Speaker

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Workshop

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Title

Brick wall excursions

Abstract

Let us consider an m-steps random walk in $\mathbb{R}^d$ that starts at the origin and consists of m independent steps of length 1, where the direction of each step is chosen uniformly at random. Take the distance to the origin (after m such steps) and compute a sequence of its even moments. As it was shown in 2015 by Borwein, Straub and Vignat, in dimensions $d = 2$ and $d = 4$ this sequence is an integer. While for $d = 2$, the nth moment is equal to the number of abelian squares of length $2n$ over an alphabet with m letters, for $d = 4$ no interpretion was known.

The aim of this talk is to provide such an interpretation, both for $d = 2$ and $d = 4$, in terms of n-step lattice paths in dimension $m - 1$. The key step is a bijection between Dyck paths with a prescribed number of peaks and words of a certain type. In addition, this bijection allows us to derive closed formulas for the number of lattice paths provided with certain statistics.

This talk is based on the ongoing work with Sergey Kirgizov and Michael Wallner.