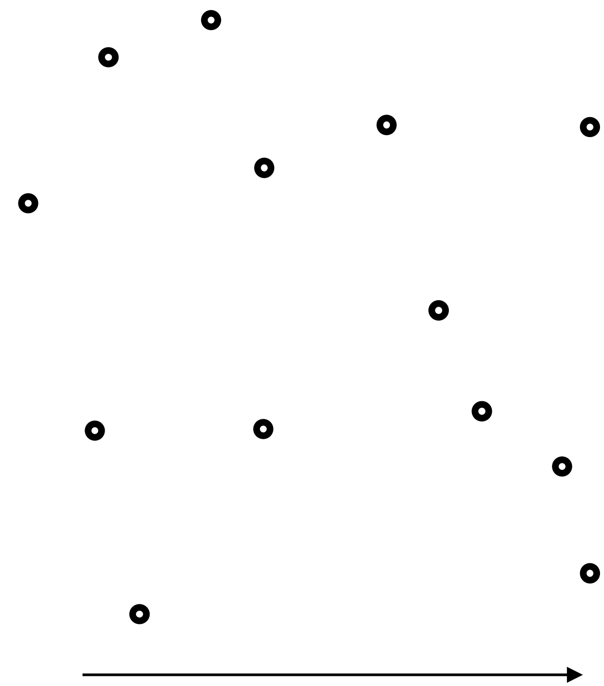

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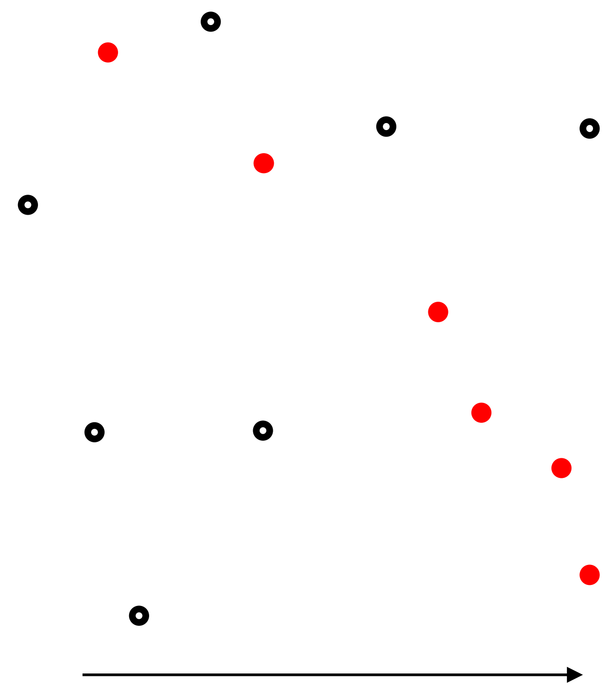

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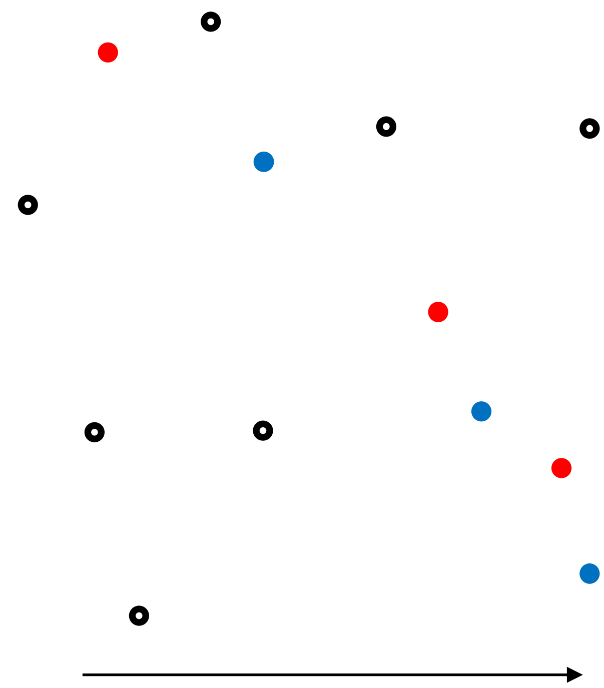
$\rightarrow$ Q: alternate points in this subsequence



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Goal : Given a set P of n points, find $Q \subseteq P$ such that

if rectangle R contains points of Q

then R **must** also contain from $P \setminus Q$

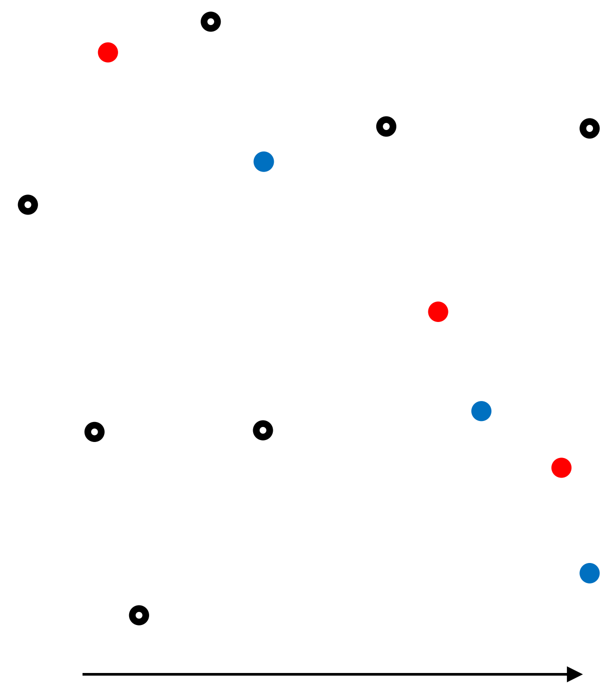
Claim : Such a Q of size $\Omega(\sqrt{n})$ exists

$\rightarrow$ sort by x-coordinate

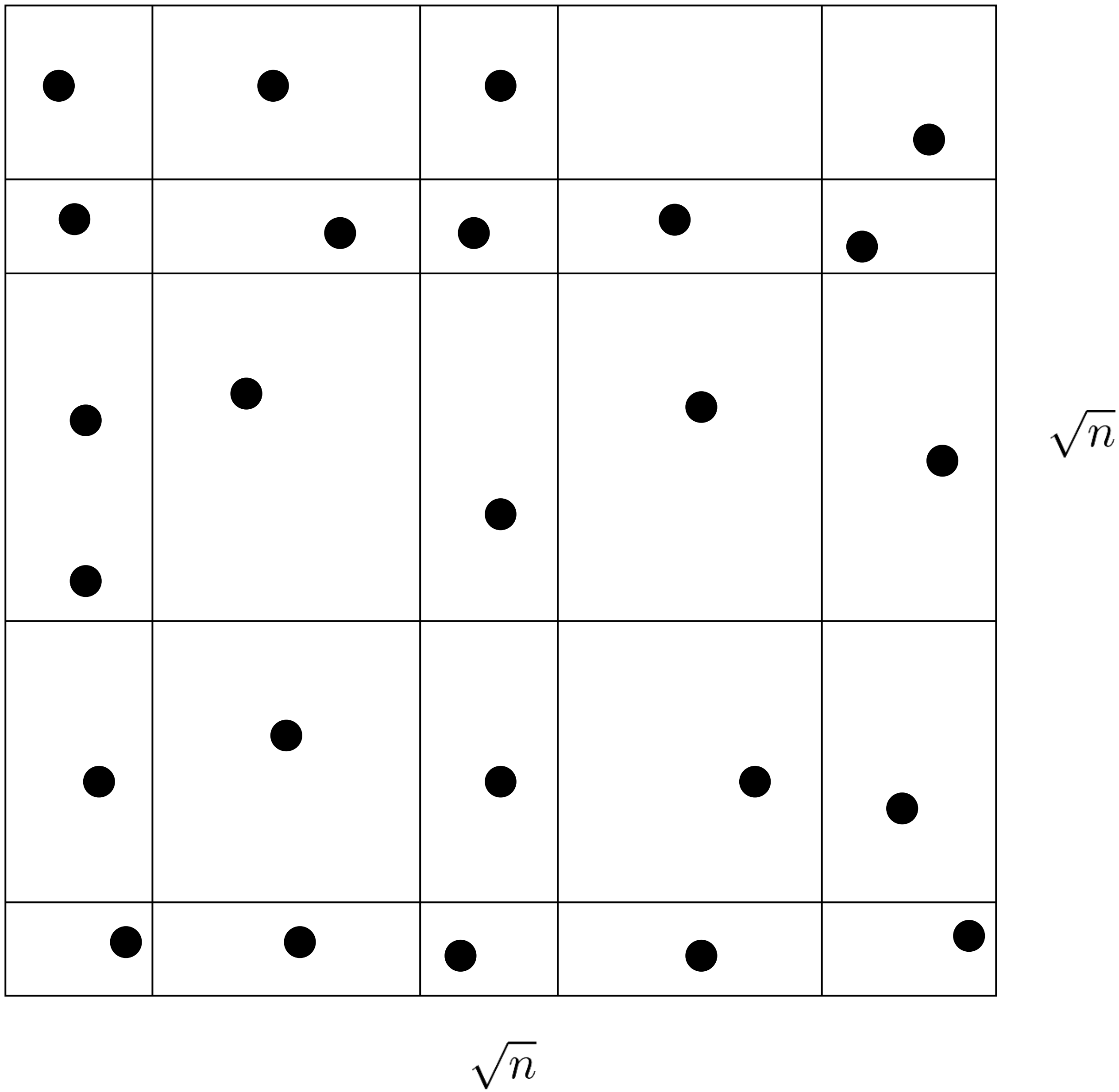
$\rightarrow$ monotone subsequence of size $\Omega(\sqrt{n})$

$\rightarrow$ Q: alternate points in this subsequence

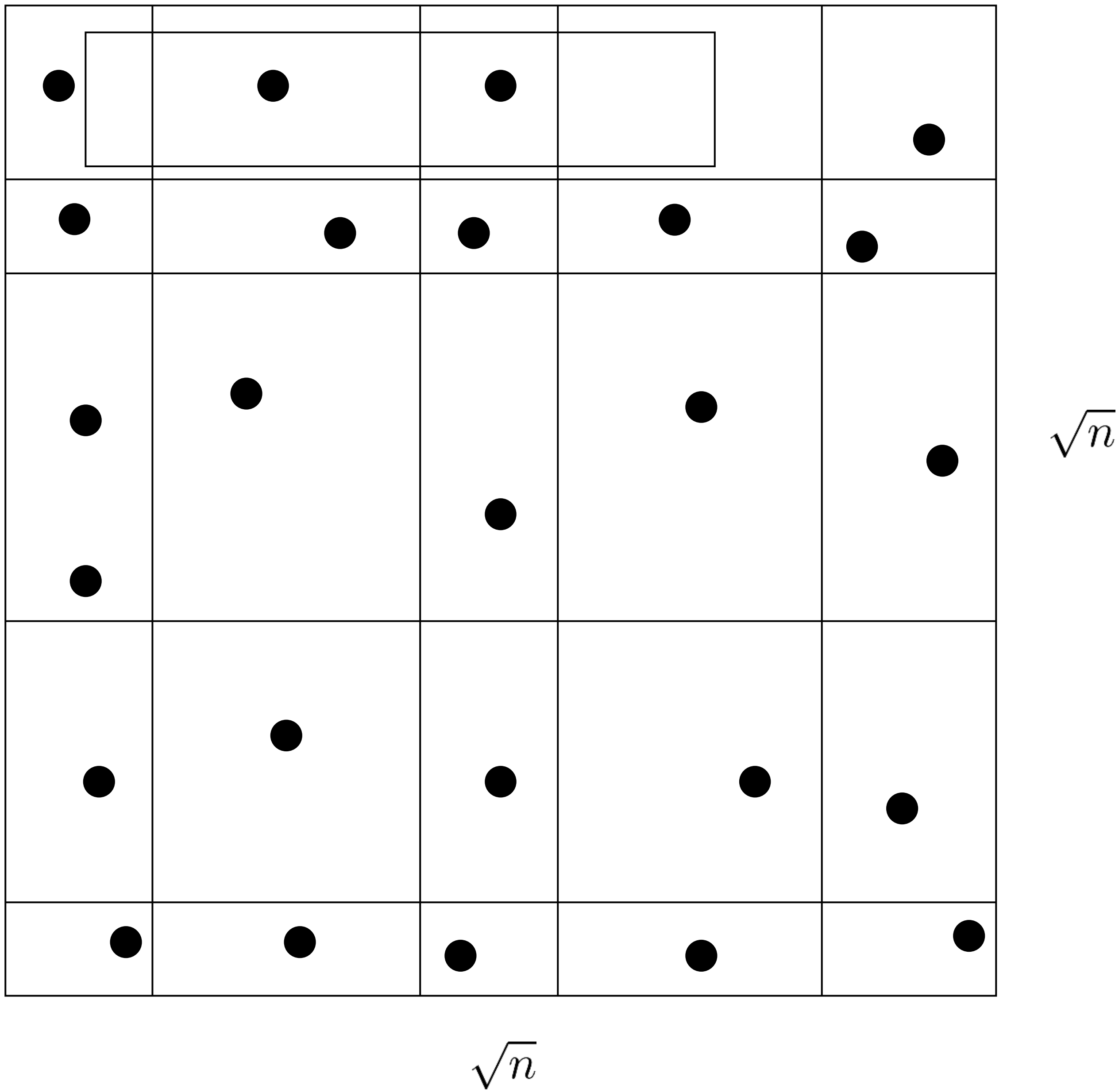
$\rightarrow$ coloring with $O(\sqrt{n})$ colors



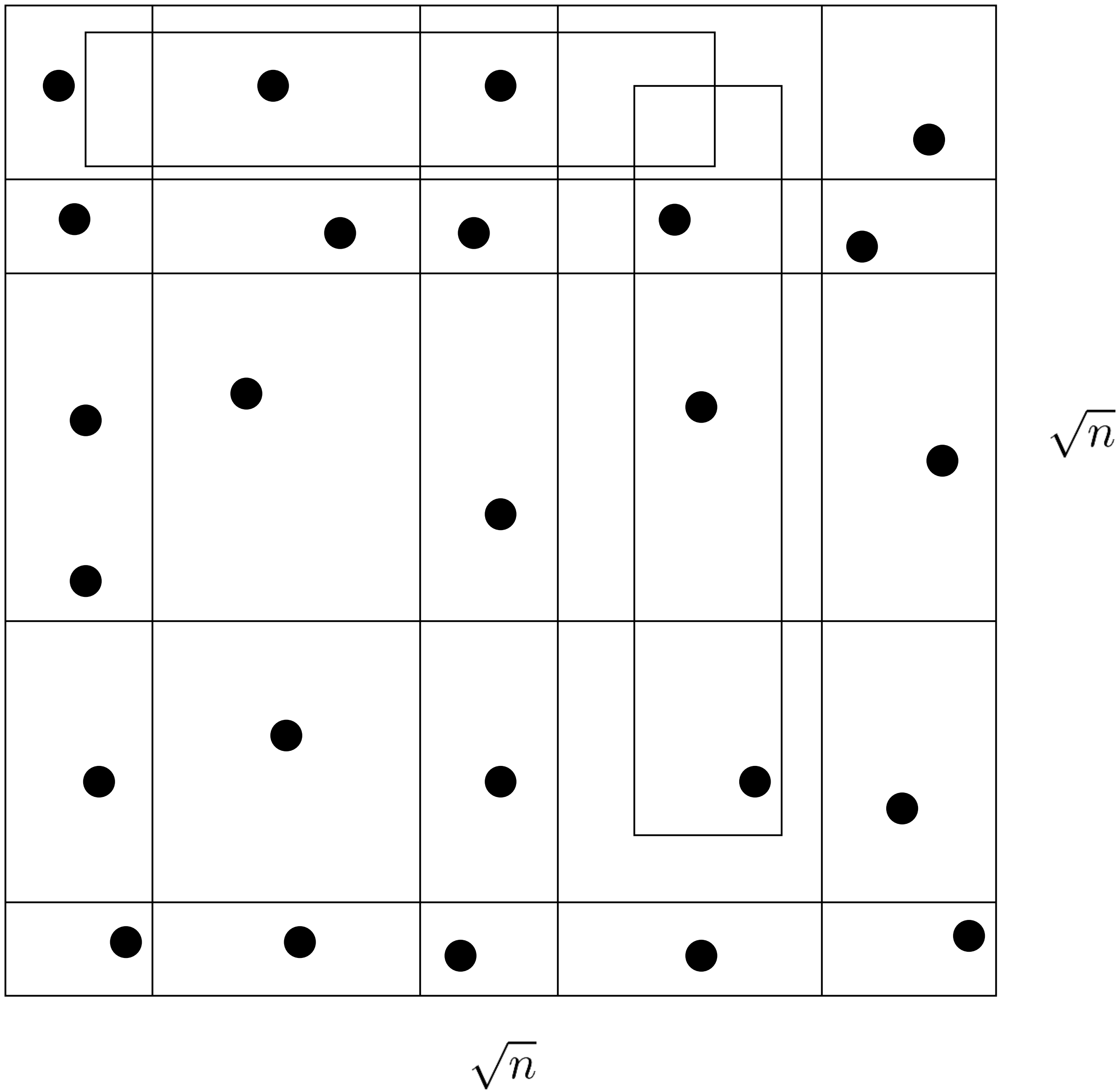
Conflict-free colorings



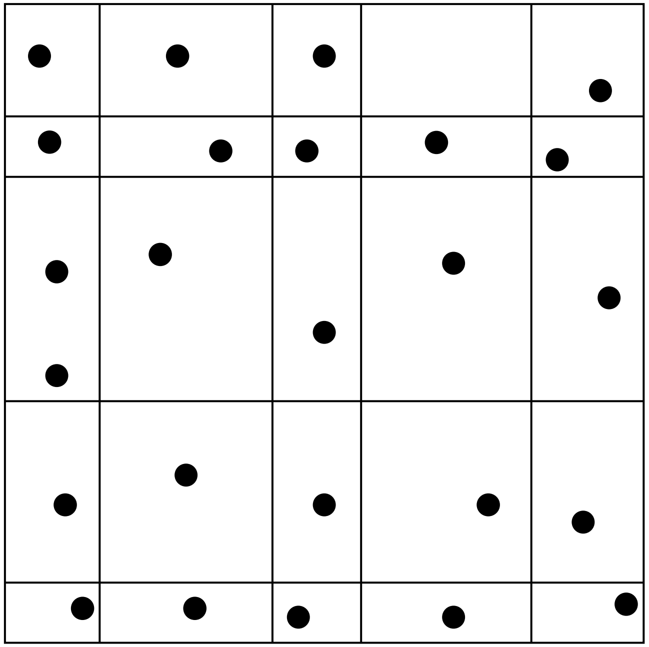
Conflict-free colorings



Conflict-free colorings

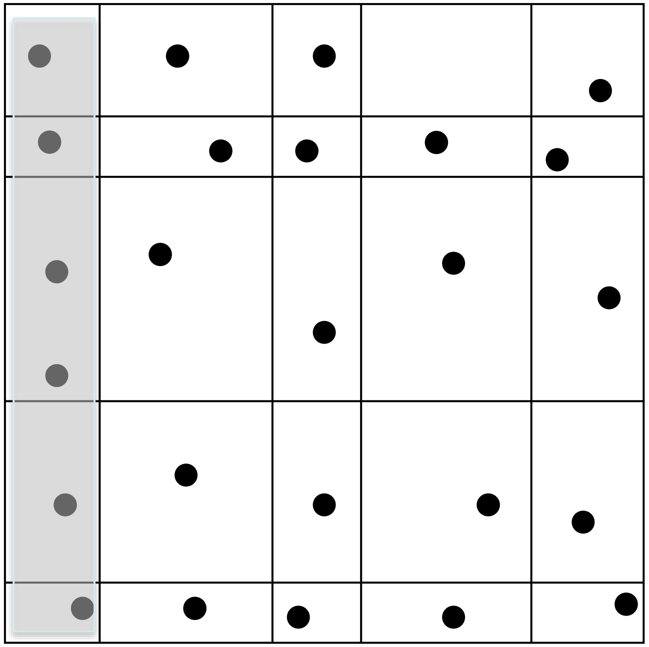


Conflict-free colorings



Conflict-free colorings

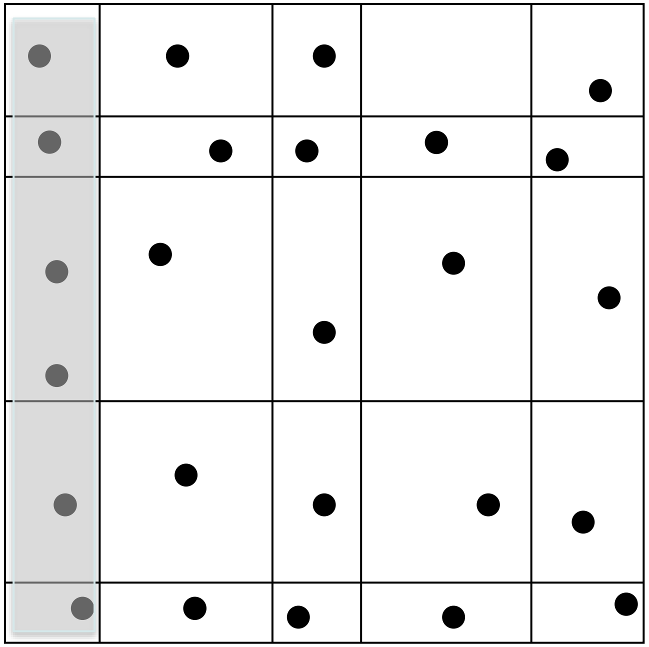
each column has $n^{\frac{1}{2}}$ points



Conflict-free colorings

each column has $n^{\frac{1}{2}}$ points

$\rightarrow$ pick a monotone subsequence of size $\Omega\left(n^{\frac{1}{4}}\right)$



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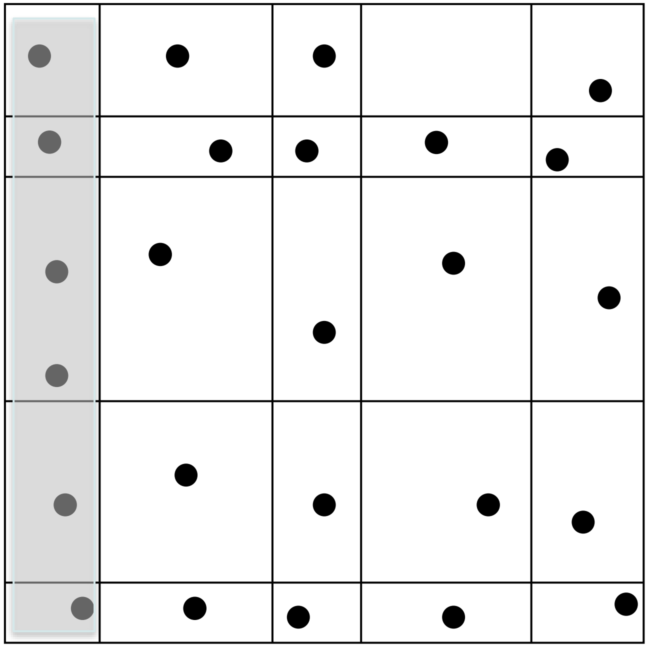
$\rightarrow$ pick a monotone subsequence of size $\Omega\left(n^{\frac{1}{4}}\right)$

for each row :



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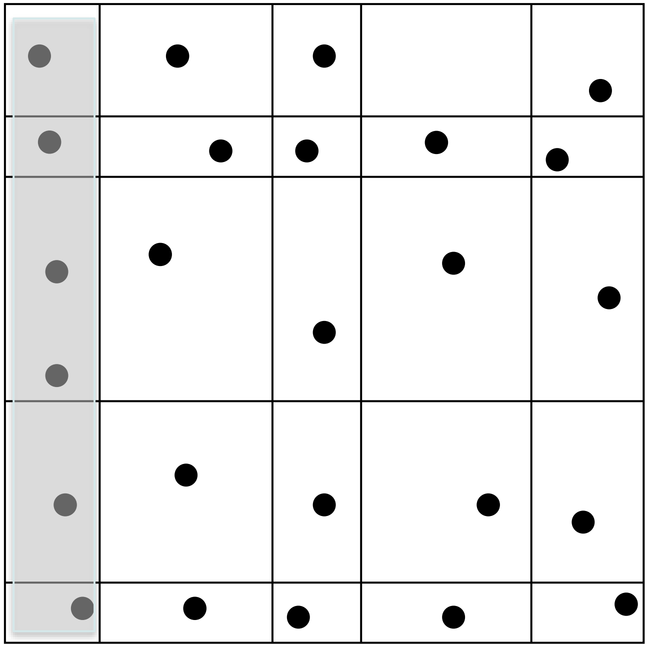
$\rightarrow$ pick a monotone subsequence of size $\Omega\left(n^{\frac{1}{4}}\right)$

for each row :

$\rightarrow$ monotonic subsequence of points in it



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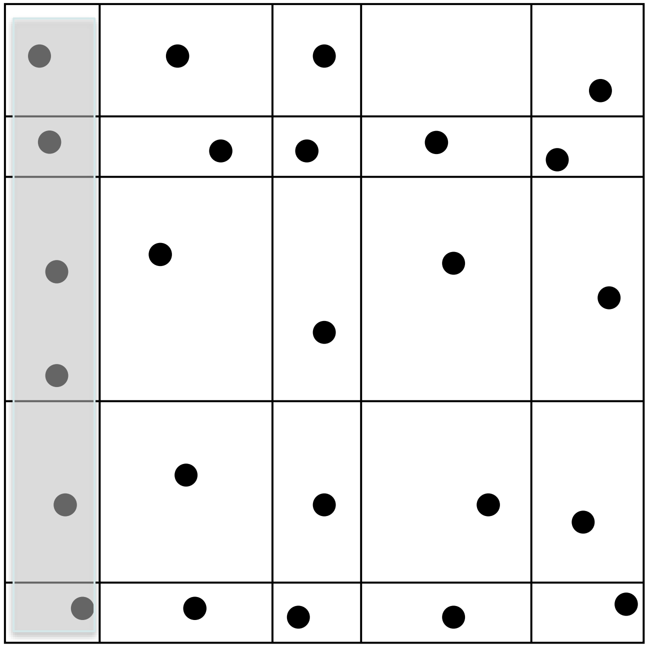
Worst case :



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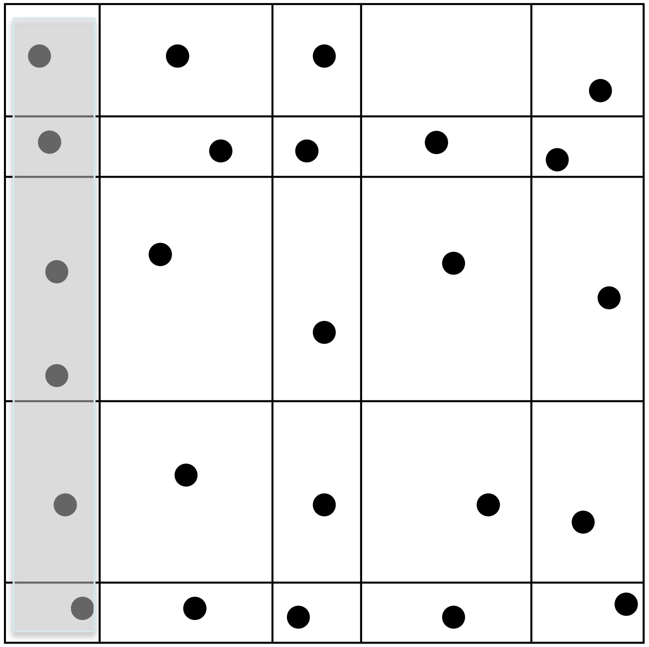
Worst case :

first $n^{\frac{1}{4}}$ rows full of chosen points for all columns



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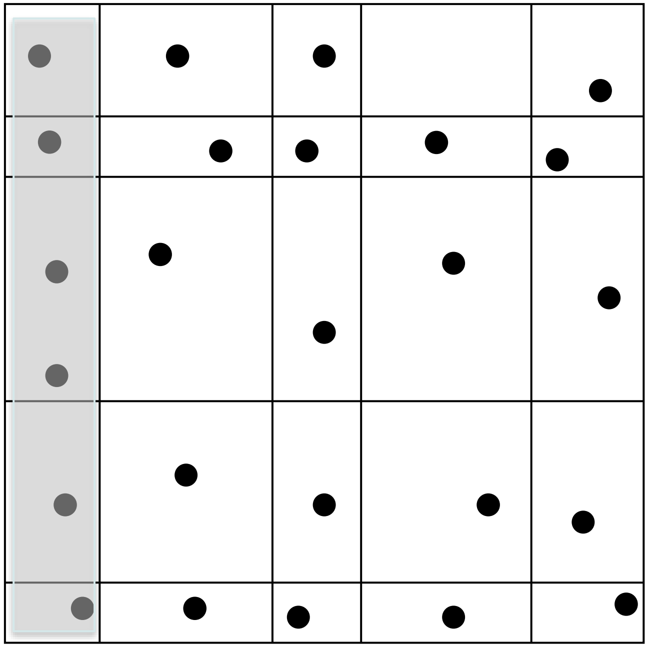
Worst case :

first $n^{\frac{1}{4}}$ rows full of chosen points for all columns

Q has size: $\Omega\left(\sqrt{n^{\frac{1}{2}}} \cdot n^{\frac{1}{4}}\right) = \Omega\left(n^{\frac{1}{2}}\right)$



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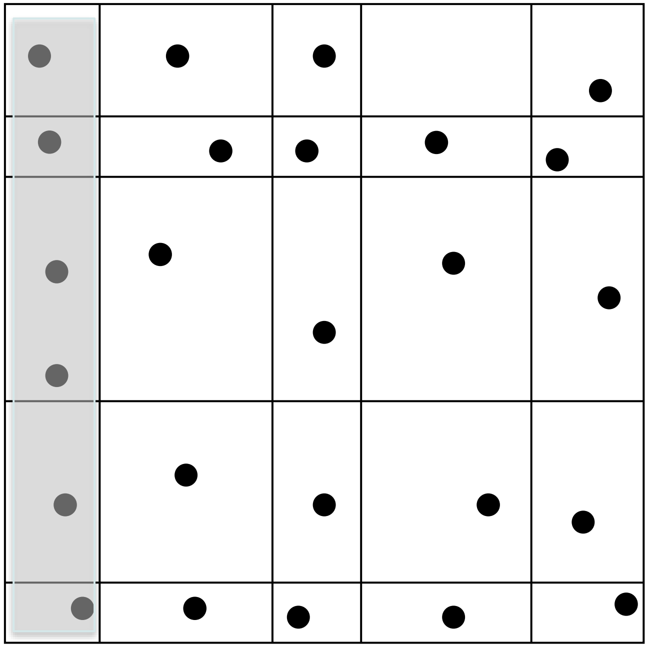
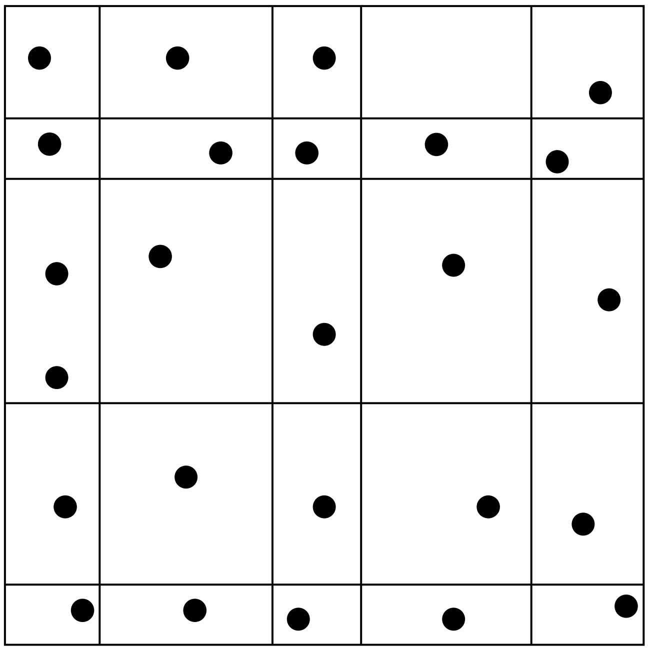


Insight : *many* monotone subsequences

Conflict-free colorings



Conflict-free colorings

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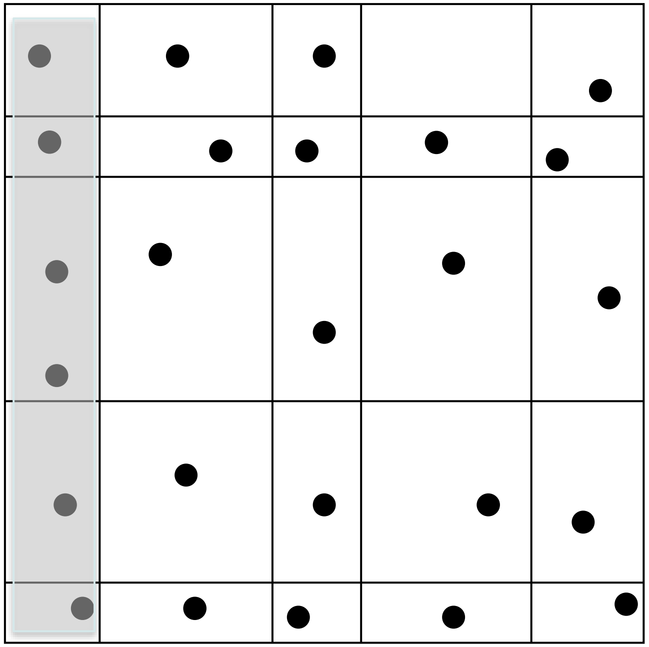
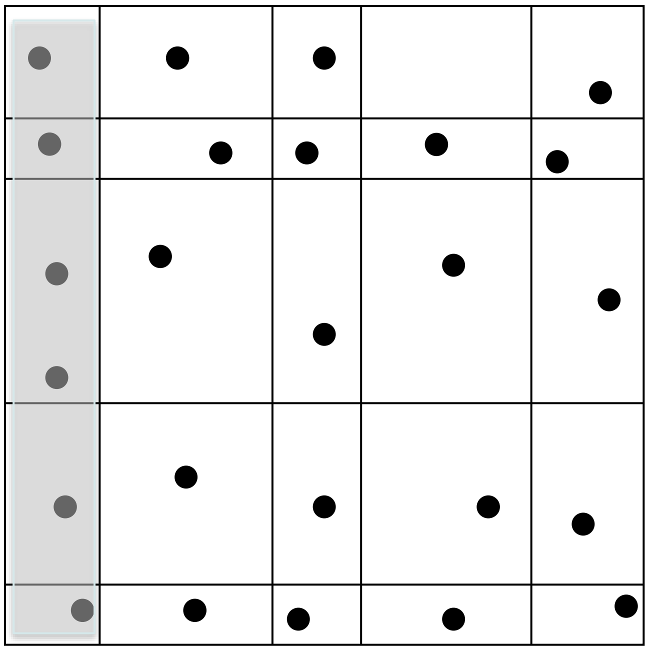


Conflict-free colorings

each column has $n^{\frac{1}{2}}$ points

$\rightarrow$ partition into $O\left(n^{\frac{1}{4}}\right)$ monotonic subsequences

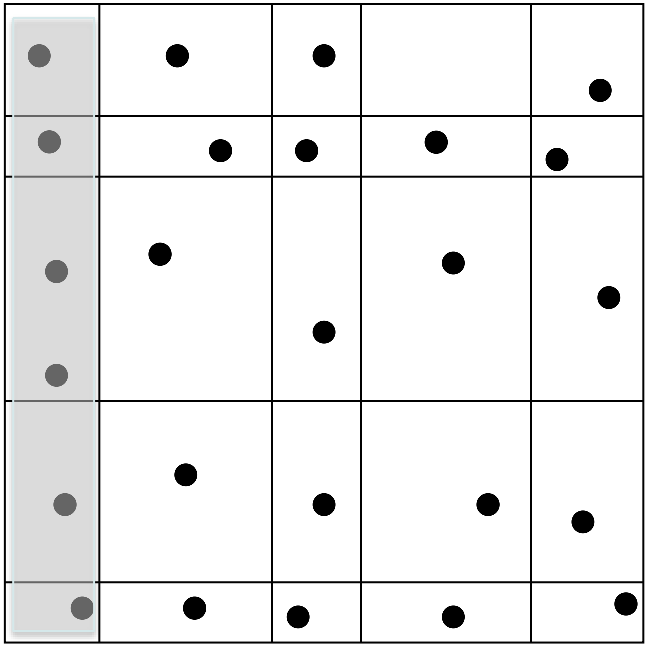


Conflict-free colorings

each column has $n^{\frac{1}{2}}$ points

$\rightarrow$ partition into $O\left(n^{\frac{1}{4}}\right)$ monotonic subsequences

$\rightarrow$ pick one uniformly at random



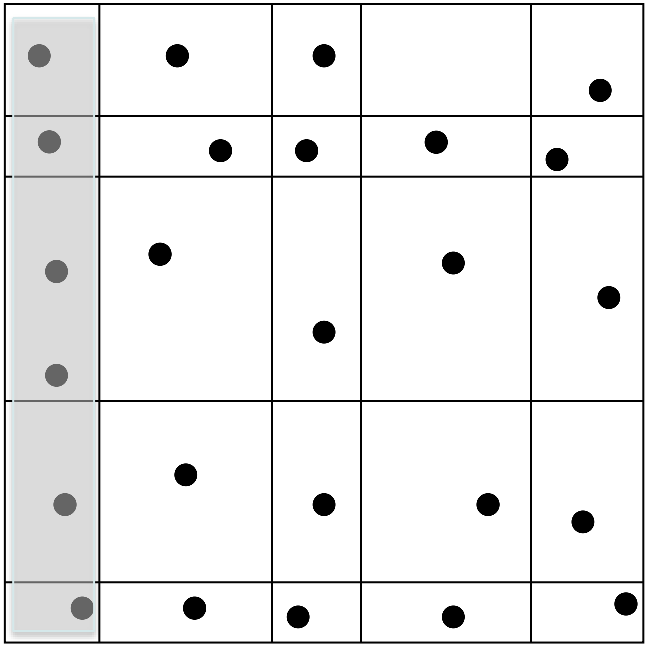
Conflict-free colorings

each column has $n^{\frac{1}{2}}$ points

$\rightarrow$ partition into $O\left(n^{\frac{1}{4}}\right)$ monotonic subsequences

$\rightarrow$ pick one uniformly at random

expected points in each row : $O\left(\frac{1}{n^{\frac{1}{4}}} \cdot n^{\frac{1}{2}}\right) = O\left(n^{\frac{1}{4}}\right)$



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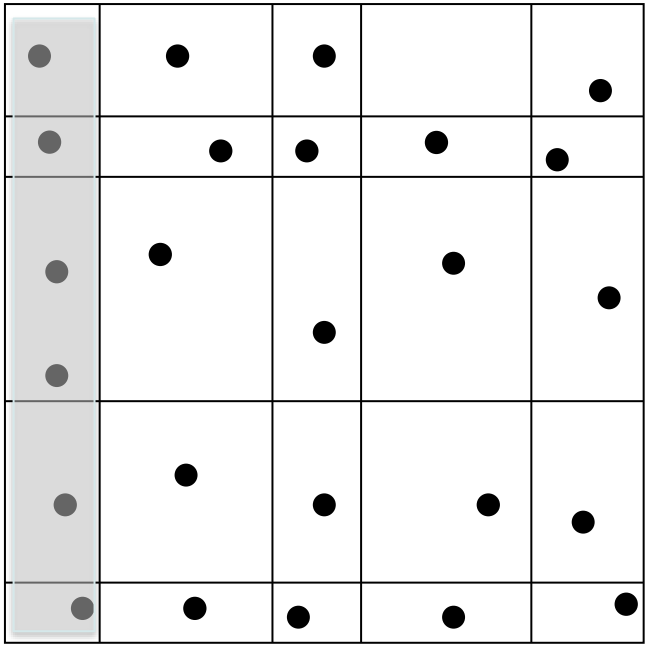
$\rightarrow$ monotonic subsequence has size $O\left(n^{\frac{1}{8}}\right)$



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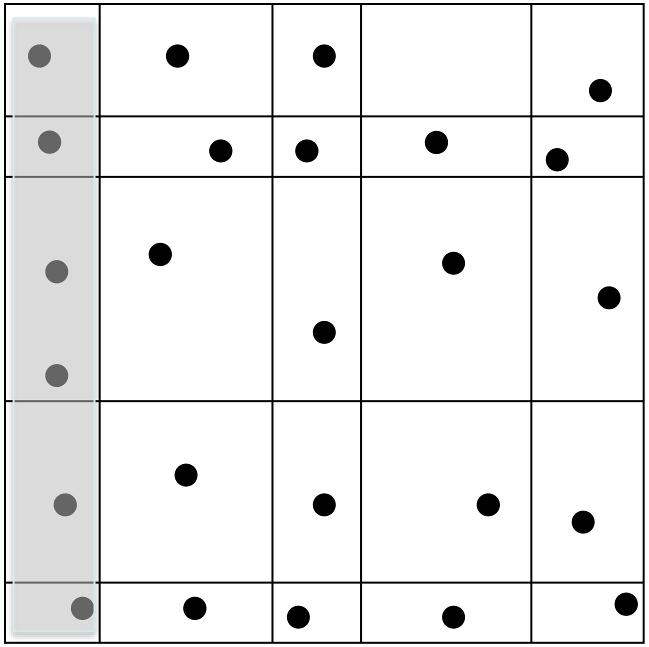
$\rightarrow$ monotonic subsequence has size $O\left(n^{\frac{1}{8}}\right)$

$\rightarrow$ strongly concentrated (Chernoff's bound)



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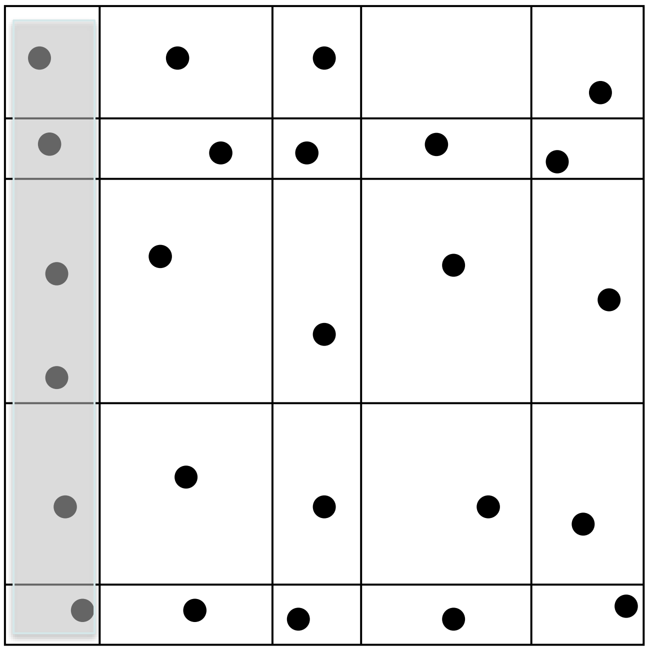
Q has size: $\tilde{O}\left(n^{\frac{1}{2}} \cdot n^{\frac{1}{8}}\right) = \tilde{O}\left(n^{\frac{5}{8}}\right)$



Conflict-free colorings

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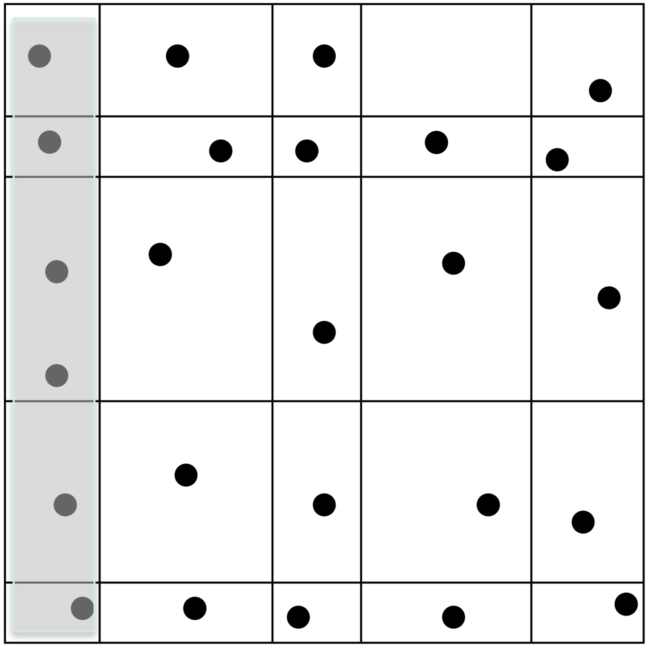
Q has size: $\tilde{O}\left(n^{\frac{1}{2}} \cdot n^{\frac{1}{8}}\right) = \tilde{O}\left(n^{\frac{5}{8}}\right)$

$\rightarrow$ coloring with $\tilde{O}\left(n^{\frac{3}{8}}\right)$ colors

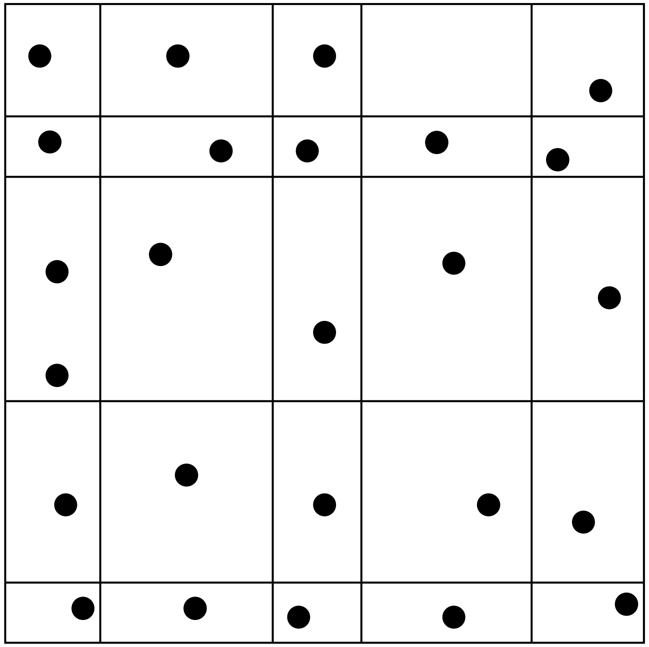
[Elbassioni, M.]



Conflict-free colorings

Grid case :

$\rightarrow$ coloring with $O\left(n^{\frac{3}{8}}\right) = O\left(n^{0.375}\right)$ colors



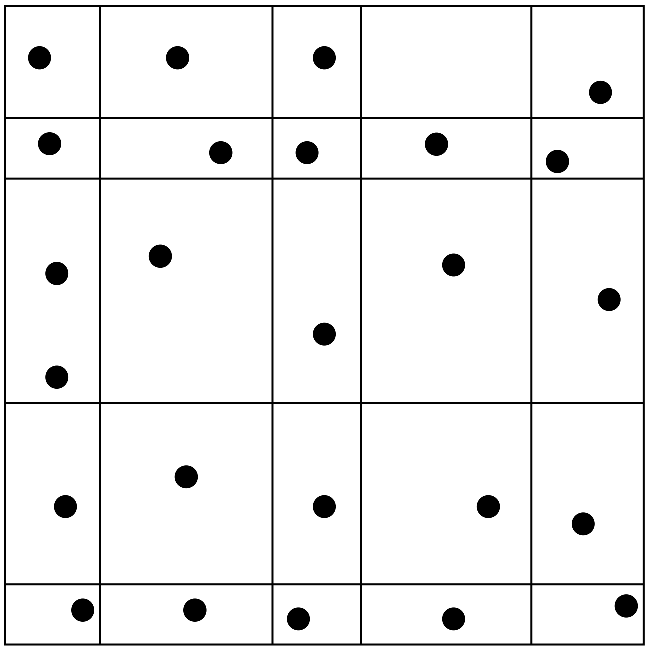
Conflict-free colorings

Grid case :

$\rightarrow$ coloring with $O\left(n^{\frac{3}{8}}\right) = O\left(n^{0.375}\right)$ colors

General case with $O(n^{1-\epsilon})$ Steiner points :

$\rightarrow$ coloring with $O\left(n^{\frac{3(1+\epsilon)}{8}}\right)$ colors



Conflict-free colorings

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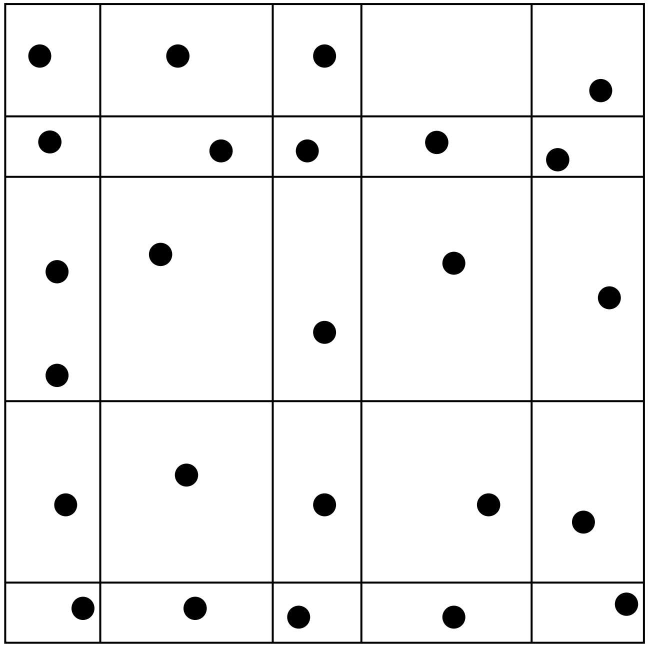
General case with $O(n^{1-\epsilon})$ Steiner points :

$\rightarrow$ coloring with $O\left(n^{\frac{3(1+\epsilon)}{8}}\right)$ colors

General case :

$\rightarrow$ coloring with $O\left(n^{0.382}\right)$ colors

[Ajwani, Elbassioni, Govindarajan, Ray]



Conflict-free colorings

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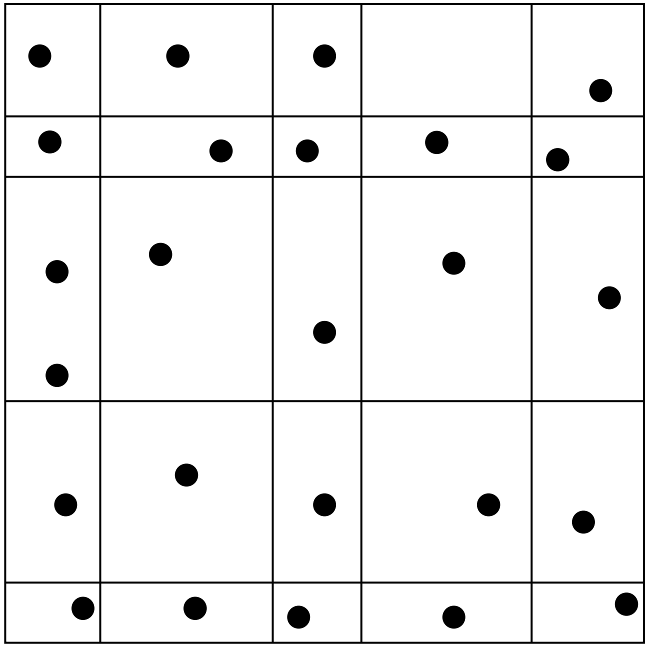


General case :

$\rightarrow$ coloring with $O\left(n^{0.382}\right)$ colors

[Ajwani, Elbassioni, Govindarajan, Ray]

$\rightarrow$ coloring with $O\left(n^{0.368}\right)$ colors

[Chan]

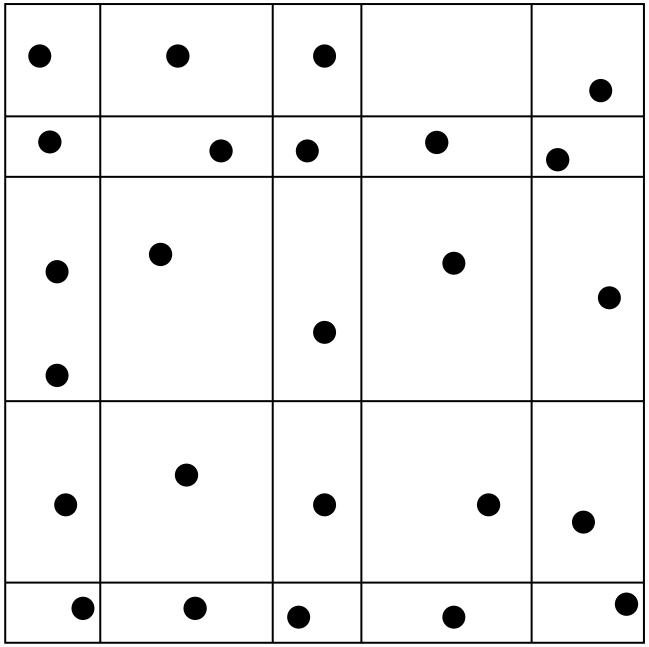
Conflict-free colorings

Grid case :

→ coloring with $O\left(n^{\frac{3}{8}}\right) = O\left(n^{0.375}\right)$ colors

General case with $O(n^{1-\epsilon})$ Steiner points :

→ coloring with $O\left(n^{\frac{3(1+\epsilon)}{8}}\right)$ colors



General case :

→ coloring with $O\left(n^{0.382}\right)$ colors

[Ajwani, Elbassioni, Govindarajan, Ray]

→ coloring with $O\left(n^{0.368}\right)$ colors

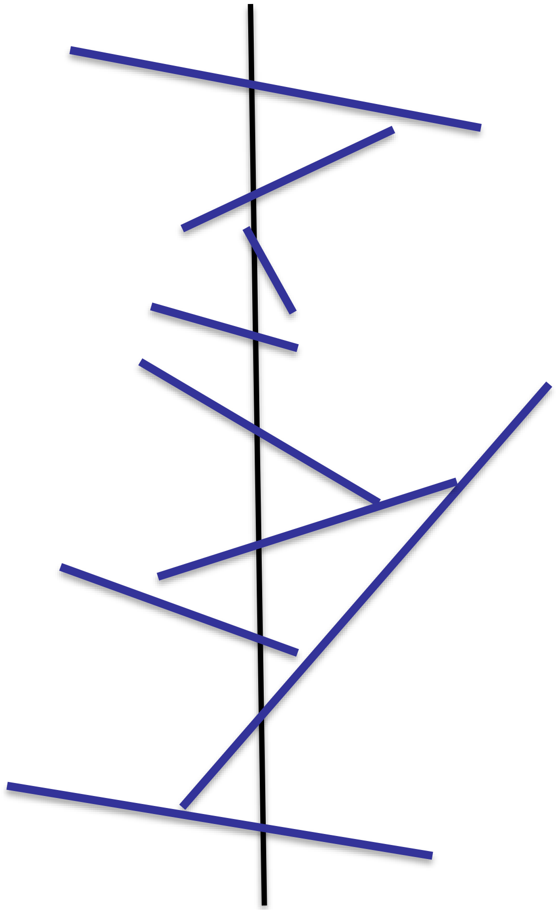
[Chan]

Open Problem 1

Independent Sets

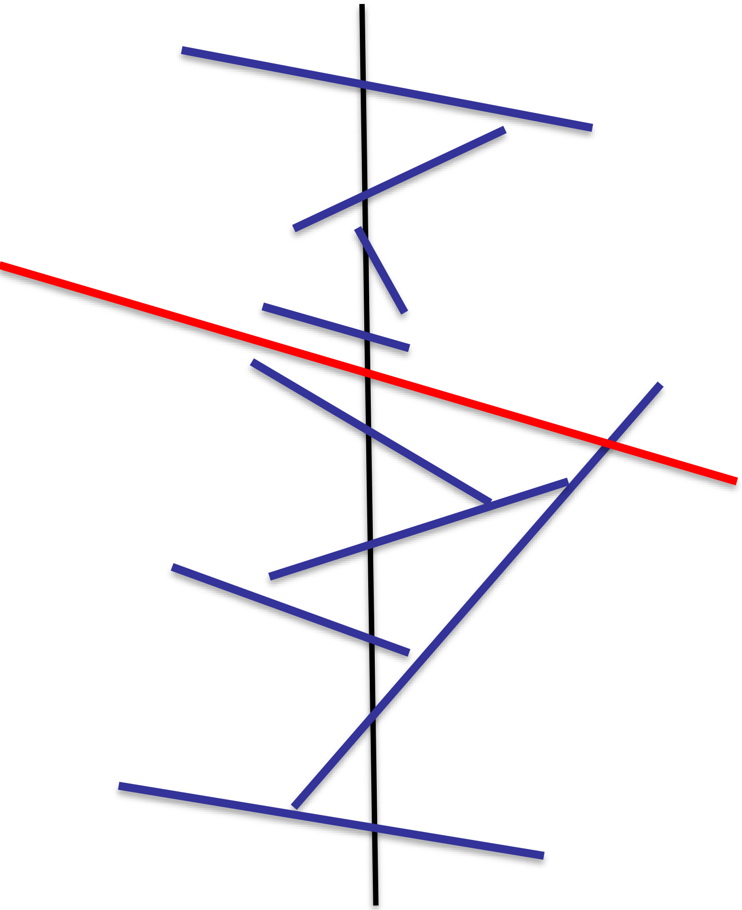
Line segments

Goal: linear separation



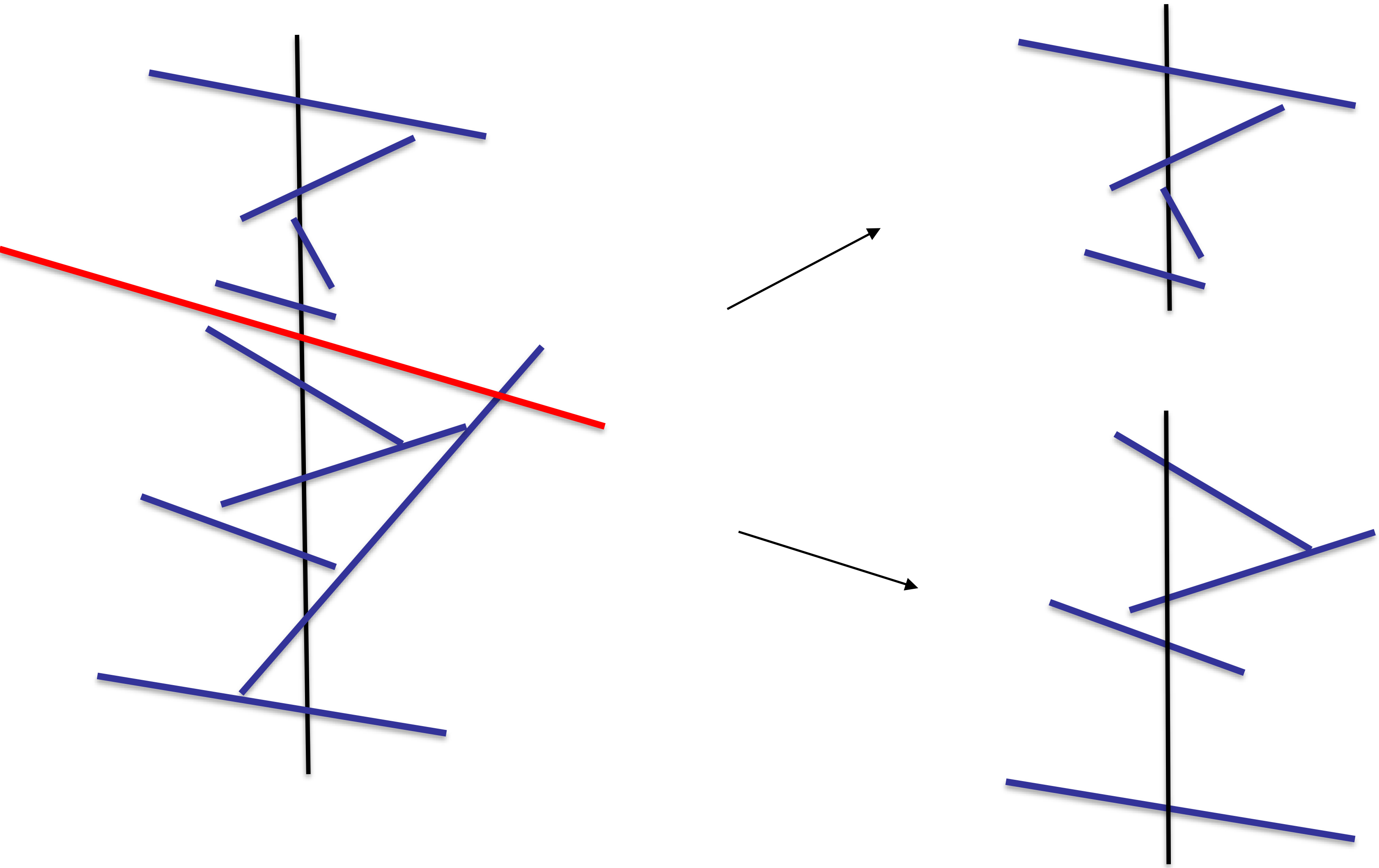
Line segments

Goal: linear separation



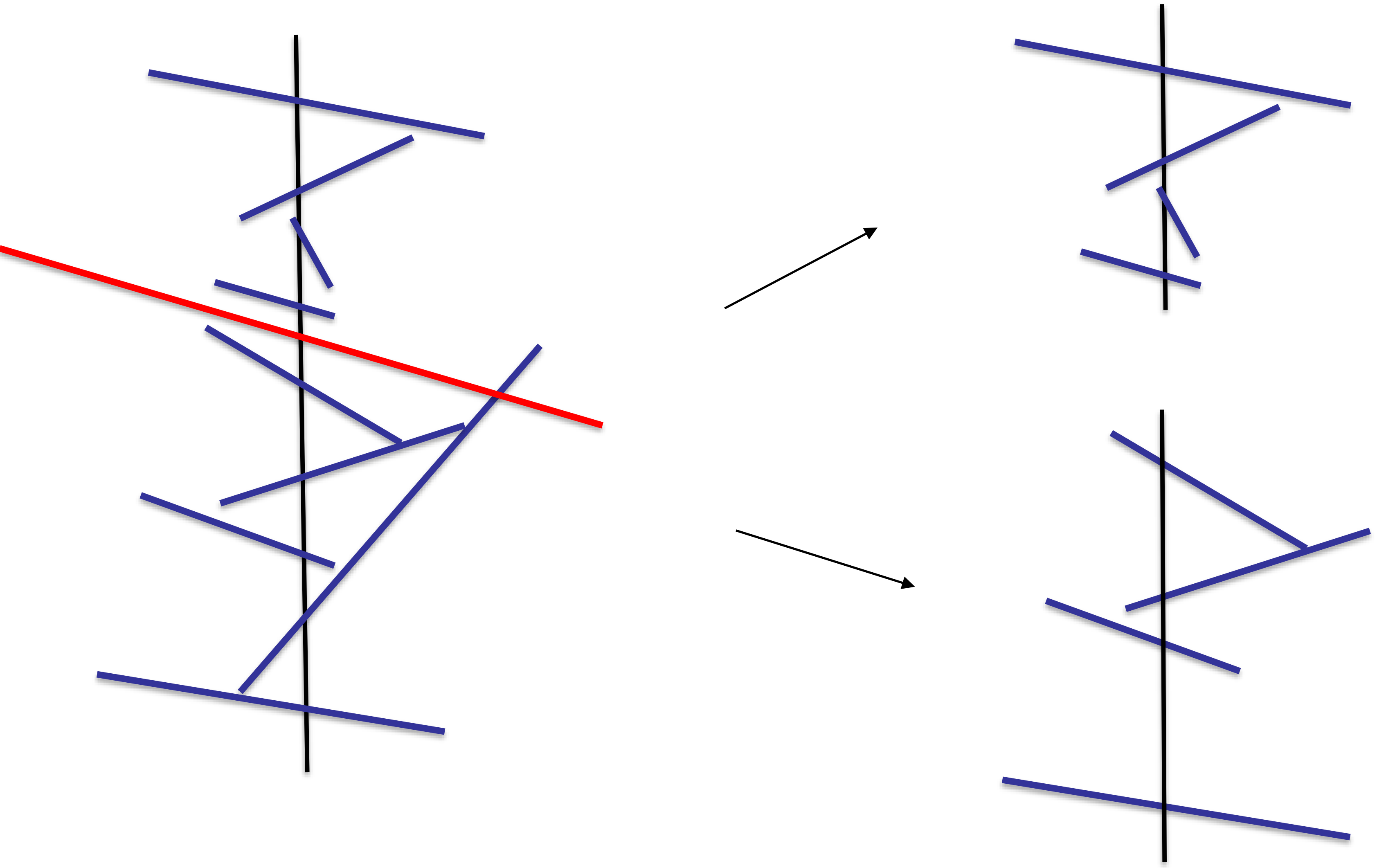
Line segments

Goal: linear separation



Line segments

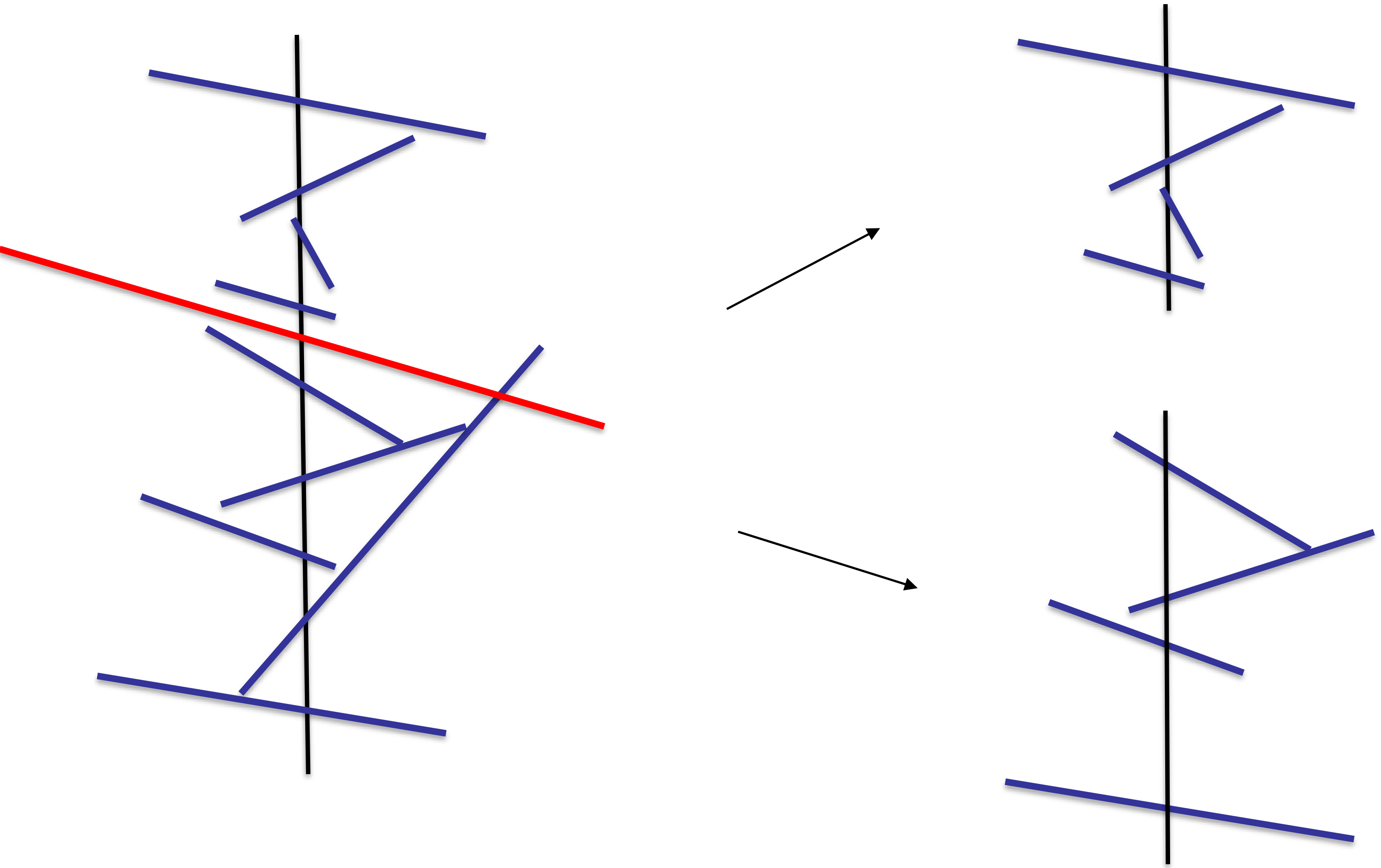
Goal: linear separation



Question: how many can be separated ?

Line segments

Goal: linear separation



Question: how many can be separated ?

→ approximation for independent set

Line segments

Claim : possible to get $\Omega\left(\sqrt{n}\right)$ segments separated

Line segments

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$\rightarrow$ sort segments by intersection with line

Line segments

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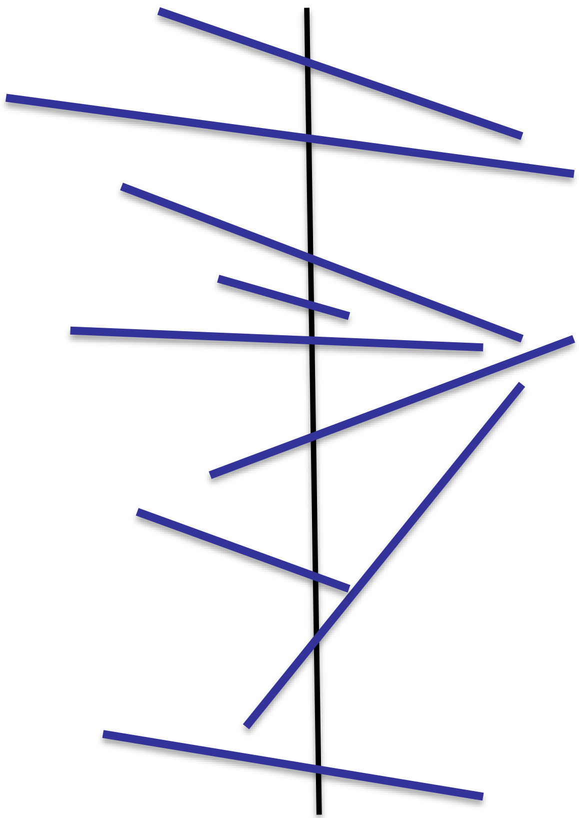
$\rightarrow$ monotonic subsequence by slopes

Line segments

Claim : possible to get $\Omega\left(\sqrt{n}\right)$ segments separated

→ sort segments by intersection with line

→ monotonic subsequence by slopes

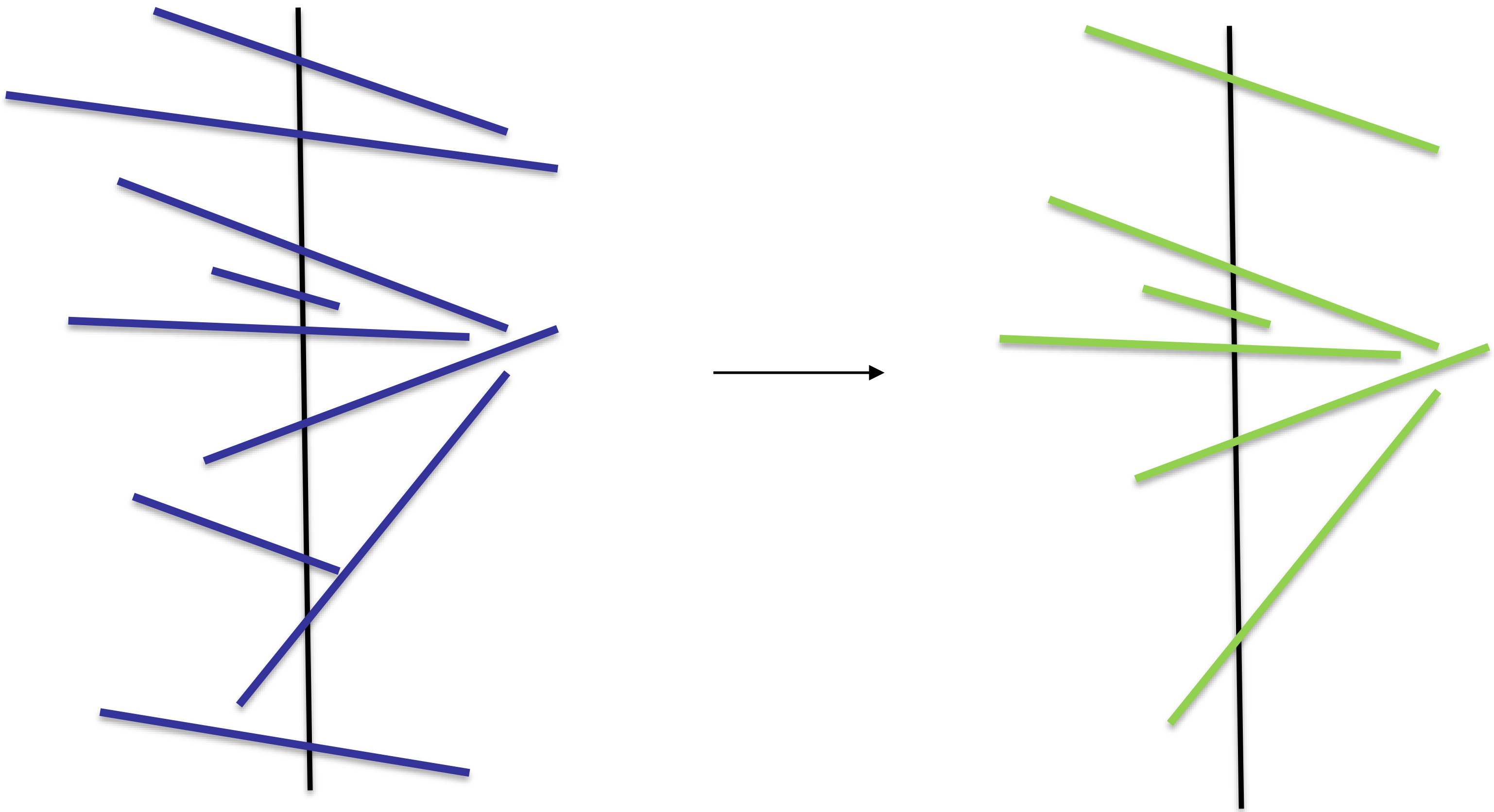


Line segments

Claim : possible to get $\Omega(\sqrt{n})$ segments separated [Pach, Tardos]

→ sort segments by intersection with line

→ monotonic subsequence by slopes



Line segments

Claim : can separate a monotonic subsequence

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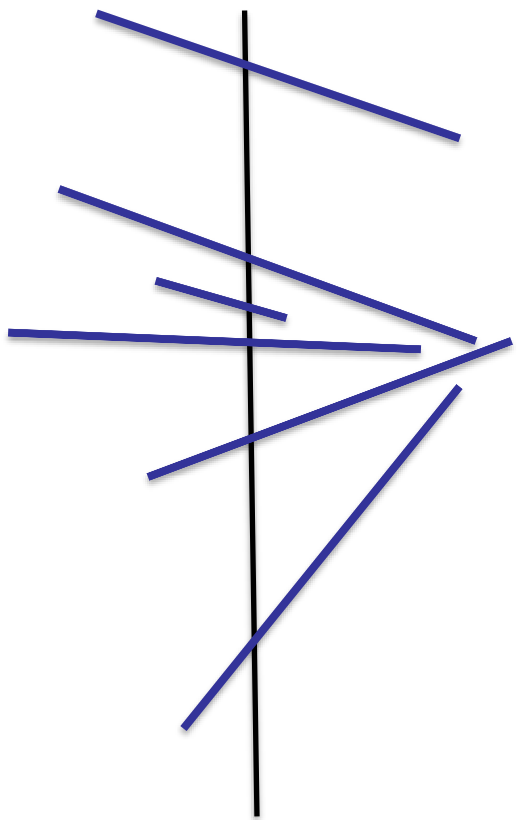

Line segments

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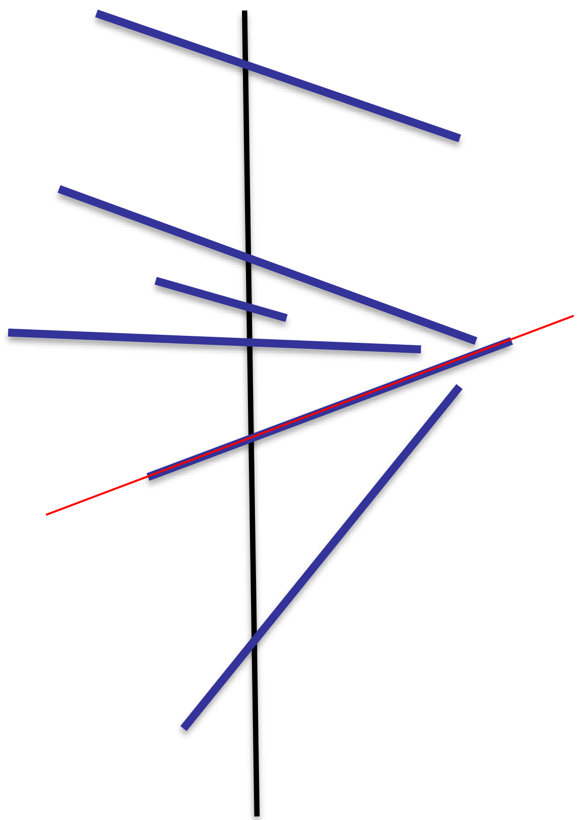

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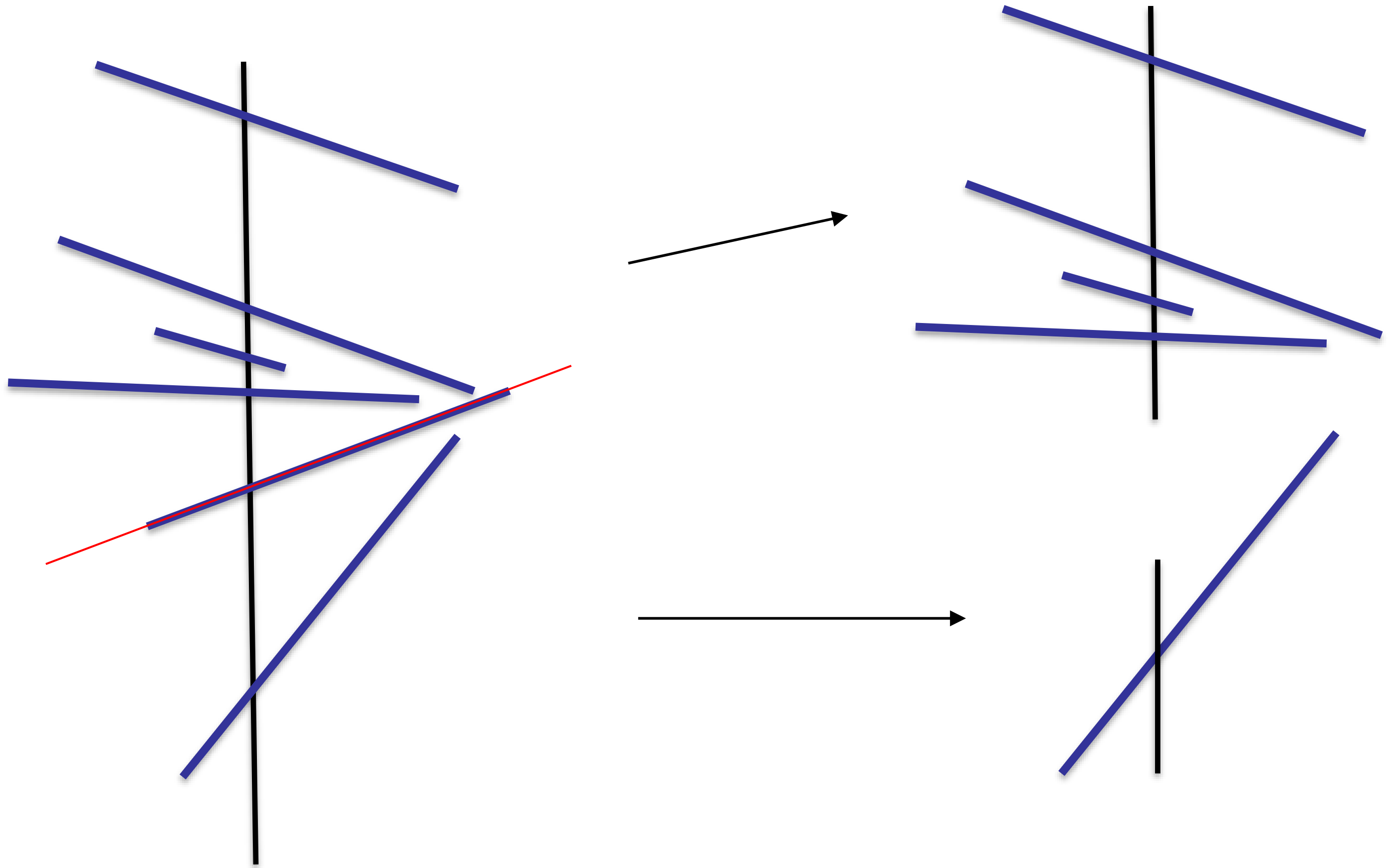

Line segments

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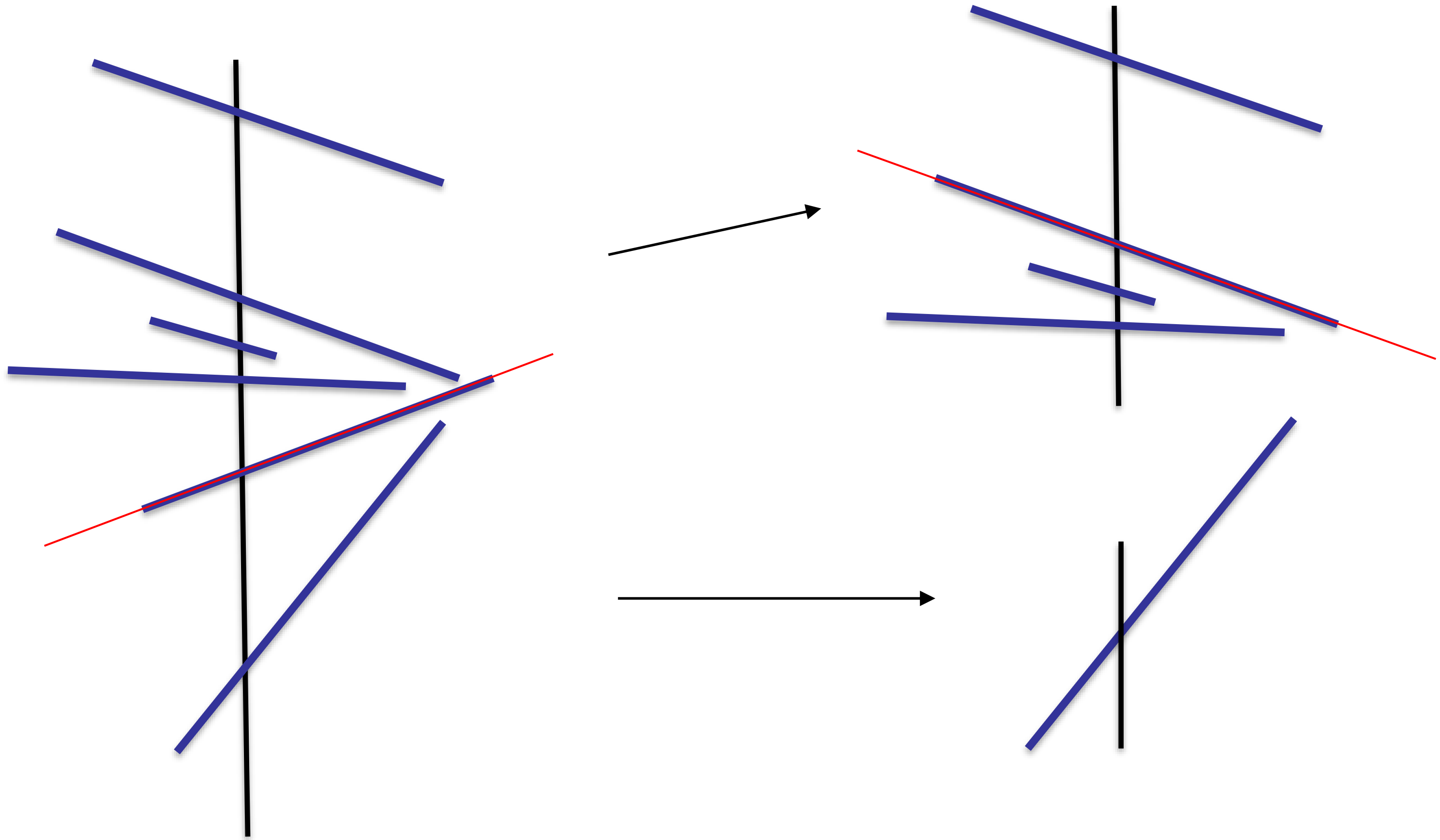

Line segments

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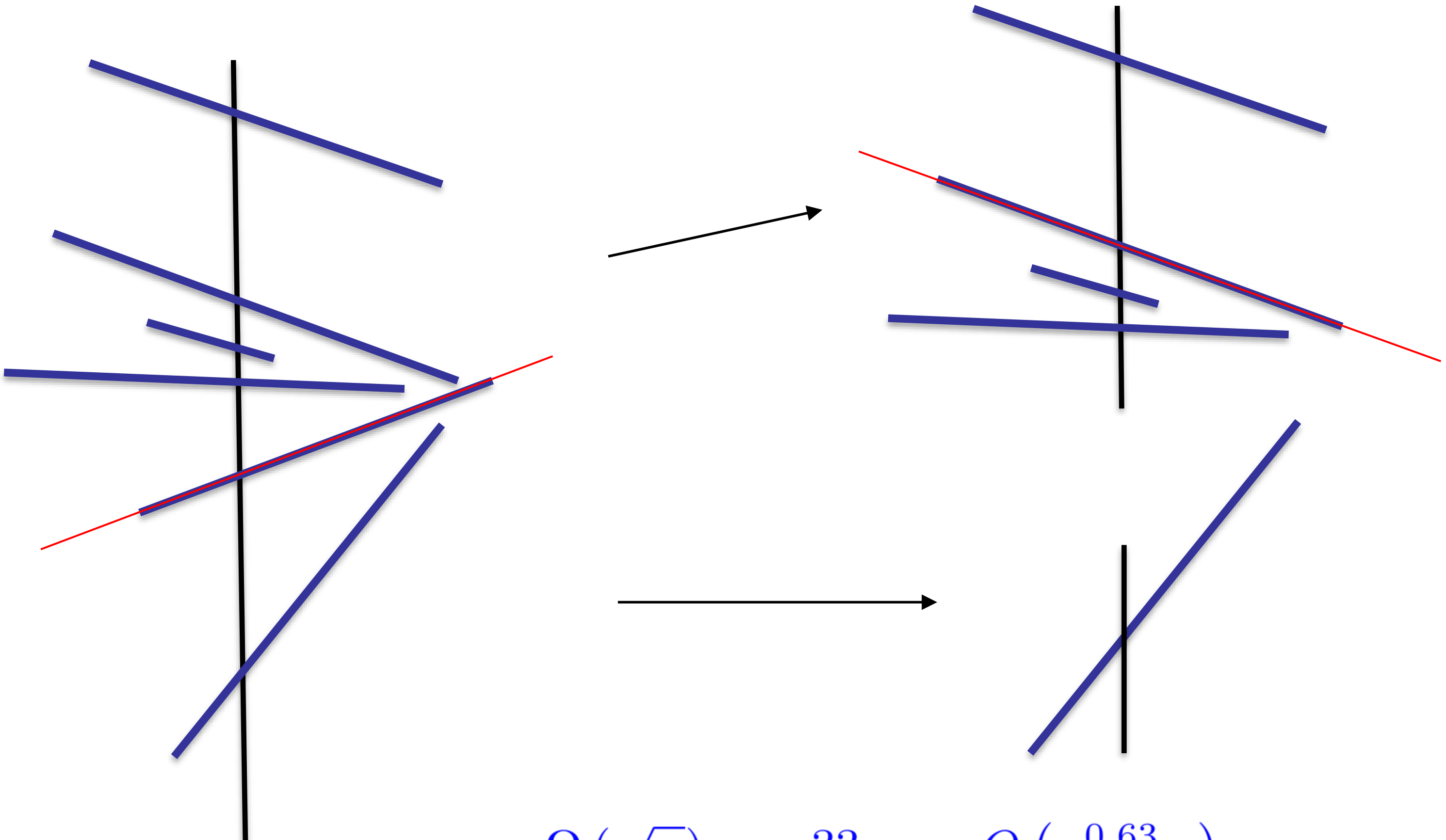
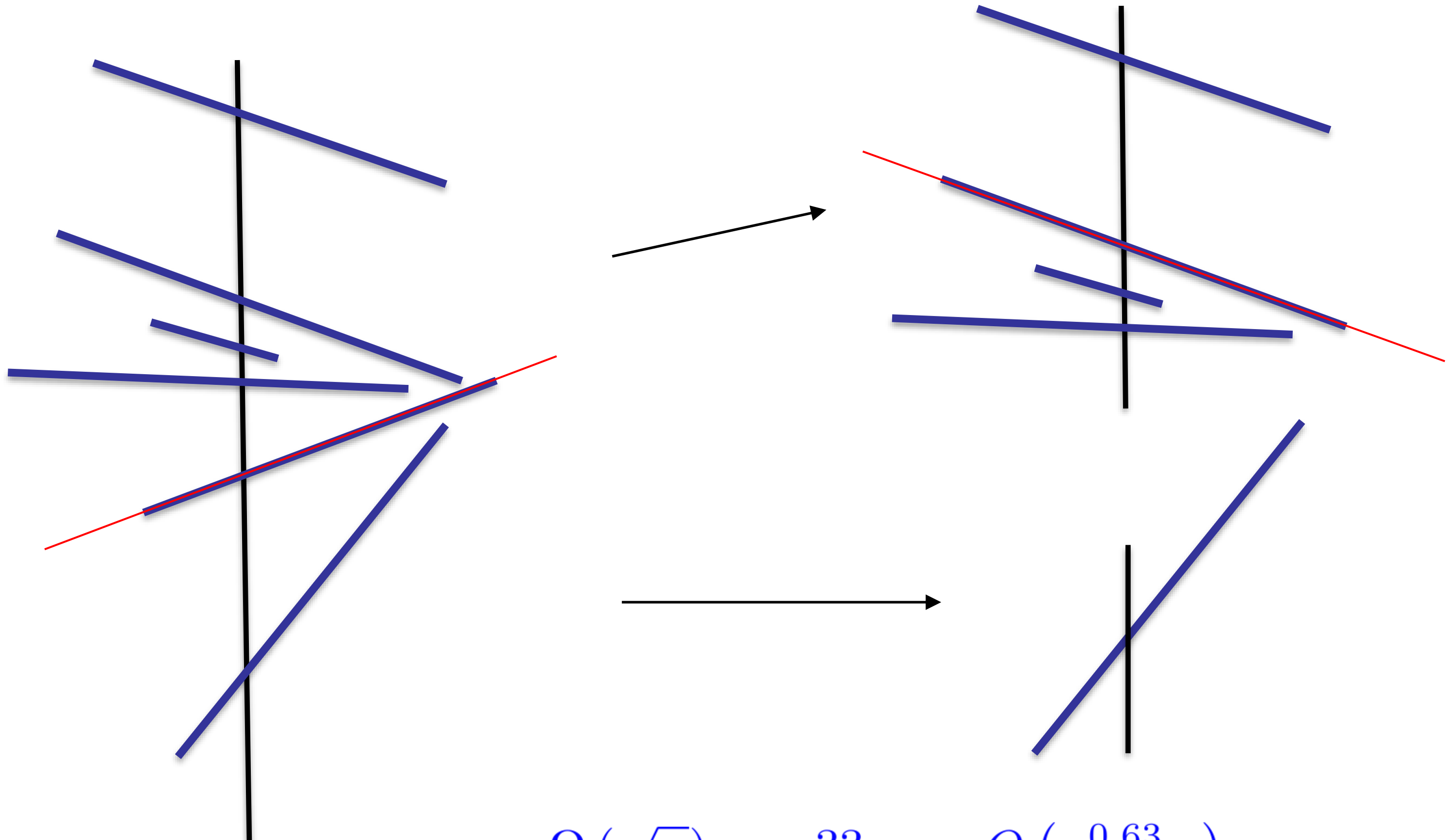


$$\Omega\left(\sqrt{n}\right) \;=\; ?? \;=\; O\left(n^{0.63\ldots}\right)$$

Line segments

Claim : can separate a monotonic subsequence

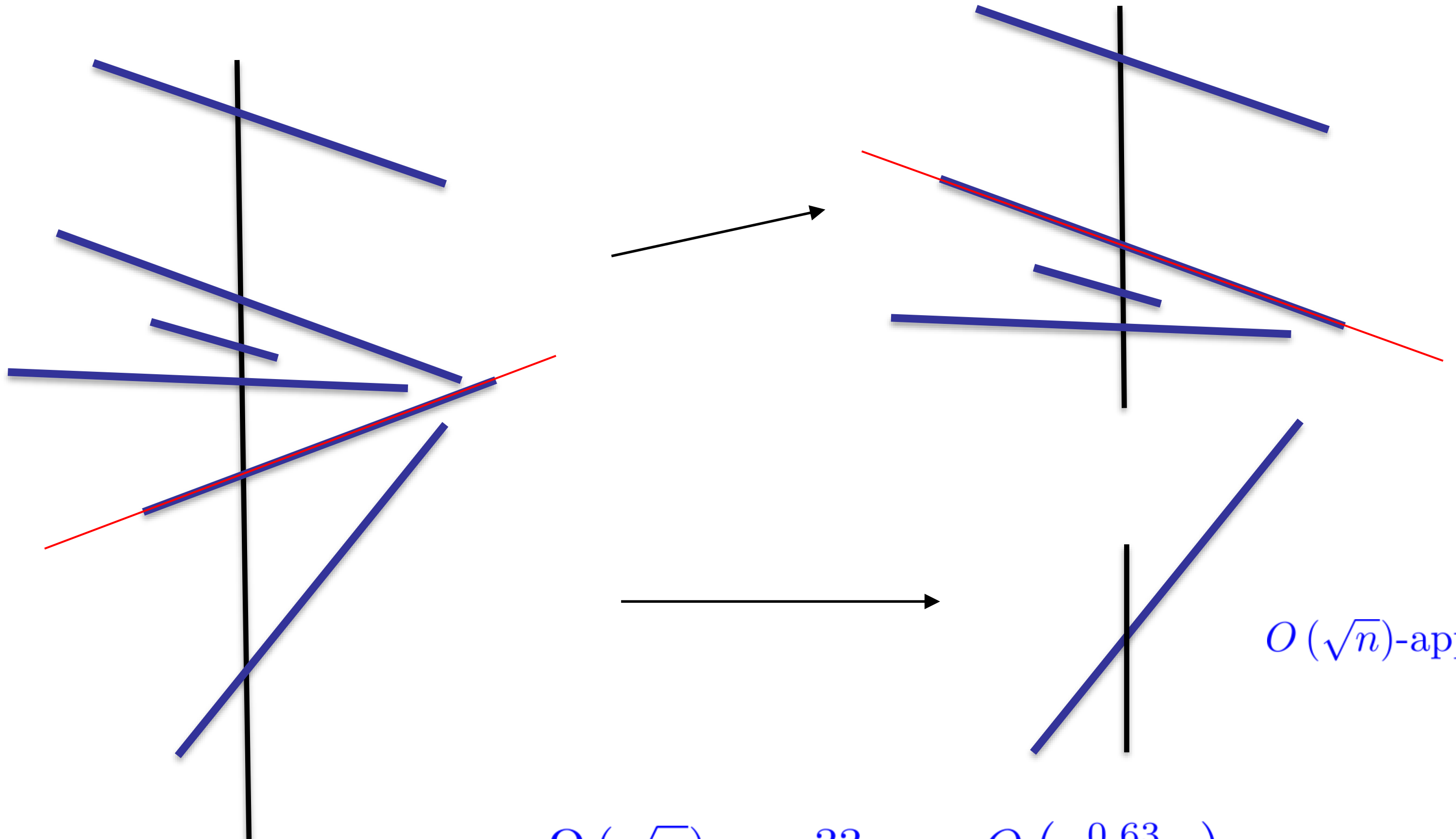


$$\Omega\left(\sqrt{n}\right) \;=\; ?? \;=\; O\left(n^{0.63\ldots}\right)$$

Open Problem 2

Line segments

Claim : can separate a monotonic subsequence

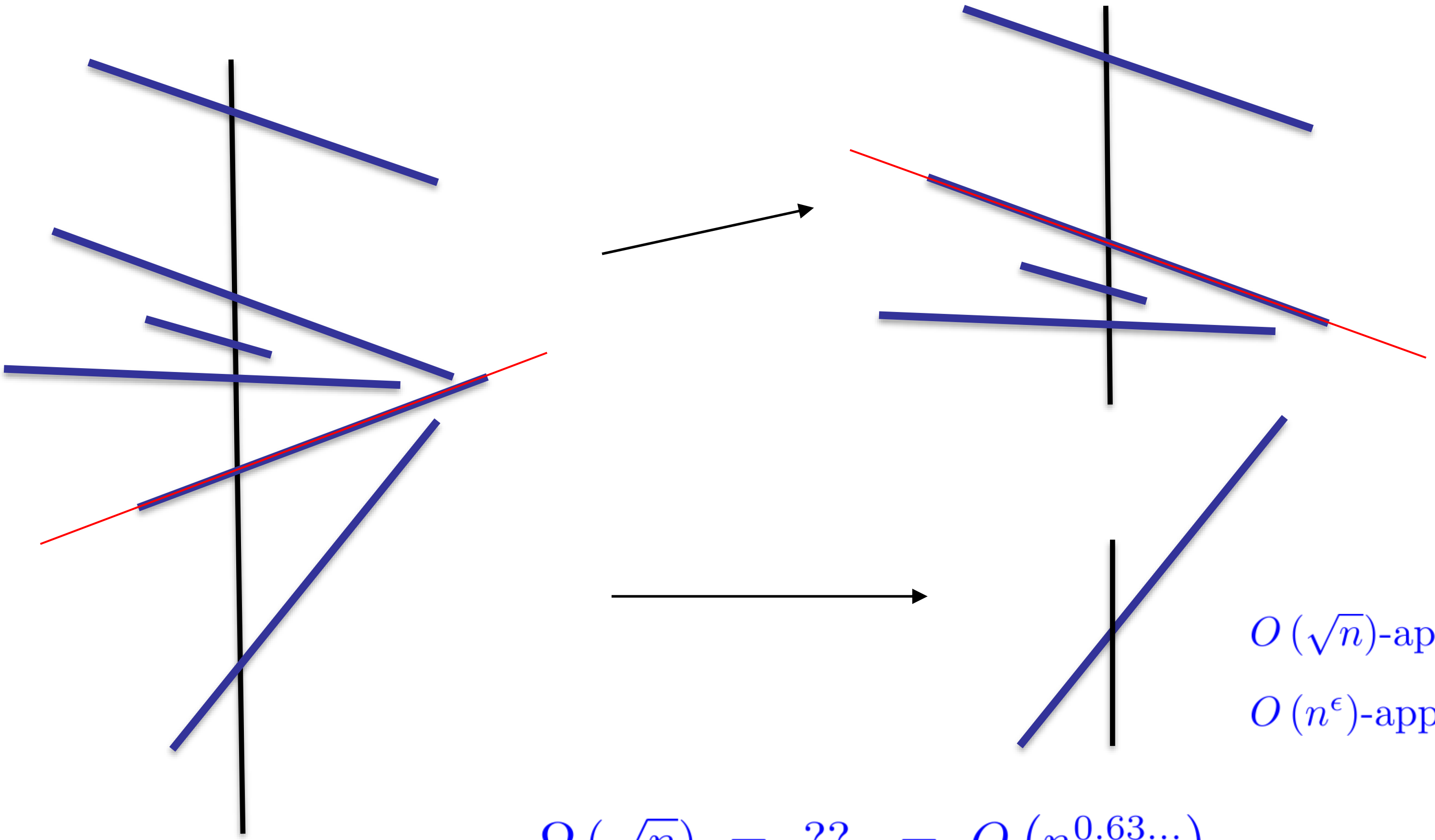


$$\Omega(\sqrt{n}) = ?? = O\left(n^{0.63\ldots}\right)$$

Open Problem 2

Line segments

Claim : can separate a monotonic subsequence



$O\left(\sqrt{n}\right)$-approximation

$O\left(n^{\epsilon}\right)$-approximation

[Fox, Pach]

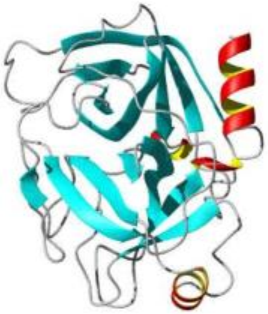
$$\Omega\left(\sqrt{n}\right) \;=\; ?? \;=\; O\left(n^{0.63\ldots}\right)$$

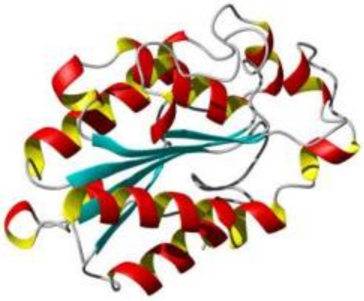
Open Problem 2

Contact-map matching

Contact-map similarity

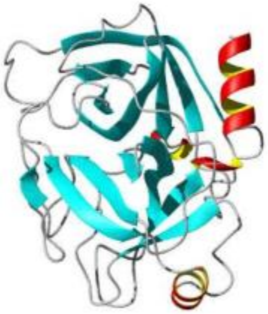
Measuring protein similarity

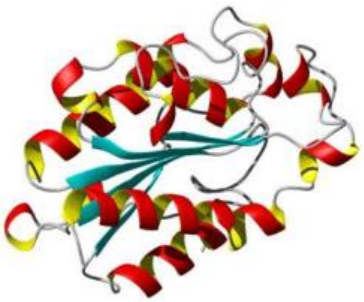




Contact-map similarity

Measuring protein similarity

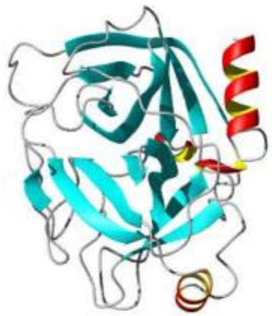


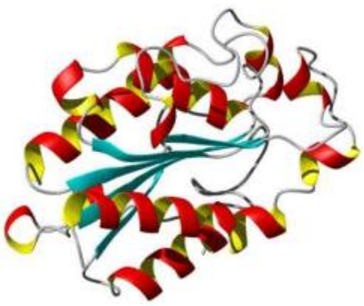


→ contact-maps

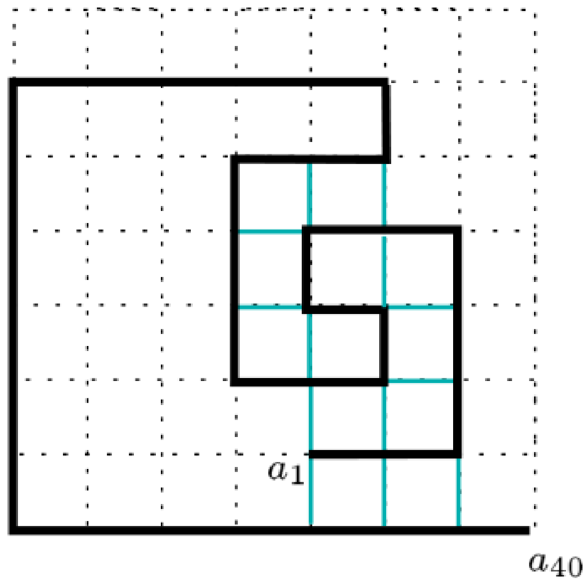
CONTACT-MAP SIMILARITY

Measuring protein similarity





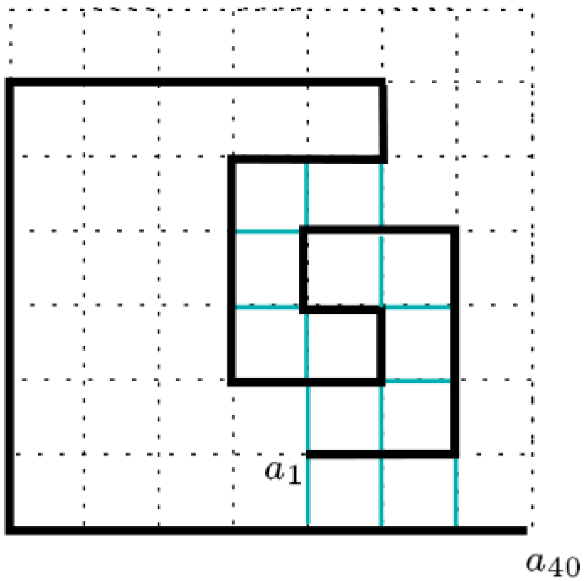
→ contact-maps

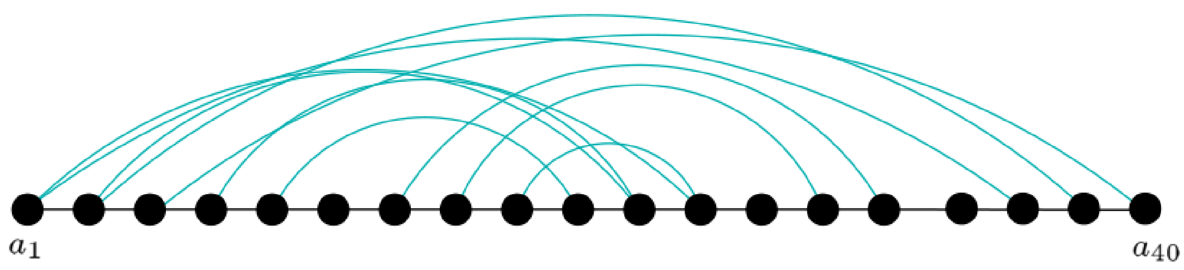


Contact-map similarity

Measuring protein similarity

→ contact-maps



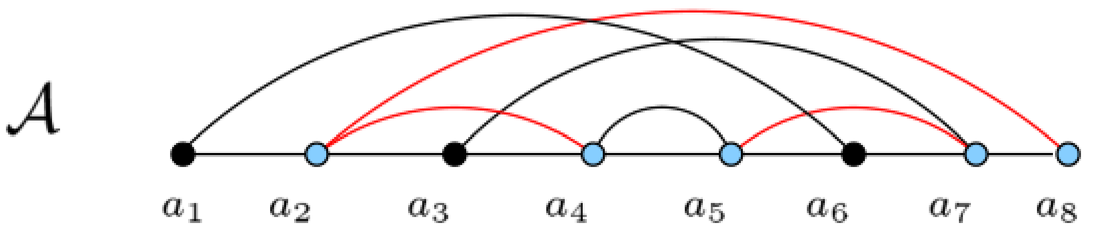


Contact-map similarity

→ order-preserving mapping $f(\cdot)$

Contact-map similarity

$\rightarrow$ order-preserving mapping $f(\cdot)$



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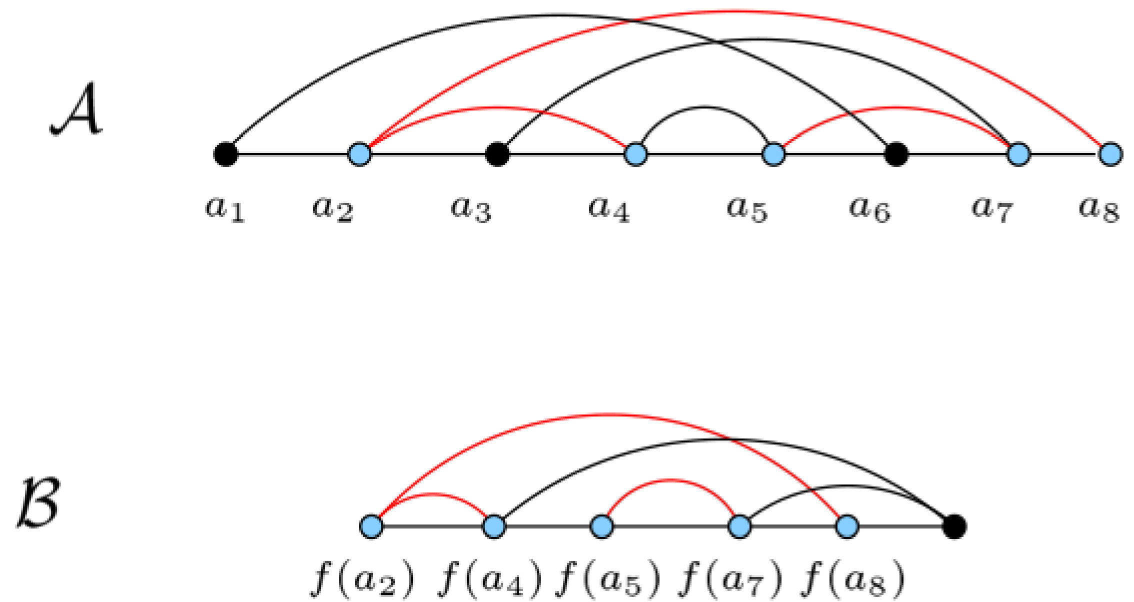

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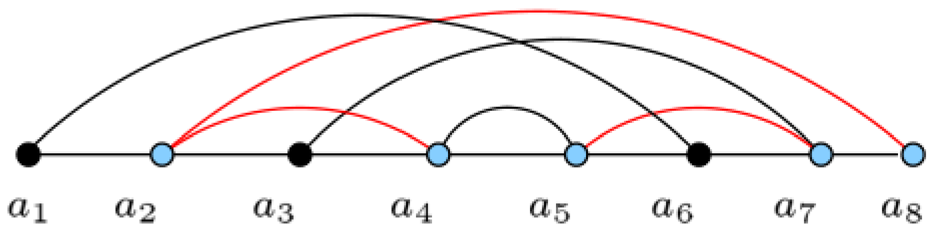
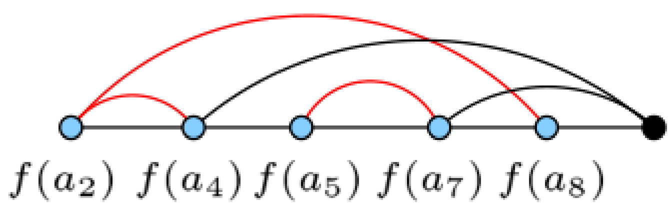
$\mathcal{A}$



$\mathcal{B}$



→ NP-hard

Contact-map similarity

$\rightarrow$ In $\mathbb{R}^2$, a nice decomposition is possible

Contact-map similarity

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Claim: Contact-map in $\mathbb{R}^2$ decomposed into 2 stacks and 1 queue

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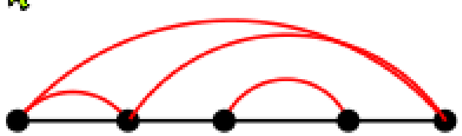

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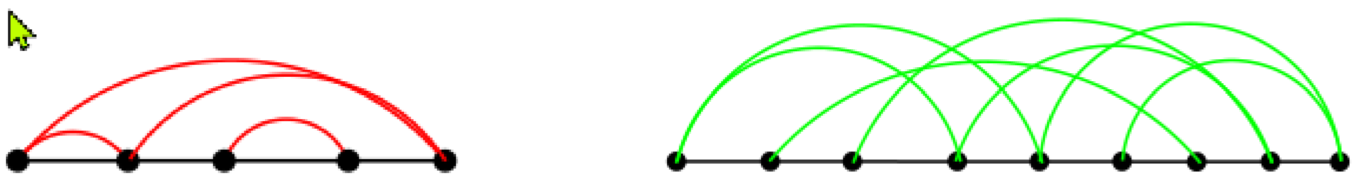

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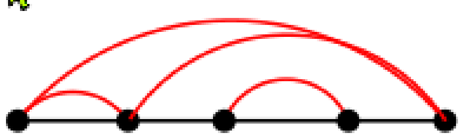

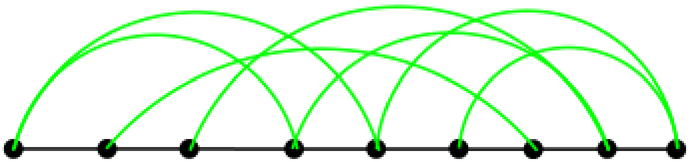

Claim: Optimal matching of a stack and a contact-map

Approximate matching of a queue and a contact-map

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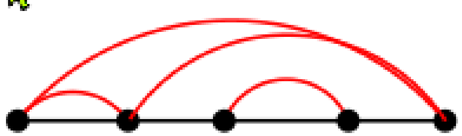
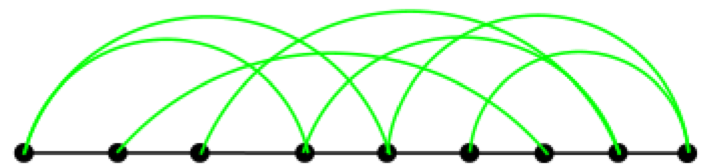
[Goldman, Istrail, Papadimitriou]





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→ 3-approximation in $\mathbb{R}^2$

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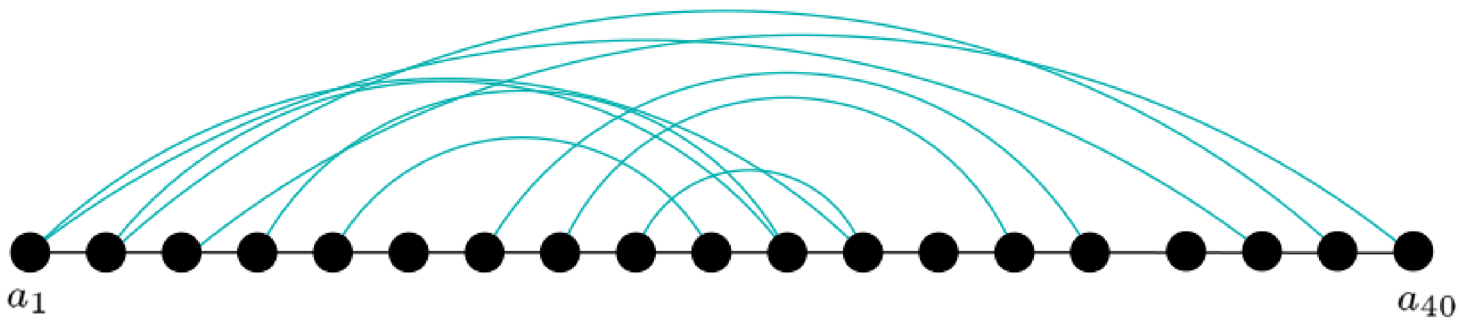

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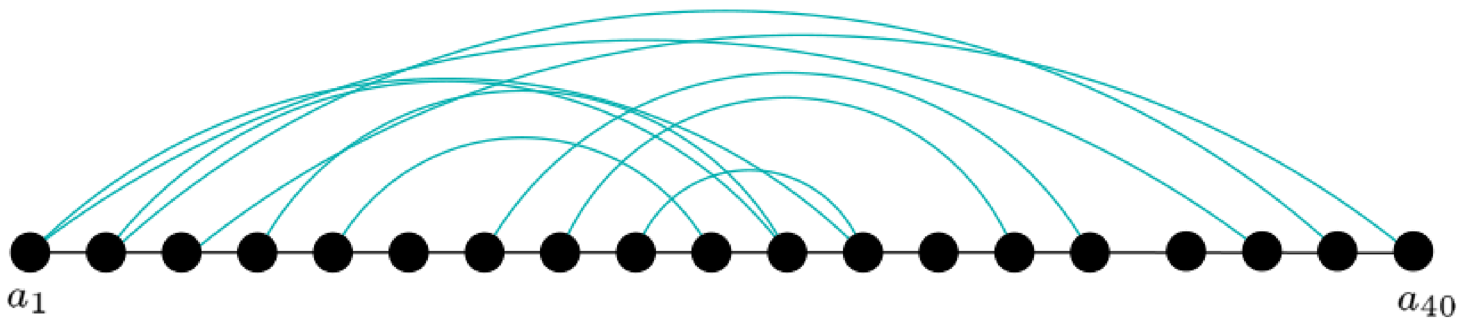

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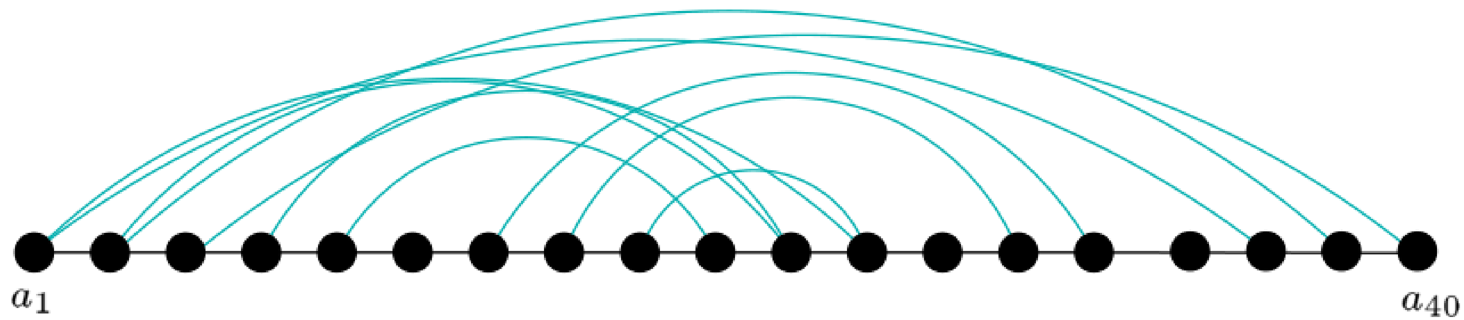

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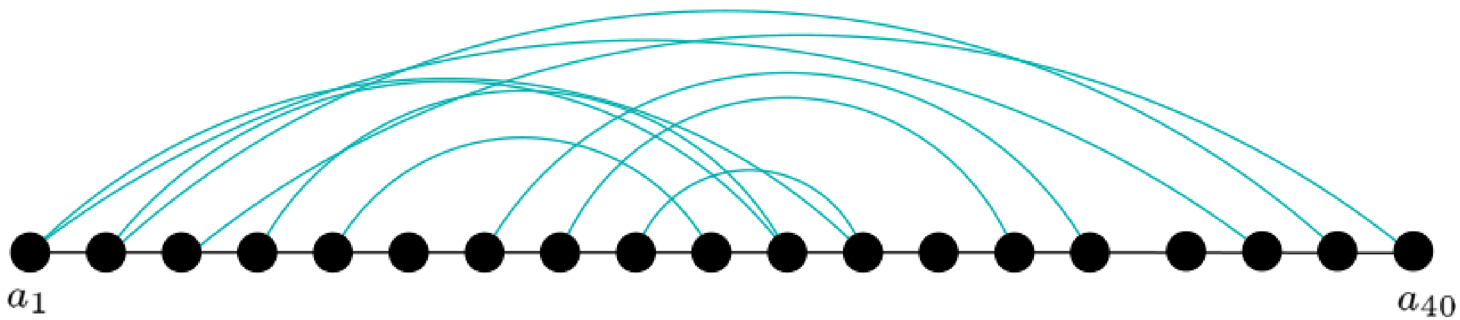

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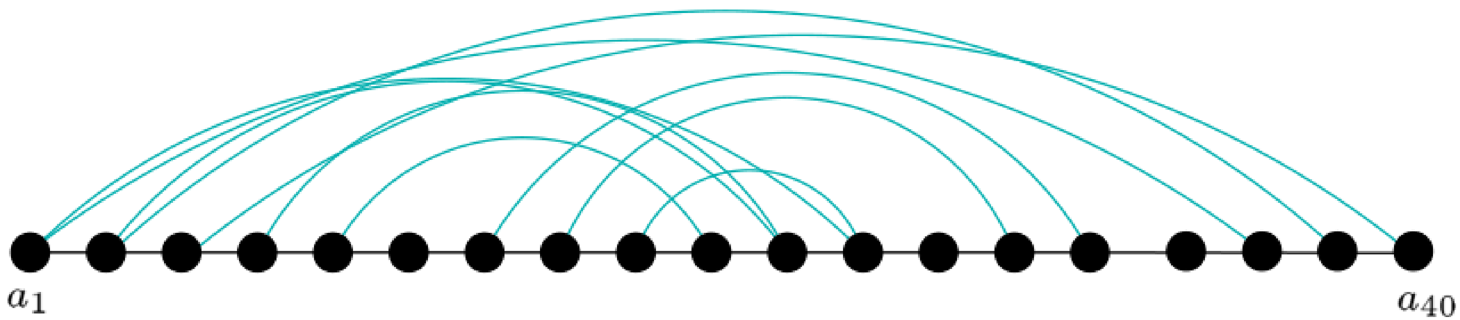

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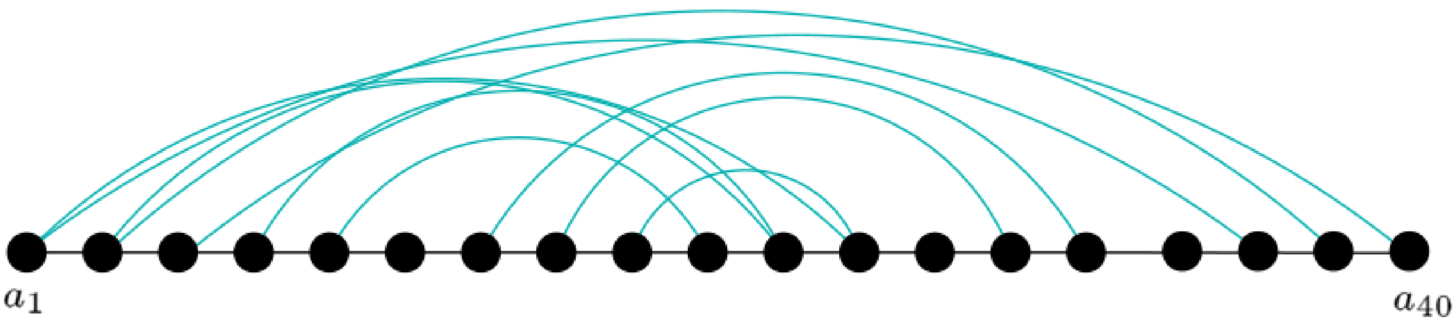

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in practice, small number of stacks and queues

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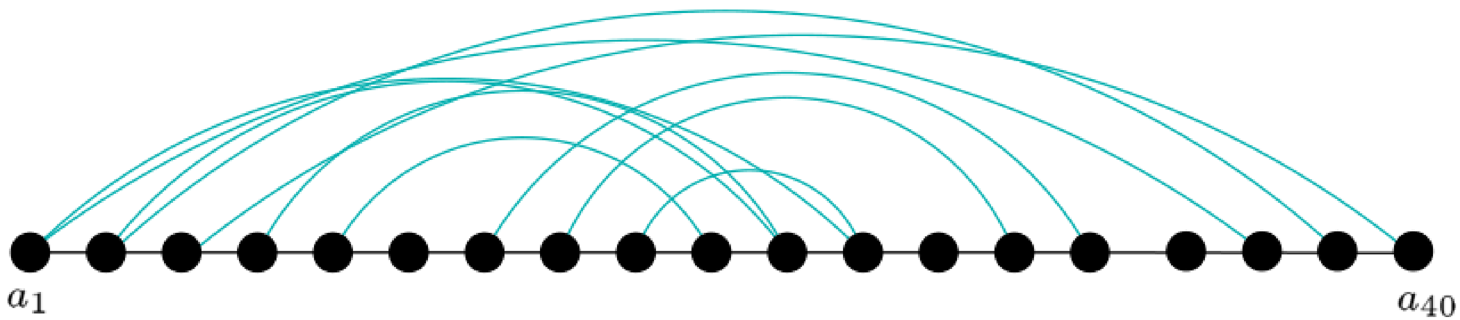

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in practice, small number of stacks and queues

Open Problem 3

Thank you