Realisability on Dynamical Systems and Complexity

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A few definitions

Abstract Programs

Definition

An *abstract model of computation* (AMC) is defined as a monoid action $\alpha : M \curvearrowright \mathbf{X}$ of a monoid $M = \mathrm{M}\langle G, \mathrm{R}\rangle$ on a space $\mathbf{X}$. I.e. a morphism $\alpha : M \to \mathrm{End}(\mathbf{X})$.

Definition (Reductions)

If $\alpha : M \curvearrowright \mathbf{X}$ and $\beta : N \curvearrowright \mathbf{Y}$ are AMCs, $\alpha \leqslant \beta$ iff there exists an automorphism $\phi : \mathbf{X} \to \mathbf{Y}$ s.t. for all $m \in M_\alpha$, $\phi(\alpha(m))$ is a glueing of a finite number of restrictions of elements $\beta(n_1), \beta(n_2), \ldots, \beta(n_k)$.

Remark

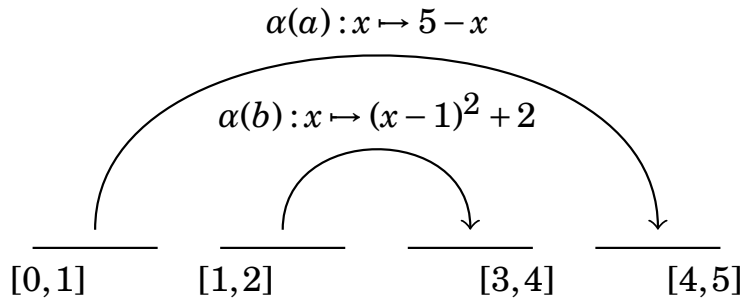
This induces an equivalence relation on AMCs (which is finer than Orbit Equivalence (OE) in the case of groups of m.p.m.).

Definition

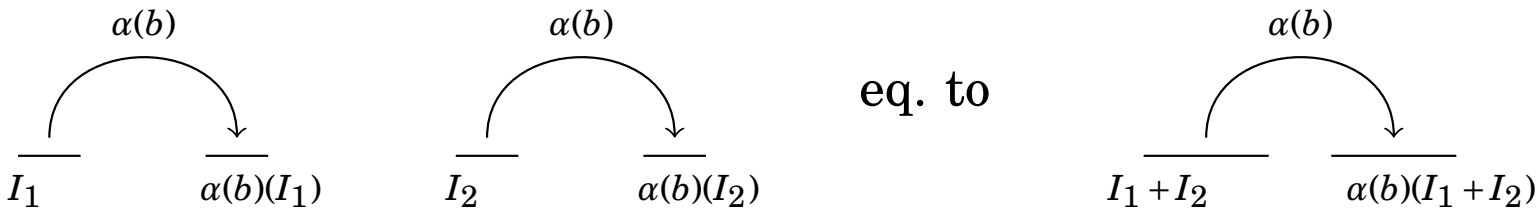
An *abstract program* is a α-graphing.

Wait. What's a graphing?

- Pick a directed graph.
- Replace vertices by (–, resp. open, resp. measurable) subsets of a fixed (discrete, resp. topological, resp. measured) space.
- Decide *how* (i.e. pick an element of M) the edges map sources to targets.



- Then quotient the set of such objects w.r.t. *refinement*:

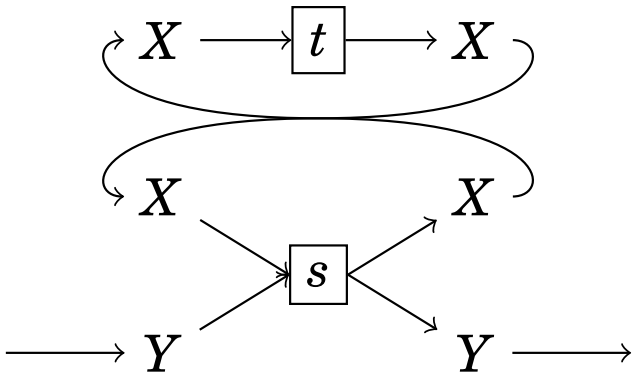


Graphings vs Lambda-Calculus

Lambda	IG	Discrete	Deterministic	Probabilistic
Grammar	AMC	(AMC)	(AMC)	(AMC)
Term	Graphing	Graph	Dyn. System	Markov P.*
Execution	"Paths"	Paths	Max. Finite Orbits	– –
Orthogonality	"Cycles"	Cycles	Infinite Orbits	– –

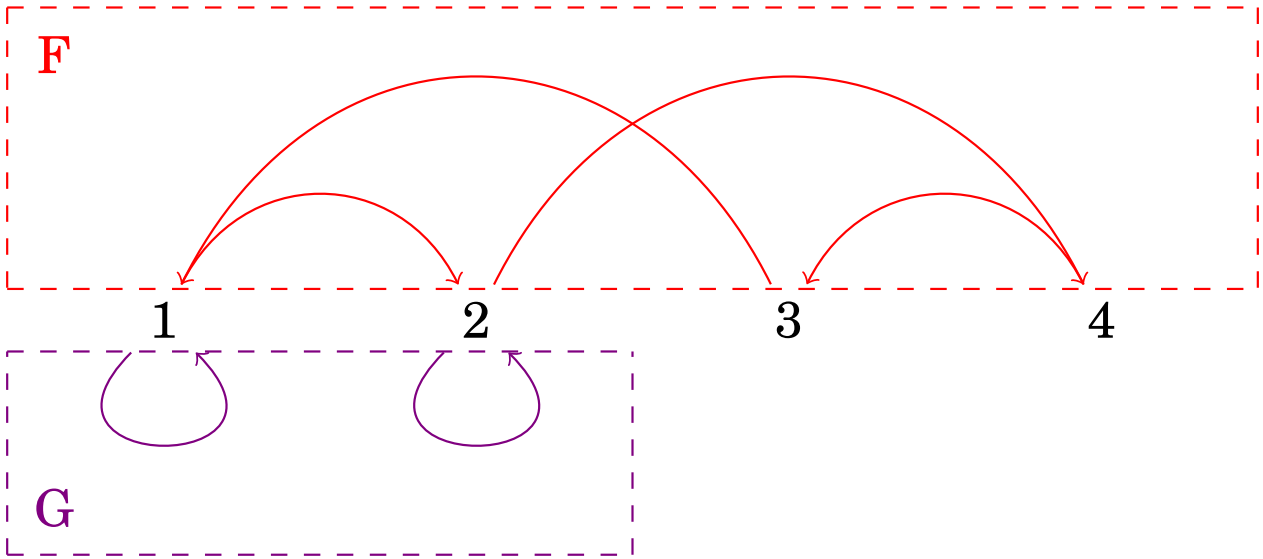
Execution

Execution is defined as follows. Given $s : X + Y \to X + Y$ and $t : X \to X$, we define $s :: t : Y \to Y$ as the fixpoint of:



Execution as Paths

The execution $F :: G$ of two graphs F, G is the graph of alternating paths of source and target in $V^F \Delta V^G$.



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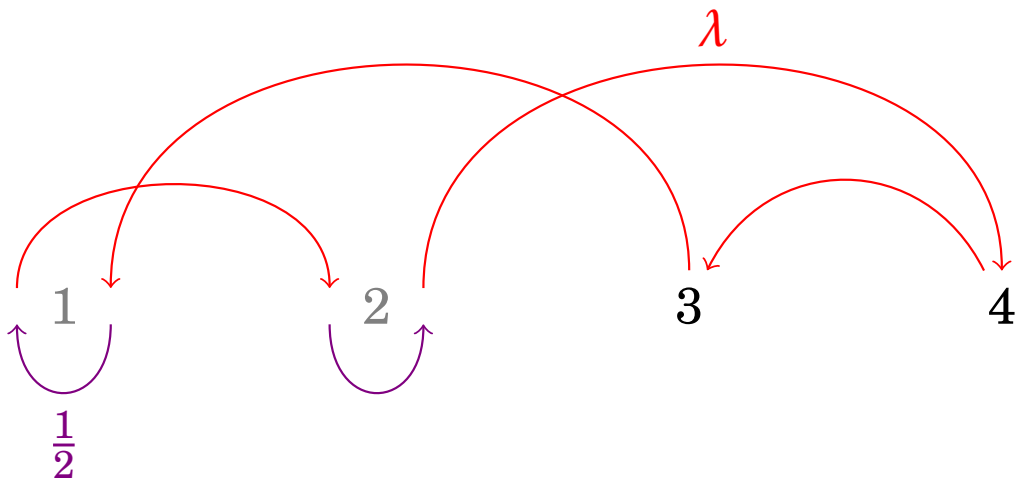

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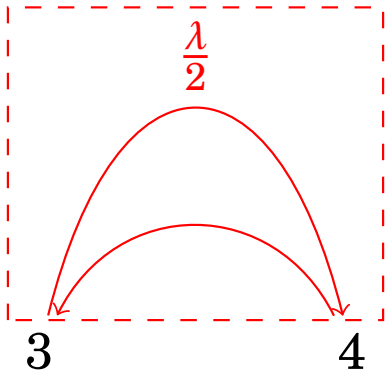
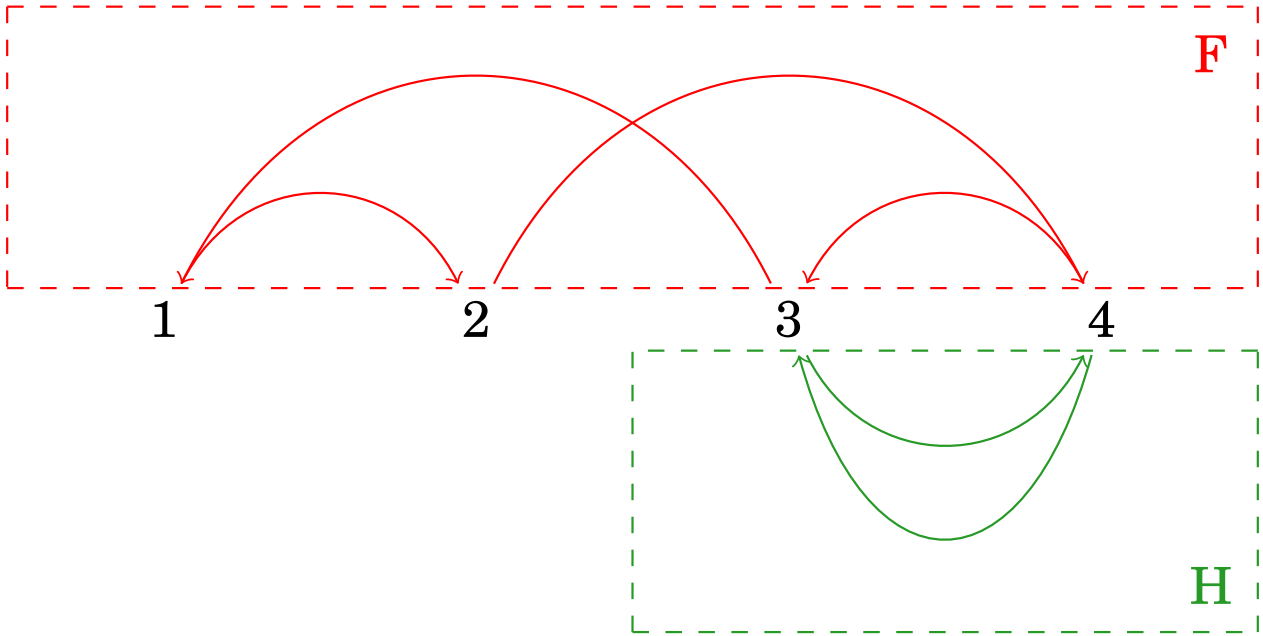


Cycles

In some cases, cycles appear during this operation.



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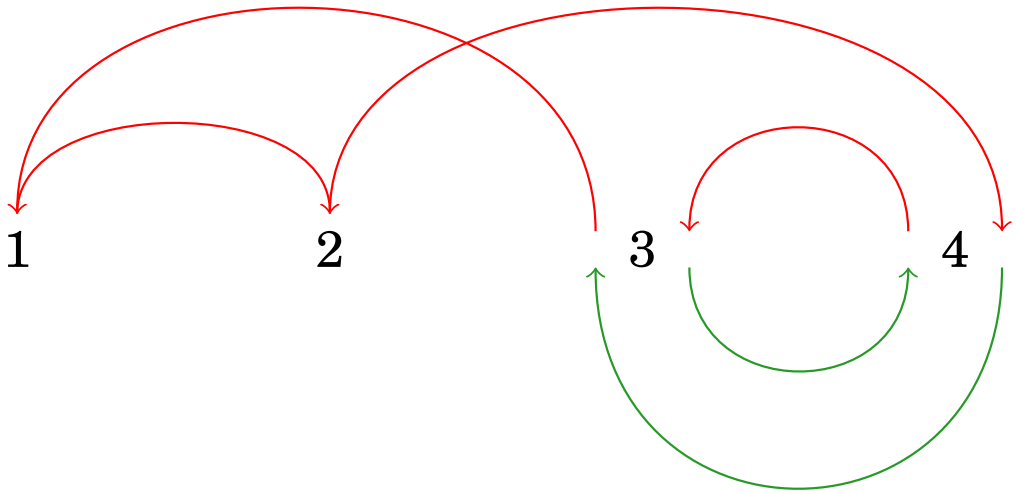

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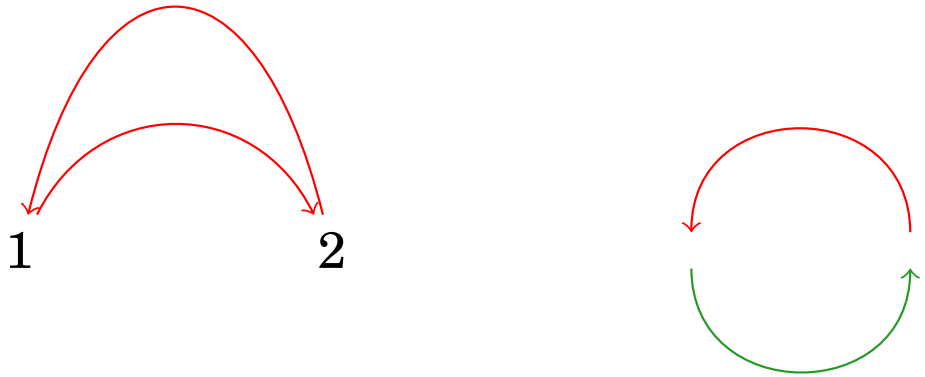
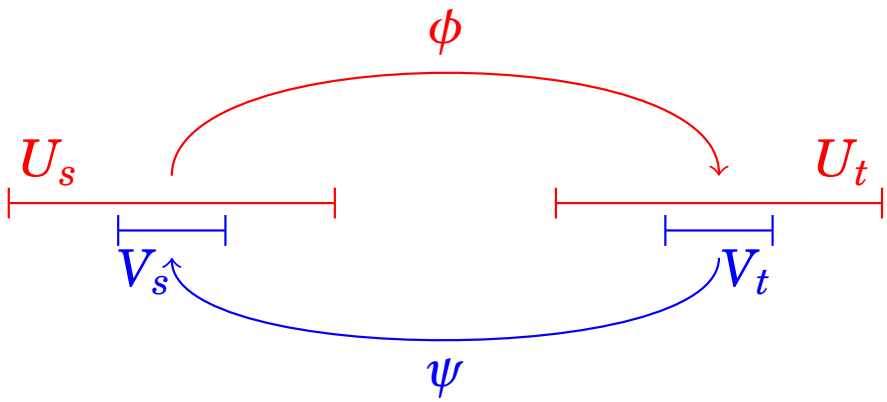
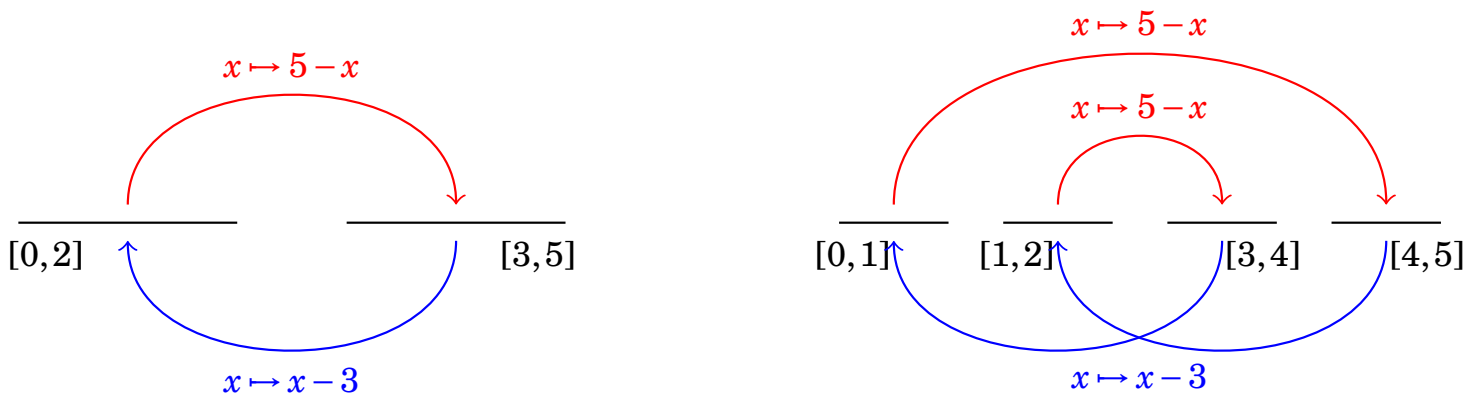


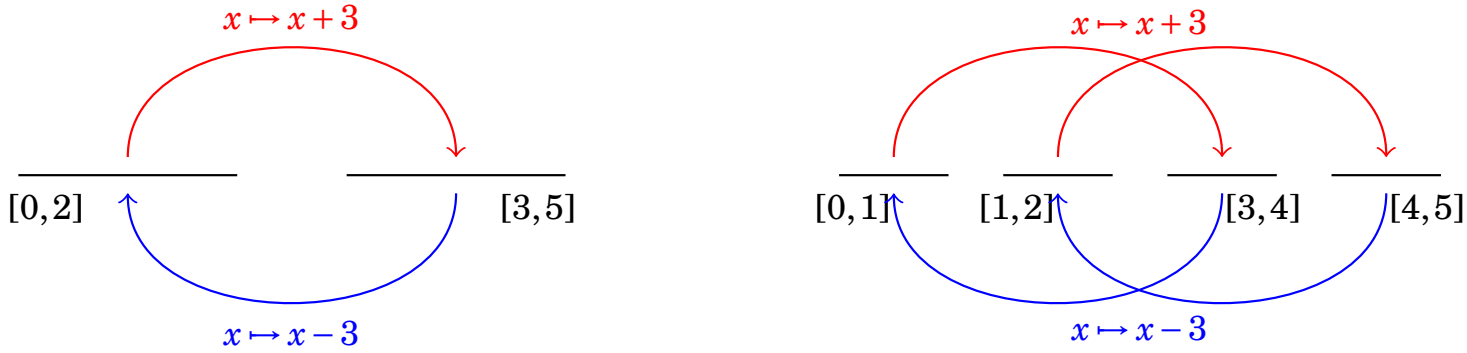
Execution in Graphings



Cycles in Graphings



(a) A cycle can become a longer cycle



(b) A cycle can become two (or more) cycles

Figure: Evolution of cycles through refinement

Orthogonality and Zeta

- Orthogonality in IG: defined by *measuring* cycles.
  - In the case of graphs:
$$[\![F,G]\!]_m = \sum_{\pi \in \mathscr{C}(F,G)} m(\pi)$$
  - In the case of dynamical systems:
$$[\![s,t]\!]_m = \sum_i m(\mathrm{Fix}((st)^i))$$
- This is related to Zeta functions of graphs (Ihara) and dynamical systems (Ruelle, Artin-Mazur):
$$\zeta_G(z) = \prod_{\pi \in \mathscr{C}(G)} (1 - z^{\omega(\pi)})^{-1}$$
$$\zeta_{f,\Phi}(z) = \exp\left(\sum_{m \geqslant 1} \frac{z^m}{m} \sum_{x \in \mathrm{Fix}(f^m)} \mathrm{tr}\left(\prod_{i=0}^{m-1} \phi(f^i(x)) \right) \right)$$

Realisability

- We want: $[\![s, t+t']\!]_m = [\![s::t, t']\!]_m$, but in fact $[\![s, t+t']\!]_m = [\![s::t, t']\!]_m + [\![s, t]\!]_m$. This can be corrected by keeping track of sets of cycles, i.e. t becomes (c, t) with c some term representing a set of cycles (can be the set of cycles itself or its measurement). We keep writing t in the following.
- From $[\![s, t+t']\!]_m = [\![s::t, t']\!]_m$, one can define orthogonality through a pole $\perp \subset \mathrm{Im}([\![\cdot, \cdot]\!]_m)$.
- Define types as sets $\mathbf{A} = \mathbf{A}^{\perp\perp}$, (eq. $\exists B$ s.t. $\mathbf{A} = B^{\perp}$);
- Define $\mathbf{A} \otimes \mathbf{B} = \{s + s' \mid s \in \mathbf{A}, s' \in \mathbf{B}\}^{\perp\perp}$;
- Define $\mathbf{A} \multimap \mathbf{B} = \{t \mid \forall s \in \mathbf{A}, t::s \in \mathbf{B}\}$.
- Check that $\mathbf{A} \multimap \mathbf{B} = (\mathbf{A} \otimes \mathbf{B}^{\perp]})^{\perp}$.

Hierarchies of models

Complexity Constraints

Quantitative Aspects
(e.g. probabilities, effects)

Theorem (Seiller, APAL 2017)

For every monoid of measurable maps $\mathfrak{m}$ *(and every monoid* Ω *, and every measurable map* $g : \Omega \to \mathbf{R}_{\geqslant 0} \cup \{\infty\}$ *), the set of α-graphings defines a non-degenerate model of Multiplicative-Additive Linear Logic .*

Geometric Measurement
(Ihara/Ruelle Zeta Functions)

AT LEAST!

Constraints on Graphings
(e.g. deterministic: (partial) measured dynamical systems,
probabilistic: (discrete time) Markov processes)

Hierarchies of models

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All Geometry of Interaction constructions are recovered as specific cases

Operators in C* / von Neumann algebras (1989,1990,2011)

Unification/Resolution clauses / Prefix Rewriting (1995,2016)

Complexity

Previous Results

AMC	det. model	non-det. model		prob. model
α_1	REGULAR	REGULAR	REGULAR	STOCHASTIC
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
α_k	D_k	N_k	co-N_k	P_k
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
α_∞	LOGSPACE	NLOGSPACE	coNLOGSPACE	PLOGSPACE
β	PTIME	PTIME	PTIME	PTIME?
γ	PTIME	NPTIME	coNPTIME	PP?

Refines and generalises both:

- a series of characterisations of complexity classes (e.g. LOGSPACE, PTIME) by sets of operators (with Aubert), logic programs (with Aubert, Bagnol and Pistone);
- the result where relating the expressivity of GoI models with a classification of *inclusions of maximal abelian sub-algebras*:

$$\ell^\infty(\mathbf{X}) \subseteq \ell^\infty(\mathbf{X}) \rtimes \mathfrak{m}$$

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- Only known correspondence between infinite hierarchies of mathematical objects and complexity classes.
- Indicates a strong connection between *geometry* and complexity: cf. AMC generalise *group actions*, use of (generalised) Zeta functions, (homotopy) equivalence between microcosms implies equality of the classes.

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Conjecture

If $\alpha : M \curvearrowright \mathbf{X}$ and $\beta : N \curvearrowright \mathbf{Y}$ are *separable* (by e.g. ℓ^2-Betti numbers), they characterise different complexity classes.

- Can be checked on classical separation results and the computation of the invariants on the AMCs $\alpha_1, \ldots, \alpha_k, \ldots, \alpha_\infty$.

Dynamic complexity

Definition

An *abstract model of computation* (AMC) is defined as a monoid action $\alpha : M \curvearrowright \mathbf{X}$ of a monoid M on a space $\mathbf{X}$. I.e. $\alpha : M \to \mathscr{M}(\mathbf{X} \to \mathbf{X})$.

Definition

An *abstract program* is a $\alpha(M)$-graphing.

Definition

Define(!) the *dynamic complexity* of a program as the *smallest* (equivalence class of) monoid action $\alpha : M \curvearrowright \mathbf{X}$ needed to interpret it as an abstract program.

Conjecture

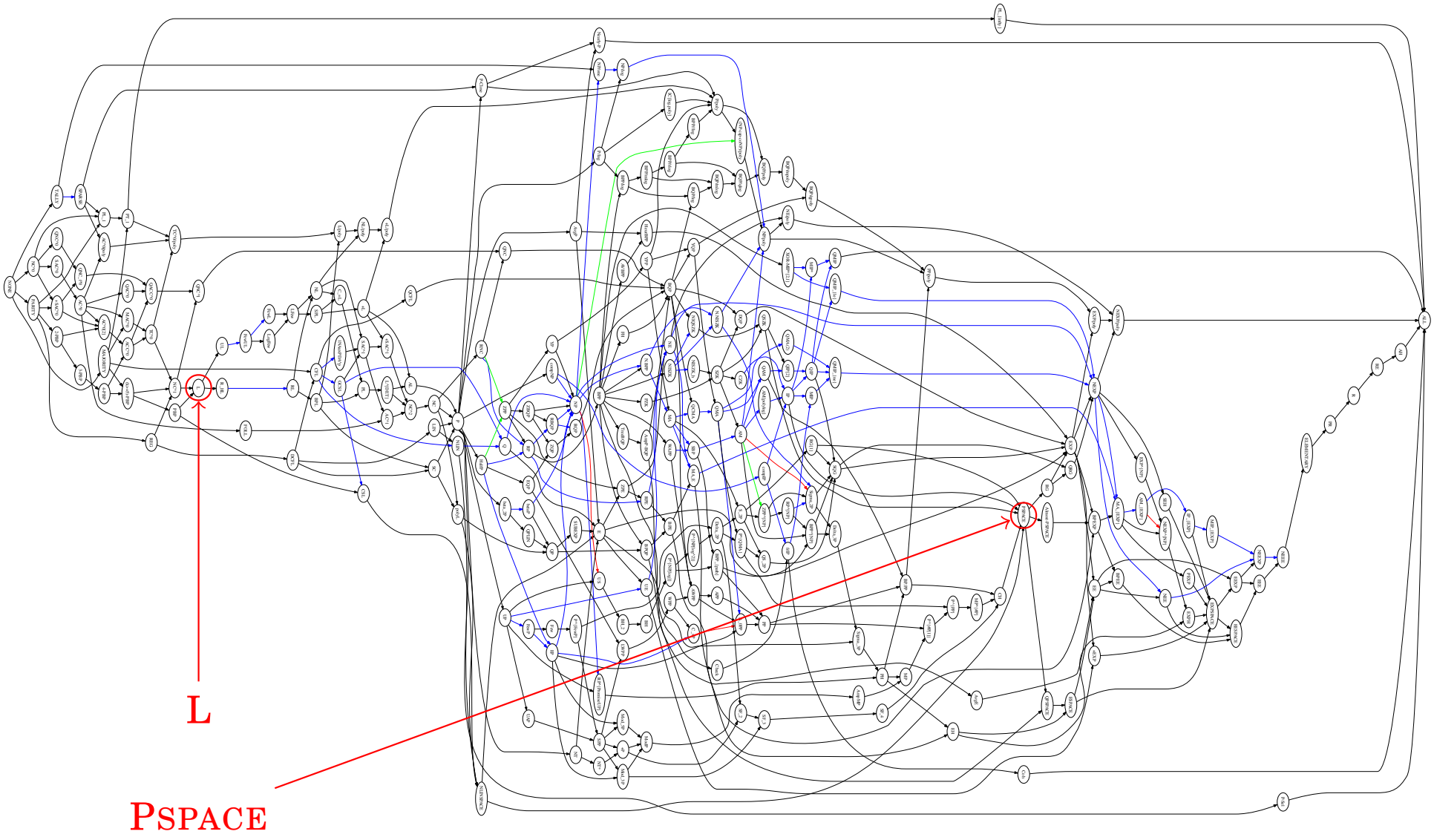
Dynamic complexity of programs coincide* with computational complexity.

Lower Bounds and Barriers

Complexity Theory, Today

- Separation results were obtained, most of them in the 70s, but a lot of questions remain open. E.g. we know $\textsc{Logspace} \subsetneq \textsc{Pspace}$, but not which of these are strict: $\textsc{Logspace} \subset \textsc{NLogspace} \subset \textsc{NC} \subset \textsc{P} \subset \textsc{NP} \subset \textsc{Pspace}$.

Complexity Theory, Today (well, in 2006)



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- In fact, the three more important results are *negative results* (called *barriers*) showing that known proof methods for separation of complexity classes are inefficient w.r.t. currently open problems. They are: relativisation (1975), natural proofs (1995), and algebrization (2008).
- **Thus: no proof methods for (new) separation results exist today.**
- (Proviso) A single research program is considered as viable for obtaining new results: Mulmuley's *Geometric Complexity Theory* (GCT). According to Mulmuley, **if** GCT produces results, it will not be during our lifetimes (and maybe not our children's lifetime either*). Recent ...

*Well, this depends on the children ages and how long they live, but it is quite unlikely.

Requiem for a Dream

State of the Art in Complexity (Separation Problem): Barriers.

- Relativization/Algebrization (SIAM J. Comp. 1975 / STOC 2008): Proof methods that are oblivious to the use/disuse of oracles are ineffective.

- Natural Proofs (J. Comp. Sys. Sci. 1997): Proof methods expressible as (**Large**, Constructible) predicates on boolean functions are ineffective.

Conclusion: Lack of proof methods for separation.

Geometric Complexity Theory

- GCT is a research program whose aim is to prove PTIME $\neq$ NPTIME using techniques from algebraic geometry (Mulmuley and Sohoni).
- Basically, the goal is to prove that the permanent cannot be embedded in the determinant[†]
- Mulmuley does not expect results within the next 100 years. Recently several drawbacks, in particular closing the easiest path to GCT.

[†]More details: tomorrow's CALIN seminar and subsequent working groups (June 12th and 19th).

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- Somehow builds on *Algebraic Complexity* (AC) which studies models of computation over arbitrary structures (e.g. the real numbers). Advantages of AC: lower bounds are easier, some successes in proving some bounds on specific computations (e.g. matrix multiplication).

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- Initiated after a proof of lower bound for a restricted algebraic PRAM model, which we note PRAM$^-$. This model defines a class NC$^-$ lying within NC (still quite large) and shows it is strictly contained within PTIME.
  This is sometimes considered as the strongest lower bounds result obtained so far.

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Barriers as Guidelines

State of the Art in Complexity (Separation Problem): Barriers.

- Relativization/Algebrization: Proof methods that are oblivious to the use/disuse of oracles are ineffective.

- Natural Proofs: Proof methods expressible as (**Constructible**, Large) predicates on boolean functions are ineffective.

Barriers as Guidelines

State of the Art in Complexity (Separation Problem): Barriers.

- Relativization/Algebrization: Proof methods that are oblivious to the use/disuse of oracles are ineffective.
  - Separation proof methods should depend on the computational principles allowed in the model.
- Natural Proofs: Proof methods expressible as (**Constructible**, Large) predicates on boolean functions are ineffective.
  - Separation proof methods should not "quotient" the set of programs too much. (by definition, complexity classes are non-decidable predicates on boolean functions)

Can we jump the barriers?

Why barriers do not apply to this approach:

- (Relativisation/Algebrization)

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  - Impact the invariants: if $\alpha : M \curvearrowright \mathbf{X}$ and $\beta : N \curvearrowright \mathbf{Y}$ are separable, there are no reasons to believe that $\alpha + o : M \curvearrowright \mathbf{X}$ and $\beta + o : N \curvearrowright \mathbf{Y}$ are separable.

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- (Natural Proofs)

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- (Natural Proofs)
  - The Natural Proofs barrier applies to non-uniform classes.
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- (Natural Proofs)
  - The Natural Proofs barrier applies to non-uniform classes.
  - As explained above, the approach should violate the constructivity axiom of the Natural Proof barrier.
  - More importantly, we can argue that if barriers exists in this setting then the separation problem is undecidable.

Entropy and Lower Bounds

Deterministic graphings

In this work, we only consider *deterministic topological graphings*.

Definition

A graphing on a space $\mathbf{X}$ is deterministic if for all $x \in \mathbf{X}$, x belongs to at most one source. (In measurable case, replace "for all" by "for almost all".)

Proposition

The set of all (–, resp. topological, resp. measurable) graphings over a space $\mathbf{X}$ *is equal to the set of all partial (discrete, resp. topological, resp. measurable) dynamical systems over* $\mathbf{X}$.

Goal

Convince you that invariants of the dynamical systems are relevant tools for the study of computational complexity.

Entropy and Cells

Definition

Let $\mathbf{X}$ be a topological space and $f : \mathbf{X} \to \mathbf{X}$ be a continuous partial map. For any finite open cover $\mathscr{U}$ of $\mathbf{X}$, we define:

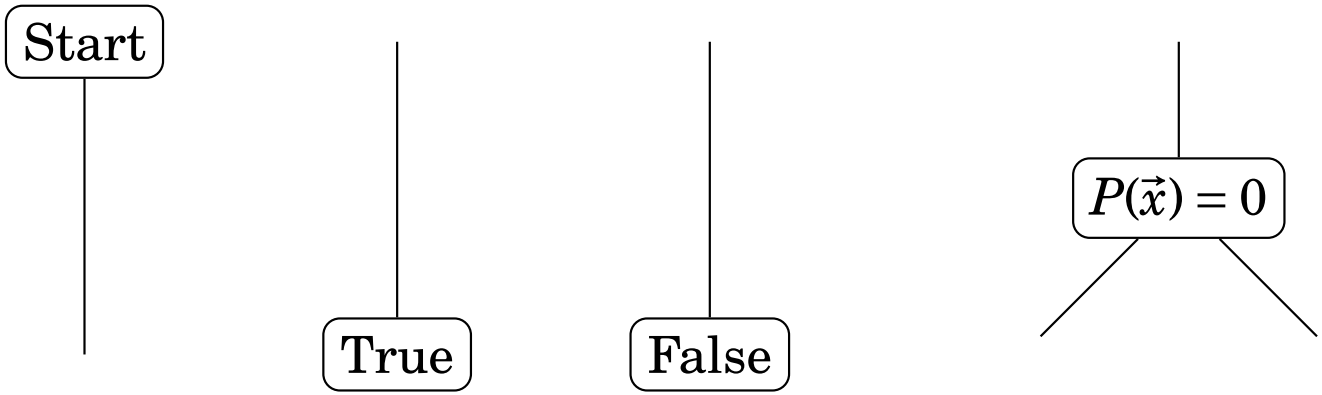
$$H^k_{\mathbf{X}}(f, \mathscr{U}) = \frac{1}{k} H^0_{f^{-k+1}(\mathbf{X})}(\mathscr{U} \vee f^{-1}(\mathscr{U}) \vee \cdots \vee f^{-(k-1)}(\mathscr{U})).$$

The *entropy* of f is then defined as $h(f) = \sup_{\mathscr{U} \in \mathrm{FCov}(\mathbf{X})} h(f, \mathscr{U})$, where $h(f, \mathscr{U})$ is again defined as the limit $\lim_{n \to \infty} H^n_{\mathbf{X}}(f, \mathscr{U})$.

Proposition

Let G be a deterministic graphing, with entropy $h(G)$. The cardinality of the k-th cell decomposition of $\mathbf{X}$ w.r.t. G, as a function $c(k)$ of k, is asymptotically bounded by $g(k) = 2^k 2^{h([G])}$, i.e. $c(k) = O(g(k))$.

Lower Bounds results I: Algebraic Decision Trees



Lemma

Let G be a regular deterministic graphing interpreting an algebraic decision tree of max degree d. The k-th cell decomposition of $\mathbf{X}$ w.r.t. G is determined by at most 2^k algebraic varieties of degrees bounded by d.

The Milnor-Thom theorem

Theorem (Milnor, Thom)

Let V be an algebraic variety $\subset \mathbf{R}^m$, defined by a polynomial of degree at most d. The sum of the Betti numbers of V is not greater than $d(2d-1)^{m-1}$.

Corollary

Let (S) be a system of s polynomial equations and inequalities in k variables, of degrees at most $d \geqslant 2$. The number of connected components of the set of solutions of (S) in $\mathbf{R}^n$ is not greater than $d(2d-1)^{k+s-1}$.

Remark

There are better bounds, e.g. Roy ($s^k O(d)^k$).

Lower Bounds results I: Algebraic Decision Trees

Lemma

Let G be a regular deterministic graphing interpreting an algebraic decision tree of max degree d. The k-th cell decomposition of $\mathbf{X}$ w.r.t. G is determined by at most 2^k algebraic varieties of degrees bounded by d.

- This gives lower bounds for deciding (semi-)algebraic sets. E.g.

Example 3. Set Disjointness. *Given two sets $A = \{x_1, \ldots, x_n\}$ and $B = \{y_1, \ldots, y_n\}$, determine whether or not $A \cap B = \emptyset$.*

For this problem set

$$W = \left\{ (x_1, \ldots, x_n, y_1, \ldots, y_n) \,\middle|\, \prod_{i,j} (x_i - y_j) \neq 0 \right\}$$

It is easy to see that $\#W \geq (n!)^2$, so again we know that $C(W), M(W), C_d(W) = \Omega(n \log n)$.

A second separation result

Lower Bounds results II: Algebraic circuits

Proposition

Let $f_n \in \mathbf{R}[X_1,\ldots,X_n], n \in \mathbf{Z}$, be a family of nonconstant irreducible polynomials such that for each n, the zero set $\mathcal{Z}(f_n)$ is a variety of dimension $n-1$. Let $d(n) = \deg(f_n)$. Then any parallel machine deciding the set $S = \{x \in \mathbf{R}^\infty \mid f_{\mathrm{size}(x)}(x) = 0\}$ has running time greater than $\log d(n)$.

Proposition

The same problem with the family of polynomials $X_1 - X_2^{2^n}$ is computable in polynomial (even linear) time in the real BSS model.

Theorem

$$\mathrm{NC}_{\mathbf{R}} \subsetneq \mathrm{Ptime}_{\mathbf{R}}$$

PRAMs w/o bit operations

Definition

A SRAM *command* is a pair (ℓ, I) of a *label* (or *line*) $\ell \in \mathbf{N}^\star$ and a *command* $\mathtt{I}$ among the following ($i,j \in \mathbf{N}$, $\ell, \ell' \in \mathbf{N}^\star$ labels):

$$\begin{array}{rcl} \text{Commands} & := & \mathtt{skip}; \quad \mathtt{X_i} := \mathtt{X_i} + 1; \quad \mathtt{X_i} := \mathtt{X_i} - 1; \\ & & \mathtt{X_i} := \mathtt{X_j}; \quad \mathtt{X_i} := \sharp\mathtt{X_j}; \quad \sharp\mathtt{X_i} := \mathtt{X_j}; \\ & & \mathtt{if}\ \mathtt{X_i} = 0\ \mathtt{goto}\ \ell\ \mathtt{else}\ \ell'; \end{array}$$

Definition

A PRAM machine M is simply given as a finite sequence of SRAM machines $M_1, \ldots, M_p$, where p is the number of *processors* of M. Each processor M_i has access to its own, private, set of registers $(\mathtt{X}^i_k)_{k \geqslant 0}$ and a *shared memory* represented as a set of registers $(\mathtt{X}^0_k)_{k \geqslant 0}$.

PRAMs w/o bit operations

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Lemma

Let G be a regular deterministic graphing interpreting a PRAM *with p processors. The k-th cell decomposition of $\mathbf{X}$ w.r.t. G is determined by at most $2^k p^k$ algebraic varieties of degree at most 2^k.*

Lower Bounds results III

The following result provides a geometric representation of a PTIME-complete problem on $\mathbf{R}^3$. (Need whiteboard here.)

Theorem (Murty, Carstensen)

1. *there exists an affine parametrization of bitsize $O(n)$ and complexity $2^{\Omega(n)}$ of combinatorial linear programming, where n is the total number of variables and constraints of the problem.*
2. *there exists an affine parametrization of bitsize $O(n^2)$ and complexity $2^{\Omega(n)}$ of the maxflow problem for directed and undirected networks, where n is the number of nodes in the network.*

Lower Bounds results III-2

Definition

Let K be a compact of $\mathbf{R}^3$.
A finite set of surfaces S on K *separates* a ρ-fan **Fan** on K if the partition on $\mathbf{Z}^3 \cap K$ induced by S is finer than the one induced by **Fan**.

Theorem (Mulmuley)

Let S be a finite set of algebraic surfaces of total degree δ.
There exists a polynomial P such that, for all $\rho > P(\delta)$, S does not separate ρ-fans.

Corollary

Let G be a deterministic graphing interpreting a PRAM *without bit operations with $2^{O(N^c)}$ processors (N is the length of the inputs and c any positive integer).*
G does not decide maxflow in $O(N^c)$ steps.

Theorem

$$\mathrm{NC}^{\mathrm{PRAM}^-} \subsetneq \mathrm{Ptime}$$

A Remark

- Algebraic Computational Trees.
  - Results of Dobkin and Lipton, Yao, Steele and Yao, Ben-Or –> 83.
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  - Result of Cucker –> 92
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