Internship proposal at Paris 13 University

Thomas Seiller

Title	Formalising Transcendental Syntax
Topics	Semantics of Programs, Logic
Location	Villetaneuse, France
Lab	Laboratoire d'Informatique de Paris Nord (LIPN) Paris 13 University
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Context. The study of denotational semantics was initiated by Scott [6], as he gave the first mathematical model of (recursive) programs where data types are interpreted as topological spaces and programs as continuous functions.

The geometry of interaction program was proposed by Girard [3] shortly after the inception of linear logic. In opposition to traditional denotational semantics – e.g. domains –, the GoI program aims at giving an account of the proofs and programs which also interprets their dynamical features, i.e. cut-elimination/execution. Among the most recent and full-fledged embodiement of this program lie Seiller's Interaction Graphs models [7, 8, 9, 10].

Aim of the internship. In a recent series of papers and preprints [5, 1, 2], Girard proposes a new framework fulfilling the geometry of interaction program. This framework, called *transcendental syntax* refines former work by proposing a better treatment of some logical aspects of the models. In particular, the last preprint [2] explains how to extend the approach to first order quantification. This novelty and the associated treatment of equality, is of particular interest, but Girard only sketches the definitions, even the basic ones, and no proofs are provided.

The core objective of this internship is to formalise and prove correct the constructions for the multiplicative fragment of linear logic, as a first step towards an full understanding of Girard's account of predicate calculus. As part of the internship, the student will explore the relationship between Transcendantal Syntax and Interaction Graphs models. If time permits, and depending on the interests of the intern, it will be possible to extend the study to one of the following (ordered by increasing workload): second order quantification, *linear dependent types* [4], or predicate calculus.

References

[1] Jean-Yves Girard. Transcendental syntax ii: non deterministic case. Logical Methods in Computer Science (to appear).

[2] Jean-Yves Girard. Transcendental syntax iii: equality.

[3] Jean-Yves Girard. Towards a geometry of interaction. In *Proceedings of the AMS Conference on Categories, Logic and Computer Science*, 1989.

[4] Jean-Yves Girard. Geometry of interaction V: Logic in the hyperfinite factor. *Theoretical Computer Science*, 412:1860–1883, 2011.

[5] Jean-Yves Girard. Transcendental syntax i: deterministic case. *Mathematical Structures in Computer Science*, 27(5):827–849, 2017.

[6] Dana Scott. Outline of a mathematical theory of computation. Technical Report PRG02, OUCL, November 1970.

[7] Thomas Seiller. Interaction graphs: Multiplicatives. *Annals of Pure and Applied Logic*, 163:1808–1837, December 2012.

[8] Thomas Seiller. Interaction graphs: Additives. *Annals of Pure and Applied Logic*, 167:95 – 154, 2016.

[9] Thomas Seiller. Interaction graphs: Full linear logic. In *IEEE/ACM Logic in Computer Science (LICS)*, 2016.

[10] Thomas Seiller. Interaction graphs: Graphings. *Annals of Pure and Applied Logic*, 168(2):278–320, 2017.